Characterizing Online Cost-Sharing Mechanisms for Demand Responsive Transport Systems

(Extended Abstract)

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Demand-responsive transport systems use shuttles to provide door-to-door shared-rides for its passengers. The operating cost of the shuttles depends on the pick-up and drop-off locations of the passengers. It is challenging to design a fair cost-sharing mechanism that offers fare quotes to potential passengers without having any knowledge of future ride requests since different passengers cause different amounts of inconvenience to other passengers. In this work, we study desirable properties of such cost-sharing mechanisms and present Proportional Online Cost Sharing.

1. INTRODUCTION

Demand-responsive transport (DRT) systems provide flexible transport services where passengers request door-to-door rides and shuttles service them in shared-ride mode without fixed routes and schedules. Sharing the operating costs of a DRT system among its passengers is a complex problem, because different passengers cause different amounts of inconvenience to other passengers. Since passengers are self-interested, it is natural to try the mechanism design approach in order to develop cost-sharing mechanisms for providing fare quotes and fares to (potential) passengers. One desirable property from such a mechanism is that all passengers feel treated fairly. It is also preferred to minimize passengers' uncertainty about whether they can be serviced or how high their fares will be, so the mechanism should provide fare quotes immediately after passengers' submit their ride requests, which forces the mechanism to make instantaneous and irreversible decisions despite having no knowledge of future ride requests. The mechanism should also provide passengers with incentives to submit their ride requests early to enable DRT systems to offer subsequent passengers lower fares due to synergies with earlier ride requests, and hence might allow them to service more passengers. In this work, we present Proportional Online Cost Sharing (POCS) [1] mechanism for DRT systems and provide a set of desired properties it satisfies.

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2. RELATED WORK

We know of no research on cost sharing for DRT systems other than [1], even though the cost-sharing literature in general is substantial. Cost sharing for DRT systems has to cope with an on-line setting (due to sequentially arriving passengers) and non-decreasing marginal costs, different from the assumptions typically made in the cost-sharing literature [2]. The operating costs of DRT systems also depend on the ride requests, different from the assumptions made in the context of recent online versions of cake-cutting and resource-allocation [3] which do not take positive and negative synergies between ride requests into account.

3. DRT PROBLEM

In a DRT problem passengers submit their ride requests sequentially by specifying their desired pick-up and drop-off locations. The arrival time of a passenger is the time when it submits its ride request. We assume, for simplicity, that all passengers arrive before the shuttles start to service passengers and that exactly one passenger arrives at each time $k = 1, \ldots, t$, namely that passenger $\pi(k)$ arrives at time k under arrival order π, where an arrival order is a function that maps arrival times to passengers. Each passenger $\pi(k)$ is associated with *alpha value* $\alpha_{\pi(k)}$ that quantifies the demand of the requested ride, that is, how much of the transport resources it requests. We assume that the alpha values are positive and independent of the arrival times of the passengers. These assumptions are, for example, satisfied by the shortest point-to-point travel distance from the pick-up to the drop-off location of a passenger.

We denote by $totalcost^t_\pi$ the *operating cost* required to service passengers $\pi(1), \ldots, \pi(t)$. We define $totalcost^0_\pi := 0$ and assume that it is non-decreasing in time and independent of the arrival order of the passengers. These assumptions are, for example, satisfied by the minimal operating cost required to service all passengers. The increase in total cost due to passenger $\pi(k)$'s arrival (under arrival order π) is its *marginal cost* and denoted by $mc_{\pi(k)} := totalcost^k_\pi - totalcost^{k-1}_\pi$. The *shared cost* of passenger $\pi(k)$ at time t under arrival order π, $c^t_{\pi(k)}$, is its share of the total cost at time t determined by a cost-sharing mechanism.

The DRT system provides (myopic) fare quotes to passengers immediately after their arrivals. The fare quoted to passenger $\pi(k)$ after its arrival at time k is $c^k_{\pi(k)}$. (A fare

	$ccpa_{\pi(i,j)}$			$cost^t_{\pi(i)}$		
	$j=1$ $\pi(1)=P_1$	$j=2$ $\pi(2)=P_2$	$j=3$ $\pi(3)=P_3$	$t=1$	$t=2$	$t=3$
$i=1$ $\pi(1)=P_1$	10	15	$7\frac{1}{2}$	20	20	15
$i=2$ $\pi(2)=P_2$		20	$6\frac{2}{3}$		40	15
$i=3$ $\pi(3)=P_3$			0			30

Table 1: POCS Values

quote of infinity means that the passenger cannot be serviced.) We assume each passenger has a *fare limit* $w_{\pi(k)}$, which is independent of time, that represents the maximum amount that it is willing to pay for its requested ride. Passenger $\pi(k)$ drops out and is not serviced if its fare limit $w_{\pi(k)}$ is lower than its fare quote, that is, $w_{\pi(k)} < c^k_{\pi(k)}$. In this case, the DRT system simply pretends that the passenger never arrived, which explains why we assume, without loss of generality, that all passengers accept their fare quotes. Otherwise, the passenger accepts its fare quote.

