Properties of Multiwinner Voting Rules

Edith Elkind
University of Oxford
Oxford, United Kingdom
elkind@cs.ox.ac.uk

Piotr Faliszewski
AGH University
Krakow, Poland
faliszew@agh.edu.pl

Piotr Skowron
University of Warsaw
Warsaw, Poland
p.skowron@mimuw.edu.pl

Arkadii Slinko
University of Auckland
Auckland, New Zealand
a.slinko@auckland.ac.nz

ABSTRACT

The goal of this paper is to propose and study properties of multiwinner voting rules (with a particular focus on rules based in some way on single-winner scoring rules). We consider, e.g., SNTV, Bloc, k-Borda, STV, and several variants of Chamberlin–Courant's and Monroe's rules, identify two natural approaches to defining multiwinner rules, and show that many of our rules can be captured by one or both of these approaches. We then put forward a number of desirable properties of multiwinner rules, and compare the rules we consider with respect to these properties.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

Keywords

multiwinner elections, axioms, voting

1. INTRODUCTION

There are many situations where societies need to select a small set of entities from a larger group. For example, in indirect democracies people choose representatives to govern on their behalf, companies select groups of products to promote to their customers [17], web search engines decide which pages to display for a given query [12], and applicants for a job (e.g., a tenure-track position at a university) are first short-listed by a group of experts. For all these tasks we need formal rules to perform the selection, and the desirable properties of such rules may depend on the task at hand.

We view these selection rules as multiwinner voting rules which, given individual preferences, output groups of winners (which we call *committees*). While there is quite some research on such rules, many results are scattered throughout the literature. The goal of this paper is to review some natural multiwinner rules (focusing on those that are, in some way, based on scoring rules), to present a uniform framework for their study, and to propose a set of natural properties (axioms) against which these rules can be judged.

Appears in: *Alessio Lomuscio, Paul Scerri, Ana Bazzan, and Michael Huhns (eds.), Proceedings of the 13th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2014), May 5-9, 2014, Paris, France.*

We focus on the setting where the voters have ordinal preferences over the candidates. We have picked ten voting rules as examples of different ideas pertaining to multiwinner elections: STV, SNTV, k-Borda, Bloc, three variants of Chamberlin–Courant's rule [2,8,17], and three variants of Monroe's rule [2,18,22]. STV and SNTV are well-known rules that are used for parliamentary elections in some countries; Bloc is a rule that asks voters to specify their favorite committee and selects k alternatives that were nominated more often than others; k-Borda picks k alternatives with the highest Borda scores and is representative of rules used for picking k finalists in a competition (indeed, Formula 1 racing and Eurovision song contest use scoring rules very similar to Borda). Chamberlin–Courant's rule and Monroe's rule are examples of rules that, like STV, focus on proportional representation, but are based on explicitly assigning a committee member to each voter. We also consider two rules based on approximation algorithms for Chamberlin–Courant's rule [17] and for Monroe's rule [22]. All these rules can be seen as being based, in some way, on single-winner scoring protocols. Naturally, there are many other multiwinner rules, based on other principles, that we do not discuss (e.g., those based on the Condrocet principle [13–15,20] and those based on approval voting [4,7]). We believe that extending our research to include these rules is an important future research direction.

We are interested in judging our multiwinner rules with respect to their applicability in the following settings:

Parliamentary Elections. Voting rules for such elections should respect the "one person, one vote" principle. This is reflected in the requirement that each elected member should represent, roughly, the same number of voters. Some such rules are based on electoral districts, i.e., separate (possibly multiwinner) elections are held in different parts of the country, while others treat the whole country as a single constituency, and focus on proportional representation of different population groups.

Shortlisting. Consider a situation where a position is filled at a university. Each faculty member ranks applicants in order to create a short-list of those to be invited for an interview. One of the important requirements in this case is that, if some candidate is shortlisted when k applicants are selected, then this candidate would also be shortlisted if the list was extended to $k+1$ applicants.

Movie selection. Based on rankings provided by different customer groups, an airline has to decide which (few) movies to offer on their long-distance flights. It is important that each passenger finds something satisfying. This task is similar to parliamentary elections, but without the need to worry that each movie would be watched by the same number of people. It is, however, quite different from shortlisting: If there are two similar candidates, then for shortlisting we should, typically, take either both or neither, whereas in the context of movie selection it makes sense to pick at most one of them.

We study properties of voting rules that are important in these settings. We introduce *committee monotonicity*, *solid coalitions property*, *consensus committee property*, and *unanimity*, and adapt the standard notions of monotonicity, homogeneity, and consistency to the multiwinner framework.

The paper is organized as follows. In Section 2 we introduce the basic terminology used in this paper; in Section 3 we define the rules that we study and put forward two ways of classifying them. In Section 4, we define several properties of multiwinner rules and in Sections 5–8 we study particular groups of these properties in detail. We present our conclusions in Section 9.

