Approximate Game Theoretic Analysis for Large Simulation-Based Games

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1. SIMULATION-BASED GAME THEORY

Multi-agent systems research raises many interesting questions about the actions of autonomous, interacting decision-makers. Game theory helps to answer some of those questions by providing a mathematical language for representing and reasoning about the incentives of self-interested autonomous agents. As is often the case with mathematical modeling, creating tractable game-theoretic models frequently requires abstracting away many aspects of the system under investigation. Simulation-based game theory constructs richer game models using agent-based simulation, but still suffers from tradeoffs between tractability and accuracy. My research agenda focuses first on improving this tradeoff by developing techniques for simulation-based game theory to conduct better analysis with limited simulation data and second on applying these techniques to understand complex interactions. My thesis will present novel methods that improve the tools available for simulation-based game theory and demonstrate the application of these tools to uncover new results in multi-agent systems of interest.

The fundamental building block of a simulation-based game model is the *environment simulator*. The simulator takes as input a strategy for each agent, simulates those strategies interacting, and outputs a payoff for each strategy. For a fixed number of agents and set of strategies, the simulator defines a *true game*, where the payoffs for a profile are the expected values of the simulator's output. In practice, this true game is unavailable and must be approximated using a finite set of observations Θ from the simulator. A *simulation-based game* is a normal form game resulting from the application of a game model function $\mathcal{M}$ to an observation set. In the most basic case, all profiles are simulated repeatedly and $\mathcal{M}$ simply calculates sample averages for every payoff. Game-theoretic analyses such as computation of Nash equilibria performed on the simulation-based game are treated approximating such analyses in the true game.

Sections 2-4 summarize thesis work that I have done so far, including two primary contributions to the methodology of simulation-based game theory and an application of simulation-based game theory to a strategic network formation domain. Sections 5 introduces current and future work in simulation-based game theory methods and applications.

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2. CREDIT NETWORK FORMATION

My thesis will present results from applying simulation-based game theory to a credit network formation game [**?**]. A credit network is a formal model of trust relationships among agents that has arisen independently in describing several phenomena. With my collaborators, I studied a game model describing how strategic agents would issue credit as a function of their expectations about future transactions and other agents' trustworthiness. When we reached the limits of tractable analysis in our explicit game models, I extended our analysis using simulation-based game theory to explore the sorts of equilibrium credit networks that arise in a much richer model.

A credit network is a weighted directed graph where an edge from node i to node j with weight c indicates that agent i extends c units of credit to agent j. My research investigates what criteria are important to strategic agents when deciding to issue credit, and what sorts of credit networks can arise in equilibrium when agents issue credit strategically. Issuing credit increases general liquidity in the network and increases the connectivity of the issuing agent, both of which potentially enable profitable transactions, but also entails risk that agents receiving credit will use it to make purchases and then default. The decision to issue credit is modeled as a one-shot game in which agents determine initial allocations of credit. Afterwards some agents randomly default and the remaining agents engage in repeated probabilistic transactions.

My co-authors showed that under extreme simplifications, one variant of a credit network formation game is a potential game. However, I showed that even slightly more-interesting variants may have no equilibria or unbounded price of anarchy. I have therefore employed simulation-based game theory to tackle more general scenarios. I found that when agents share common information about default probabilities, star-like networks often arise in equilibrium. In such networks, credit from the most trustworthy agents acts like a central currency.

I also tested environments with limited information, and found that central currency equilibria do not arise because central agents may not be trustworthy, and that issuing credit to only the most trustworthy known agents fails because those agents are no longer central. In such settings, the only equilibria other than empty networks arose from agent strategies that issued credit to agents with whom it was profitable to transact directly. Empty-network equilibria occurred in all circumstances except those where transacting was most profitable and defaults least likely.

3. DEVIATION-PRESERVING REDUCTION

Because the size of a game's payoff matrix grows exponentially with the number of players in a game, simulating all profiles in games with a large number of players is infeasible. Instead, it is sometimes possible to approximate a large game using another game with a smaller number of players. Deviation-preserving reduction improves the fidelity of such approximations relative to previous techniques, with only a small increase in the number of profiles simulated [**?**].