4. POCS

POCS [1] is a cost-sharing mechanism for DRT systems that provides low fare quotes to passengers directly after they submit their ride requests and, if they accept their fare quotes, then calculates their actual fares directly before their rides. The separation of fare quotes and fares gives the flexibility to optimize the shuttle schedules after it has received additional ride requests. The fares of passengers are guaranteed to never exceed their fare quotes but can be even lower since passengers form "cost-sharing coalitions" with one or more passengers who submit their ride requests directly after them if that lowers their fares. Each cost-sharing coalition pays its marginal cost, which is divided proportionally among the passengers in the cost-sharing coalition. Formally, for all times k_1, k_2 and t and all arrival orders π with $k_1 \leq k_2 \leq t$, the *coalition cost per alpha value* of passengers $\pi(k_1), \ldots, \pi(k_2)$ at time t under arrival order π is

$$ccpa_{\pi(k_1,k_2)} := \frac{\sum_{j=k_1}^{k_2} mc_{\pi(j)}}{\sum_{j=k_1}^{k_2} \alpha_{\pi(j)}}.$$

and the *shared cost* of passenger $\pi(k)$ determined by POCS at time t under arrival order π is

$$cost^t_{\pi(k)} := \alpha_{\pi(k)} \min_{k \leq j \leq t} \max_{1 \leq i \leq j} ccpa_{\pi(i,j)}.$$

5. DESIRABLE PROPERTIES

POCS satisfies several desirable properties of cost-sharing mechanisms for DRT systems[1]. *Budget balance* means that the total cost is shared by all serviced passengers (that is, there are no profits or subsidies). *Individual rationality* implies that the fares of serviced passengers cannot exceed their fare limits (to prevent them from dropping out). *Immediate response* guarantees that the shared costs of serviced passengers are monotonically nonincreasing in time. This property prevents any serviced passenger (that is, any passenger that accepted its initial offer) to drop out at a later time because its shared cost increased beyond its fare limit. Finally, *Online fairness* implies that the shared costs per alpha value of passengers never exceed the shared costs per alpha value of all passengers that arrive after them.

A simple example is illustrated in Figure 1. There is one shuttle that can transport up to three passengers and starts

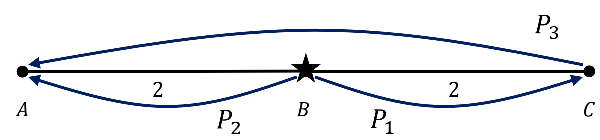

Figure 1: DRT Example

		$k=1$ $\pi(k)=P_1$	$k=2$ $\pi(k)=P_2$	$k=3$ $\pi(k)=P_3$
Alpha Value:	$\alpha_{\pi(k)}$	2	2	4
Fare Limit:	$w_{\pi(k)}$	60	60	60
Total Cost:	$totalcost^k_\pi$	20	60	60
Marginal Cost:	$mc_{\pi(k)}$	20	40	0

Table 2: DRT Values

at the star. The shuttle incurs an operating cost of 10 for each unit of distance traveled and does not need to return to its initial location. There are three passengers with arrival order $\pi(1) = P_1$, $\pi(2) = P_2$ and $\pi(3) = P_3$ and their ride requests are shown in Figure 1. The passengers' alpha values and fare limits provided in Table 2. It also shows the total costs after the arrival of each passenger and the marginal costs of all passengers under the assumption that all passengers are serviced. For example, $\alpha_{\pi(2)}$ is the point-to-point travel distance from its pick-up location B to its drop-off location A (B-A), and the total cost after the arrival of Passenger P_2 is 10 times the minimal travel distance of the shuttle required to service Passengers P_1 and P_2 from its initial location (B-C-B-A or B-A-B-C). Assume that Passenger P_1 arrives and is serviced. Budget balance requires $c^1_{\pi(1)} = totalcost^1_\pi = 20$. Now assume that Passenger P_2 arrives. If it is serviced, budget balance requires $c^2_{\pi(1)} + c^2_{\pi(2)} = totalcost^2_\pi = 60$. Cost-sharing mechanisms that satisfy budget balance, individual rationality, immediate response and online fairness can service Passenger P_2, for example, with $c^2_{\pi(1)} = 0$ and $c^2_{\pi(2)} = 60$ or with $c^2_{\pi(1)} = 20$ and $c^2_{\pi(2)} = 40$. We would like the fare quote $c^2_{\pi(2)}$ of Passenger P_2 to be as low as possible to maximize the likelihood of it accepting its fare quote and being serviced.

Table 1 shows all coalition costs per alpha value and shared costs for the DRT example from Figure 1.

6. CONCLUSIONS

In this work, we investigated a cost-sharing mechanisms for DRT systems, called POCS, which satisfy budget balance, individual rationality, immediate response and online fairness properties.

7. REFERENCES

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