Our paper is a preliminary attempt to give a formal framework for the study of multiwinner rules. Thus we use the word *axiom* quite freely, without meaning that it should be a *normative requirement*. We omit many proofs due to space constraints.

2. PRELIMINARIES

An *election* is a pair $E = (C, V)$, where $C = \{c_1, \dots, c_m\}$ is a set of *candidates* and $V = (v_1, \dots, v_n)$ is a sequence of *voters*. Each voter is described by a *preference order*, which is a ranking of the candidates from the most desirable one to the least desirable one. We denote the position of a candidate $c \in C$ in the preference order of a voter $v \in V$ by $\mathrm{pos}_v(c)$. If V_1 and V_2 are two sequences of voters over the same candidate set C, then $V_1 + V_2$ denotes the concatenation of V_1 and V_2. If V is a sequence of voters and t is an integer, then tV denotes the concatenation of t copies of V. For $E_1 = (C, V_1)$ and $E_2 = (C, V_2)$, we write $E_1 + E_2$ to denote $(C, V_1 + V_2)$, and for $E = (C, V)$ and a positive integer t, we write tE to denote (C, tV). For an integer n, we denote $\{1, \dots, n\}$ by $[n]$.

A *multiwinner voting rule* $\mathcal{R}$ is a function that, given an election $E = (C, V)$ and a positive integer k, $k \le \|C\|$, returns a set $\mathcal{R}(E, k)$ of k-element subsets of C, which we call *committees*. That is, a rule returns a set of committees that are tied-for-winning. Brams and Fishburn [3] refer to these rules as *choose-k rules*. In practice, one would need to combine such a rule with a tie-breaking mechanism but, for simplicity, we mostly disregard this issue here.

We stress that if $\mathcal{R}$ is a multiwinner rule, then given an election $E = (C, V)$ and a positive integer k, $k \le \|C\|$, $\mathcal{R}(E, k)$ must output committees of size exactly k; when we need to emphasize the size of the committee, we use the term *k-committee selection rules*. This is a natural requirement if, for example, the goal is to elect a parliament whose size is fixed by external rules. However, as a consequence, we are sometimes forced to elect Pareto-dominated candidates (e.g., if all voters unanimously rank the candidates in the same order and $k > 1$). Alternatively, we could require $\mathcal{R}(E, k)$ to return committees of up-to-k members. The latter approach is also studied in the literature (either explicitly or implicitly), but we adopt the former one due to its simplicity and applicability in our settings of interest. Note that Brams and Fishburn [3] require a choose-k rule to select a committee of size at least k.

3. MULTIWINNER VOTING RULES

We now provide definitions of our multiwinner rules and discuss two general ways of classifying them.

3.1 Definitions of Multiwinner Rules

Many multiwinner rules rely on ideas from single-winner rules, so let us review these first.

Plurality score. The plurality score of a candidate c is the number of voters that rank c first.

***t*-approval score.** Let t be a positive integer. The t-approval score of a candidate c is the number of voters that rank c among top t positions.

Borda score. Let v be a vote over a candidate set C. The Borda score of a candidate $c \in C$ in v is $\|C\| - \mathrm{pos}_v(c)$. The Borda score of c in an election $E = (C, V)$ is the sum of c's Borda scores from all voters in V.

We are now ready to describe several multiwinner rules. Let $E = (C, V)$ be an election and let $k \in [\|C\|]$ be the size of the committee that we seek. We assume the parallel-universes tie-breaking [9], i.e., our rules return all the committees that could result from breaking an intermediate tie during the computation of the rule.

Single Transferable Vote (STV). STV is a multistage elimination rule that works as follows. If there is a candidate c whose Plurality score is at least $q = \lfloor \frac{\|V\|}{k+1} \rfloor + 1$ (the so-called Droop quota), we do the following: (a) include c in the winning committee, (b) delete q votes where c is ranked first, and (c) remove c from all the remaining votes. If each candidate's Plurality score is less than q, a candidate with the lowest Plurality score is deleted from all votes.

There are many other variants of STV; we point the reader to the work of Tideman and Richardson [23] for details.

Single Nontransferable Vote (SNTV). Under SNTV, we return the k candidates with the highest Plurality scores (thus one can think of SNTV as simply k-Plurality).

Bloc. Under Bloc, we return the k candidates with the highest k-approval scores.

***k*-Borda.** Under k-Borda, we return the k candidates with the highest Borda scores. Debord [10] provided an axiomatic characterization of this rule.

Chamberlin–Courant's and Monroe's Rules. These rules explicitly aim at proportional representation. The main idea is to provide an optimal assignment of committee members to voters by using a satisfaction function to measure the quality of the assignment.

A *satisfaction function* is a monotonically nonincreasing mapping $\alpha \colon \mathbb{N} \to \mathbb{N}$. Intuitively, $\alpha(i)$ is a voter's satisfaction from being represented by a candidate that this voter ranks in position i. We focus on the Borda satisfaction function, which for m candidates is defined as $\alpha_{\mathrm{B}}^m(i) = m - i$.