A symmetric game with N agents and $|S|$ strategies contains $\binom{N+|S|-1}{N}$ profiles. For a sense of how great a burden this imposes, consider that a symmetric game with 15 agents and 15 strategies contains over 77 million profiles, so if estimating a profile's payoff through simulation required one second, constructing the full game would take more than two years. The largest simulation-based game study to date is my work on credit network formation games, in which roughly 166,000 profiles were simulated.

Under deviation-preserving reduction, each reduced game player views itself as controlling one full game agent, while the opponents in the reduced game summarize the opponents in the full game. Across experiments on congestion, local effect, and credit network games, deviation-preserving reduction significantly out-performed previous player reduction methods in terms of full-game regret of reduced-game equilibria and in minimizing the number of strategies erroneously found to be dominated in the reduced game. Since my paper appeared, deviation-preserving reduction has been applied in multiple simulation-based game analyses.

4. BOOTSTRAP STATISTICS FOR GAMES

Virtually all published simulation-based game studies have been based on simulated environments with substantial randomness. As a result, it has been common practice to gather many samples of each simulated profile. Despite this, statistical methods for simulation-based game theory are underdeveloped, and the few tools that exist are rarely applied. I will be presenting work at AAMAS 2014 describing the first method to combine bootstrap statistics and game theory [**?**]. This work method enables derivation of statistical confidence bounds on the regret of approximate Nash equilibria in simulation-based games.

The bootstrap treats a sample set as representative of the population from which it was drawn for the purpose of computing distributional statistics. To derive a sampling distribution for a statistic, the bootstrap resamples the sample set to simulate drawing many samples from the population. If the original sample has size k, then each resample is a set of size k drawn with replacement from that sample. The statistic is then computed on each resample set, giving a bootstrap distribution for the statistic that can be used in place of a sampling distribution for computing confidence intervals.

My method to compute bootstrap confidence intervals on regret constructs a large number of bootstrap games by simultaneously resampling every payoff in the observation set Θ. Each bootstrap game is based on a resampled observation set $\hat{\Theta}$, where for each payoff, a resample set is drawn with replacement from its samples to have equal size. The bootstrap game $\mathcal{M}(\hat{\Theta})$ is then constructed by applying the game model creation function to the resampled observations. The regret of a profile of interest can then be computed in every bootstrap game, giving a bootstrap distribution for the regret statistic. The 95^{th} percentile of this distribution is then used as a 95% confidence bound on the true-game regret of the profile.

I demonstrated the validity of bootstrap confidence intervals for regret through experiments involving a large set of true games: randomly generated instances several game classes, of varying numbers of players and strategies. I emulated a simulator by adding artificial noise drawn from several distributions with widely varying magnitudes to the true games. Equilibria were computed in simulation-based games constructed from the noisy data and regret of those equilibria in the corresponding true games was compared to the bootstrap distribution estimates. These experiments show that the bootstrap distribution provides a good estimate of true game regret.

5. FUTURE WORK

I propose to improve approximation of large symmetric games by learning payoff functions with regression. The key insight I hope to exploit is that in symmetric games, representing profiles as vectors of strategy counts provides sufficient structure to support regression methods that learn utility functions. With large N, there should be enough data available to apply machine learning. The primary goal of my proposed research is to find regression methods appropriate to the task of learning payoff functions for each strategy in a game from simulation data. Because the full-game payoff matrix is generally too large to represent, the expected value of a symmetric mixed strategy must be computable in the model without enumerating all full-game profiles.

I have also done preliminary work studying optimal stopping games. These games are inspired by a classic problem [**?**], in which a manager interviews candidates from a finite set C about whom she has no prior information. After each interview the manager knows the relative rank of all candidates seen so far and has the option to either hire the current candidate and stop or reject the current candidate and continue interviewing. The manager receives utility 1 if the single best candidate is hired or 0 otherwise, and therefore wishes to maximize the probability of hiring that candidate. My co-authors and I are studying an extension to multiple competing managers in which pressure to hire immediately arises strategically and information gleaned from hiring decisions by other managers can create strategic reasons to alter the order of interviews.

6. REFERENCES

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