Let k be the target committee size. A function $\Phi \colon V \to C$ is an *assignment function* if $\|\Phi(V)\| \le k$. Intuitively, in the elected committee voter v is represented by candidate $\Phi(v)$. There are several ways to compute the societal satisfaction from the assignment; we focus on the following two:

$$\ell_1(\Phi) = \sum_{v \in V} \alpha(\mathrm{pos}_v(\Phi(v))) \ , \ \ell_{\min}(\Phi) = \min_{v \in V}(\alpha(\mathrm{pos}_v(\Phi(v))).$$

The former one, $\ell_1(\Phi)$, is a utilitarian measure, which sums the satisfactions of all the voters, and the latter one, $\ell_{\min}(\Phi)$, is an egalitarian measure, which consider the satisfaction of the least satisfied voter only.

Let α be a satisfaction function and let ℓ be one of ℓ_1 and $\ell_{\min}$. Chamberlin–Courant's rule for ℓ and α (ℓ-α-CC) finds an assignment function Φ that maximizes $\ell(\Phi)$ and declares the candidates in $\Phi(V)$ to be the winning committee. If $\|\Phi(V)\| < k$, the rule fills in the missing committee members in an arbitrary way. ℓ-α-Monroe's rule is defined in the same way, except that we optimize over assignment functions that additionally satisfy the so-called Monroe criterion, which requires that $\lfloor \frac{n}{k} \rfloor \leq \|\Phi^{-1}(c)\| \leq \lceil \frac{n}{k} \rceil$ for each elected candidate c. To simplify notation, we omit α_{B}^m when referring to Monroe/CC rule with Borda function.

For Chamberlin–Courant's rule, for each set of candidates $C' \subseteq C$ we define the assignment function $\Phi^{\mathrm{CC}}(C')$ so that for each voter v, $\Phi^{\mathrm{CC}}(C')(v)$ is v's top candidate in C'. If W is a winning committee under Chamberlin–Courant's rule, then $\Phi^{\mathrm{CC}}(W)$ is an optimal assignment function.

The utilitarian variants of the rules (i.e., ℓ_1-CC and ℓ_1-Monroe) were introduced by Chamberlin and Courant [8] and by Monroe [18], respectively. The egalitarian variants were introduced by Betzler et al. [2]. Unfortunately, these rules are hard to compute, irrespective of tie-breaking, both for Borda satisfaction function [2,17] and for various approval-based satisfaction functions [2,19].

Approximate Variants of ℓ_1-Monroe and ℓ_1-CC. Hardness results for ℓ_1-CC and ℓ_1-Monroe inspired research on designing efficient approximation algorithms for these rules [17,22]. Here, in the spirit of Caragiannis et al. [6], we consider these algorithms as full-fledged multiwinner rules.

We refer to the rules based on approximation algorithms for ℓ_1-CC and ℓ_1-Monroe as Greedy-CC and Greedy-Monroe, respectively. Greedy-CC was proposed by Lu and Boutilier [17] and Greedy-Monroe by Skowron et al. [22]. Both rules proceed in k iterations, in which they build sets $\emptyset = W_0 \subset W_1 \subset W_2 \subset \cdots \subset W_k$, and declare W_k to be the winning committee. In the i-th iteration, $i \in [k]$, Greedy-CC picks a candidate $c \in C \setminus W_{i-1}$ that maximizes $\ell_1(\Phi^{\mathrm{CC}}(W_{i-1} \cup \{c\}))$ and sets $W_i = W_{i-1} \cup \{c\}$ ($\ell_1(\cdot)$ is computed using Borda satisfaction function). Greedy-Monroe, in addition to the sets $W_0, \dots, W_k$, also maintains sets of voters $\emptyset = V_0 \subset V_1 \subset \cdots \subset V_k = V$, such that, after the i-th iteration, V_i is the set of voters for which the rule has already assigned candidates. In the i-th iteration, the rule picks a number $n_i \in \{\lceil \frac{n}{k} \rceil, \lfloor \frac{n}{k} \rfloor\}$ (see below for the choice criterion) and then picks a candidate $c \in C \setminus W_{i-1}$ and a group V' of n_i voters from $V \setminus V_{i-1}$ that together maximize the Borda score of c in V'. The rule sets $W_i = W_{i-1} \cup \{c\}$ and $V_i = V_{i-1} \cup V'$ (intuitively, Greedy-Monroe assigns c to the voters in V'). Regarding the choice of n_i, if n is of the form $kn' + n''$, where $0 \leq n'' < k$, then Greedy-Monroe picks $\lceil \frac{n}{k} \rceil$ for the first n'' iterations and picks $\lfloor \frac{n}{k} \rfloor$ for the remaining ones.

Greedy-CC and Greedy-Monroe output committees that approximate those output by ℓ_1-CC and ℓ_1-Monroe. In particular, Greedy-CC finds a committee W such that the satisfaction of the voters is at least $1 - \frac{1}{e}$ of the satisfaction achieved under ℓ_1-CC [17], and Greedy-Monroe finds a committee that achieves at least a $1 - \frac{k}{2m-1} - \frac{H_k}{k}$ fraction of the satisfaction given by ℓ_1-Monroe, where $H_k = \sum_{i=1}^{k} \frac{1}{k}$ [22]. These rules are efficiently computable in the sense that we can output some winning committee in polynomial time; however, their computational complexity under parallel-universes tie-breaking is not known.

3.2 Two Types of Multiwinner Rules

Perhaps surprisingly, it turns out that many of the rules introduced so far have very similar internal structure. Below we present two natural ways of identifying these similarities.

Best-k Rules. SNTV and k-Borda are natural extensions of Plurality and Borda to the multiwinner setting: We sort the candidates in the order of decreasing scores (with further parallel-universes tie-breaking if needed) and pick the top k ones. In general, we say that a given multiwinner rule $\mathcal{R}$ is a *best-k rule* if there is a social preference function F (i.e., a function that given an election $E = (C, V)$ returns a set of tied linear orders over C) such that for each m-candidate election $E = (C, V)$ and each $k \in [m]$, a set W is in $\mathcal{R}(E, k)$ if and only if $\|W\| = k$ and there is an order $\succ$ in $F(E)$ such that $c \succ d$ for each $c \in W$ and $d \in C \setminus W$.

SNTV and k-Borda are best-k rules. We can define a best-k rule based on the social preference function known as the Kemeny rule [16], and, somewhat surprisingly, we note that Greedy-CC is a best-k rule. Thus, best-k rules are a more diverse group than one might at first expect.

Committee Scoring Rules. Both k-Borda and ℓ_1-CC can be viewed as generalizations of the Borda rule to the multiwinner case. Here we introduce a class of *committee scoring rules*, which generalize single-winner scoring rules. This class captures k-Borda, ℓ_1-CC, and many other rules. We believe that identifying committee scoring rules is an important conceptual contribution of this paper.

Consider an election $E = (C, V)$ where we want to pick a committee of size k out of $m = \|C\|$ candidates. A k-winner committee scoring rule is defined via a *committee scoring function* f, $f \colon [m]^k \to \mathbb{N}$, as follows. Given a committee S and a voter v, we define $\mathrm{pos}_v(S)$ to be the vector $(i_1, \dots, i_k)$ resulting from sorting the set $\{\mathrm{pos}_v(c) \mid c \in S\}$ in the non-decreasing order. The winning committees are the ones that maximize the sum $\sum_{v \in V} f(\mathrm{pos}_v(S))$.

Just as for single-winner scoring rules, we require a certain form of monotonicity with respect to the values of f. Specifically, let $I = (i_1, \dots, i_k)$ and $J = (j_1, \dots, j_k)$ be two increasing sequences of numbers from $[m]$. We say that $I \succeq J$ if and only if $(i_1 \leq j_1) \wedge \ldots \wedge (i_k \leq j_k)$, and we require that $I \succeq J$ implies $f(I) \geq f(J)$.

EXAMPLE 1. *Let m be the number of candidates and let k be the target committee size. Define $\alpha_\ell \colon [m] \to \{0, 1\}$ by setting $\alpha_\ell(i) = 1$ if $i \leq \ell$ and $\alpha_\ell(i) = 0$ otherwise. For each $i \in [m]$, define $\beta(i) = m - i$. SNTV, Bloc, k-Borda, and ℓ_1-CC are committee scoring rule defined by:*

$$f_{\mathrm{SNTV}}(i_1, \dots, i_k) = \textstyle\sum_{t=1}^{k} \alpha_1(i_t) = \alpha_1(i_1),$$
$$f_{\mathrm{Bloc}}(i_1, \dots, i_k) = \textstyle\sum_{t=1}^{k} \alpha_k(i_t),$$
$$f_{k\text{-Borda}}(i_1, \dots, i_k) = \textstyle\sum_{t=1}^{k} \beta(i_t), \text{ and}$$
$$f_{\mathrm{CC}}(i_1, \dots, i_k) = \beta(i_1).$$

The form of the committee scoring function has significant impact on the properties of the rule, e.g., on its complexity.

Proposition 1. *Let $\alpha = (\alpha^1, \dots, \alpha^m, \dots)$ be a family of nonincreasing polynomial-time computable functions, $\alpha^m : [m] \to \mathbb{N}$. For each number of candidates m and each $k \in [m]$, the function $f(i_1, \dots, i_k) = \sum_{t=1}^{k} \alpha^m(i_t)$ defines a polynomial-time computable k-committee selection rule.*

We refer to committee scoring rules of the form given in Proposition 1 as *additively separable*. Under such rules we separately compute the "score" of each candidate, and pick k candidates with the highest scores. In other words, they are best-k rules. The first three rules in Example 1 are of this kind.

On the other extreme, we have committee scoring rules defined through functions $f(i_1, \dots, i_k)$ whose values depend solely on i_1 (like the fourth rule in Example 1). Such rules seem to focus on voter representation. As in ℓ_1-CC, each voter is assigned to her most preferred candidate in the selected committee, and only contributes towards the score of this candidate. We call such committee scoring rules *representation-focused*.

Note that SNTV is both additively separable and representation-focused.

4. AXIOMS

We will now put forward some properties (axioms) that multiwinner rules may or may not satisfy. We use the standard axioms for single-winner rules as our starting point, and augment them with ideas from the literature that are specific to the multiwinner domain. (Due to our choice of focus, we do not include properties based on the Condorcet principle, such as, e.g., the stability of Barberà and Coelho [1].) We stress that, since multiwinner rules have a very diverse range of applications, our properties should not necessarily be understood in the normative way: the desirability of a particular property can only be evaluated in the context of a specific application. Below, $\mathcal{R}$ denotes a multiwinner rule.

Our first axiom is *nonimposition*. It requires that each size-k set of candidates can win. This is a basic requirement that is trivially satisfied by all rules that we consider.

Nonimposition. For each set of candidates C, and each k-element subset W of C, there is an election $E = (C, V)$ such that $\mathcal{R}(E, k) = \{W\}$.

The next three axioms—consistency, homogeneity, and monotonicity—are adapted from the single-winner setting. For the first two, the adaptation is straightforward.

Consistency. For every pair of elections $E_1 = (C, V_1)$, $E_2 = (C, V_2)$ and each $k \in [\|C\|]$, if $\mathcal{R}(E_1, k) \cap \mathcal{R}(E_2, k) \neq \emptyset$ then $\mathcal{R}(E_1 + E_2, k) = \mathcal{R}(E_1, k) \cap \mathcal{R}(E_2, k)$.

Homogeneity. For each election $E = (C, V)$, each $k \in [\|C\|]$, and each $t \in \mathbb{N}$ it holds that $\mathcal{R}(tE, k) = \mathcal{R}(E, k)$.

We now consider monotonicity. If c belongs to a winning committee W then, generally speaking, we cannot expect W to remain winning when c is moved forward in some vote, as this shift may hurt other members of W. Indeed, none of our rules satisfies this variant of monotonicity. However, there are two natural relaxations of this condition. One option is to require that after the shift c belongs to *some* winning committee. Alternatively, we may restrict forward movements of c, prohibiting it to overtake other members of W. We point the reader to [21] for an extensive discussion of monotonicity in the context of irresolute voting rules.

Monotonicity. For each election $E = (C, V)$, each $c \in C$, and each $k \in [\|C\|]$, if $c \in W$ for some $W \in \mathcal{R}(E, k)$, then for each E' obtained from E by shifting c one position forward in some vote v it holds that: (1) for *candidate monotonicity*: $c \in W'$ for some $W' \in \mathcal{R}(E', k)$, and (2) for *non-crossing monotonicity*: if c was ranked below some $b \notin W$, then $W \in \mathcal{R}(E', k)$.

Our next axiom, *committee monotonicity*, is specific to multiwinner elections, as it deals with changing the desired committee size. Intuitively, it requires that when we increase the committee size, none of the already selected candidates should be dropped. Our phrasing is somewhat involved because $\mathcal{R}$ returns sets of committees.

Committee Monotonicity. For each election $E = (C, V)$ the following conditions hold: (1) For each $k \in [m-1]$, if $W \in \mathcal{R}(E, k)$ then there exists a $W' \in \mathcal{R}(E, k+1)$ such that $W \subseteq W'$; (2) for each $k \in [m-1]$, if $W \in \mathcal{R}(E, k+1)$ then there exists a $W' \in \mathcal{R}(E, k)$ such that $W' \subseteq W$.

The second condition in the definition above is aimed to prevent the following situation. Consider an election E with candidate set $C = \{a, b, c, \dots\}$. Without condition (2) a committee-monotone rule $\mathcal{R}$ would be allowed to output the following winning committees: $\mathcal{R}(E, 1) = \{\{a\}\}$, $\mathcal{R}(E, 2) = \{\{a, b\}, \{b, c\}\}$, and so on. Note that the committee $\{b, c\}$, which suddenly appears in $\mathcal{R}(E, 2)$, breaks what we would intuitively think of as committee monotonicity, but is not ruled out by condition (1) alone.

The final three axioms represent three implementations of Dummett's condition known as *proportionality for solid coalitions* [11]. Dummett's original proposal is as follows: Consider an election with n voters where the goal is to pick k candidates. If for some $\ell \in [k]$ there is a group of $\frac{\ell n}{k}$ voters that all rank the same ℓ candidates on top, these ℓ candidates should be in a winning committee. This requirement, which tries to capture the idea of proportional representation, seems to be remarkably strong: We are not aware of a single rule that satisfies it.[1] The following three axioms are weaker and reflect the same idea.

Solid Coalitions. For each election $E = (C, V)$ and each $k \in [\|C\|]$, if at least $\frac{\|V\|}{k}$ voters rank some candidate c first then c belongs to every committee in $\mathcal{R}(E, k)$.

Consensus Committee. For each election $E = (C, V)$ and each $k \in [\|C\|]$, if there is a k-element set W, $W \subseteq C$, such that each voter ranks some member of W first and each member of W is ranked first by $\lfloor \frac{\|V\|}{k} \rfloor$ or $\lceil \frac{\|V\|}{k} \rceil$ voters then $\mathcal{R}(E, k) = \{W\}$.

Unanimity. For each election $E = (C, V)$ and each $k \in [\|C\|]$, if each voter ranks the same k candidates W on top (possibly in different order), then $\mathcal{R}(E, k) = \{W\}$ (strong unanimity) or $W \in \mathcal{R}(E, k)$ (weak unanimity).

5. COMMITTEE MONOTONICITY

The desirability of the committee monotonicity property is strongly dependent on the application: if we are choosing

[1] There is a variant of Dummett's condition known as *Droop Proportionality Criterion*, which is geared toward STV [24]; STV can be shown to satisfy Dummett's condition whenever Droop quota is smaller than $\frac{n}{k}$.

Rule	Committee Monotonicity	Solid Coalitions	Consensus Committee	Unanimity	Monotonicity	Homogeneity	Consistency
STV	×	√ (◇)	√ (◇)	strong	×	√(♡)	×
SNTV	√	√	√	weak	C/NC	√	√
Bloc	×	×	×	strong	C/NC	√	√
k-Borda	√	×	×	strong	C/NC	√	√
ℓ_1-CC	×	×	√	weak	C	√	√
$\ell_{\min}$-CC	×	×	√	weak	C	√	×
Greedy-CC	√	×	×	weak	×	√	×
ℓ_1-Monroe	×	×	√	strong	×	√ (♣)	×
$\ell_{\min}$-Monroe	×	×	√	strong	×	√ (♣)	×
Greedy-Monroe	×	√	√	strong	×	√ (♠)	×

Table 1: Summary of results. √ and × indicate that the rule has/does not have the respective property. C means candidate monotonicity and NC means non-crossing monotonicity (C/NC means satisfying both conditions). The properties marked with (◇) hold for STV when $n \geq k(k+1)$; property marked with (♡) requires STV to use non-rounded Droop quota and fractional votes. Properties marked with (♣) hold if n is divisible by k and (♠) in addition requires a specific intermediate tie-breaking rule.

finalists of a competition, then it is imperative to use a rule that has this property, but in the context of proportional representation requiring committee monotonicity may interfere with selecting a truly representative committee.

It turns out that committee monotonicity axiomatically characterizes the class of best-k rules.

THEOREM 2. *A k-committee selection rule satisfies committee monotonicity if and only if it is a best-k rule.*

PROOF. Let $\mathcal{R}$ be a best-k rule and let F be the underlying social preference function. Consider an election $E = (C, V)$. Pick $k \in [\|C\| - 1]$ and $W \in \mathcal{R}(E, k)$. By definition of a k-best rule, there is an order $\succ$ in $F(E)$ such that $w \succ c$ for each $w \in W$ and each $c \in C \setminus W$. Clearly, there is a candidate $w' \in C \setminus W$ such that for each $w \in W$ and each $c \in C \setminus (W \cup \{w'\})$ we have $w \succ w' \succ c$. Hence, $W \cup \{w'\} \in \mathcal{R}(E, k+1)$. A similar argument shows that $\mathcal{R}$ satisfies the second committee-monotonicity condition.

Conversely, assume that $\mathcal{R}$ satisfies committee monotonicity. We will show that it is a best-k rule by deriving the underlying social preference function F. Let $E = (C, V)$ be some election where $C = \{c_1, \ldots, c_m\}$. We define $F(E)$ to contain all linear orders $\succ$ that satisfy the following condition: If π is a permutation of $[m]$ and $c_{\pi(1)} \succ c_{\pi(2)} \succ \cdots \succ c_{\pi(m)}$ then there is a sequence of sets $W_1 = \{c_{\pi(1)}\}, W_2 = \{c_{\pi(1)}, c_{\pi(2)}\}, \ldots, W_m = \{c_{\pi(1)}, \ldots, c_{\pi(m)}\}$ such that $W_1 \in \mathcal{R}(E, 1), W_2 \in \mathcal{R}(E, 2), \ldots, W_m \in \mathcal{R}(E, m)$. Using the two conditions from the definition of committee monotonicity, it is easy to verify that F indeed defines $\mathcal{R}$. □

Thus SNTV, k-Borda, Greedy-CC, and all additively separable committee scoring rules satisfy committee monotonicity.

Indeed, for additively separable committee scoring rules their underlying social preference functions can be shown to be based on social welfare functions that always return a single weak order (defined by candidates' scores). This distinguishes them from rules such as, e.g., Greedy-CC, whose social preference function may not be based on a social welfare function.

STV, Bloc, ℓ_1-CC, $\ell_{\min}$-CC, ℓ_1-Monroe, $\ell_{\min}$-Monroe, and Greedy-Monroe are not best-k rules, and thus none of these rules satisfies committee monotonicity.

PROPOSITION 3. *STV, Bloc, ℓ_1-CC, $\ell_{\min}$-CC, ℓ_1-Monroe, $\ell_{\min}$-Monroe, and Greedy-Monroe do not satisfy committee monotonicity.*

6. DUMMETT'S PROPORTIONALITY

Properties based on Dummett's proportionality condition (with the exception of unanimity) are geared toward rules that aim to achieve proportional representation of the voters. Thus, in this section, we judge multiwinner rules from this perspective.

We start by considering the solid coalitions property. It is easy to see that it is satisfied by both SNTV and STV. On the other hand, even though this property seems to be very much in spirit of Monroe's and Chamberlin–Courant's rules, ℓ_1-Monroe, ℓ_1-CC, $\ell_{\min}$-Monroe, and $\ell_{\min}$-CC fail to satisfy it. Yet, it is satisfied by Greedy-Monroe.

THEOREM 4. *ℓ_1-CC, $\ell_{\min}$-CC, ℓ_1-Monroe, and $\ell_{\min}$-Monroe do not have the solid coalitions property, but Greedy-Monroe does have it.*

PROOF. For the first part of the theorem, we consider ℓ_1-CC and ℓ_1-Monroe (we omit the constructions for other rules due to space restriction). Take an election with candidate set $C = \{a, b, c, d, e\}$ and nine voters whose preference orders are *aedbc*, *aedbc*, *bedac*, *bedac*, *cedab*, *cedab*, *daebc*, *dbeac*, and *dceab*. The reader can verify that none of our rules elects a size-3 committee that contains d, even though this would be required by the solid coalitions property.

For the second part of the theorem, we consider Greedy-Monroe. Take some election with n voters, where we seek a committee of size k. Suppose that some candidate c is ranked first by at least $\frac{n}{k}$ voters. Greedy-Monroe starts by picking candidates ranked first by at least $\frac{n}{k}$ voters. By the time it considers c, each of the voters that rank c first remains unassigned, so it will pick c. □

We believe that the solid coalitions property is desirable, but not crucial for applications that require proportional representation (e.g., parliamentary elections). In contrast, the consensus committee property, which we discuss next, seems to be fundamental. Indeed, it is satisfied by almost all rules that aim to achieve proportional representation.

In essence, the consensus committee property is satisfied by every rule that has the solid coalitions property.[2] In particular, it is satisfied by SNTV, STV (if there are sufficiently many voters) and Greedy-Monroe. It is also satisfied by ℓ_1-CC, $\ell_{\min}$-CC, ℓ_1-Monroe, and $\ell_{\min}$-Monroe, but, interestingly, not by Greedy-CC. This reveals a major deficiency of the latter rule: It makes decisions regarding the inclusion of some candidate c into the committee based on the preferences of the voters to whom c would not be assigned.

Proposition 5. *Bloc, k-Borda and Greedy-CC do not have the consensus committee property (nor the solid coalitions property).*

Proof. Consider an election with $C = \{a, b, c, d\}$ and two voters with preference orders $b \succ c \succ d \succ a$ and $a \succ c \succ d \succ b$. We seek a committee of size $k = 2$. Each of these rules includes c in each winning committee and thus fails the consensus committee property. $\square$

For SNTV, ℓ_1-CC, k-Borda and Bloc, the above results can also be seen as incarnations of the following two more general results regarding committee scoring rules.

Proposition 6. *Let $\mathcal{R}$ be an additively separable committee scoring rule, let $k < m$, and let $f(i_1, \dots, i_k) = \sum_{t=1}^{k} \alpha(t)$ be the respective committee scoring function. Then $\mathcal{R}$ fails the consensus committee property if $0 < \alpha(1) \le k\alpha(2)$, but satisfies it if $\alpha(1) > k\alpha(2)$ and there are sufficiently many voters.*

Proof. Due to space constraints, we consider the second claim only. Assume that $\alpha(1) > k\alpha(2)$. Consider an arbitrary election $E = (C, V)$ with $\|C\| = m$, where there is a group W of k candidates such that each voter ranks some member of W first and each member of W is ranked first by either $\lfloor \frac{n}{k} \rfloor$ or $\lceil \frac{n}{k} \rceil$ voters. Consider a candidate $x \notin W$. Its α-score is at most $n\alpha(2)$. On the other hand, the score of each candidate in W is at least $\lfloor \frac{n}{k} \rfloor \alpha(1)$. By assumption, $\lfloor \frac{n}{k} \rfloor \alpha(1) > n\alpha(2)$ for sufficiently large n and, thus, $\mathcal{R}$ satisfies the consensus committee property for such n. $\square$

Proposition 7. *Let $\mathcal{R}$ be a representation-focused k-committee scoring rule with committee scoring function $f(i_1, \dots, i_k) = \beta(i_1)$. Then $\mathcal{R}$ has the consensus committee property if and only if $\beta(1) > \beta(2)$.*

Our final instantiation of Dummett's proportionality for solid coalitions is the unanimity property. Every committee scoring rule satisfies its weak variant.

Theorem 8. *Every committee scoring rule $\mathcal{R}$ satisfies weak unanimity.*

Proof. Consider an election $E = (C, V)$ where every voters ranks candidates from some set W, $\|W\| = k$, on top. Let f be the committee scoring function for $\mathcal{R}$. By definition, for each voter v in V and each size-k set Q of candidates, we have $f(\mathrm{pos}_v(W)) \ge f(\mathrm{pos}_v(Q))$. Thus, we have $W \in \mathcal{R}(E, k)$. $\square$

It is immediate that ℓ_1-Monroe, $\ell_{\min}$-Monroe, Greedy-Monroe, Bloc and k-Borda satisfy strong unanimity, and that SNTV, ℓ_1-CC, $\ell_{\min}$-CC, and Greedy-CC do not.

Finally, we note that STV satisfies strong unanimity: If there is some set W of k candidates that each of the n voters ranks on top, then in every round of STV there is a candidate from W that is ranked first by at least $\lfloor \frac{n}{k+1} \rfloor + 1$ voters.

[2] This is not a theorem due to rounding-related issues.

7. MONOTONICITY

Monotonicity is a natural and easily satisfiable condition for single-winner rules. Among the few examples of prominent non-monotone single-winner rules are STV and Dodgson's rule [5]. In contrast, for multiwinner rules monotonicity is a rather demanding property. However, all committee scoring rules satisfy candidate monotonicity, and all additively separable committee scoring rules satisfy non-crossing monotonicity.

Theorem 9. *Let $\mathcal{R}$ be a k-committee scoring rule. Then $\mathcal{R}$ satisfies candidate monotonicity.*

Proof. Consider an election $E = (C, V)$ with $\|C\| = m$. Let f be the k-committee scoring function defining $\mathcal{R}$ for m candidates and k winners.

Let W be a committee in $\mathcal{R}(E, k)$ and let c be a candidate in W. Consider a vote $v \in V$ that does not rank c first, and replace it with a vote v' obtained from v by shifting c one position forward. Denote the resulting election by E'.

By construction, we have $f(\mathrm{pos}_{v'}(W)) \ge f(\mathrm{pos}_v(W))$. On the other hand, for each committee $S \subseteq C \setminus \{c\}$, we have $f(\mathrm{pos}_{v'}(S)) \le f(\mathrm{pos}_v(S))$. Since W was a winning committee for E, this means that either W is also a winning committee for E' or some committee $W' \in \mathcal{R}(E', k)$ has a higher score. We must have $c \in W'$, since only committees with c can have a higher score in E', compared to E. $\square$

Theorem 10. *Let $\mathcal{R}$ be an additively separable k-committee scoring rule. Then $\mathcal{R}$ satisfies non-crossing monotonicity.*

However, committee scoring rules that are not additively separable (such as ℓ_1-CC) do not always satisfy non-crossing monotonicity.

Proposition 11. *ℓ_1-CC, ℓ_1-Monroe, Greedy-CC, and Greedy-Monroe fail non-crossing monotonicity.*

While $\ell_{\min}$-CC is not a committee scoring rule, it also satisfies candidate monotonicity, but fails non-crossing monotonicity; the proof is similar to that of Theorem 9.

Theorem 12. *$\ell_{\min}$-CC satisfies candidate monotonicity, but both $\ell_{\min}$-CC and $\ell_{\min}$-Monroe fail non-crossing monotonicity.*

The remaining multiwinner rules studied in this paper fail each of our monotonicity criteria. For STV this is well-known to happen even for $k = 1$. For the rest of the rules, we provide the next result.

Proposition 13. *ℓ_1-Monroe, $\ell_{\min}$-Monroe, Greedy-Monroe, and Greedy-CC fail candidate monotonicity.*

Proof. We only give the proof for ℓ_1-Monroe due to space constraints. Consider an election with candidate set $C = \{a, b, c, d\}$ and four voters whose preference orders are *acdb*, *dacb*, *bcda*, and *dbac*. Let $k = 2$. Under ℓ_1-Monroe, there are two winning committees, $\{a, b\}$ and $\{c, d\}$, both with satisfaction $4\|C\| - 6 = 10$.

If we shift a forward by one position in the last vote, then $\{c, d\}$ is the only committee that wins in the modified election. Thus, ℓ_1-Monroe is not candidate monotone. $\square$

8. CONSISTENCY AND HOMOGENEITY

For single-winner rules, a famous theorem of Young says that only scoring rules and their compositions satisfy consistency [25]. While we do not know how to extend this result to multiwinner rules, the situation seems to be similar: We show that every committee scoring rule satisfies consistency, whereas other rules fail it (for STV, $\ell_{\min}$-CC and $\ell_{\min}$-Monroe this follows from Young's result, as for $k = 1$ these rules are not scoring rules).

PROPOSITION 14. *Every committee scoring rule satisfies consistency. STV, $\ell_{\min}$-CC, Greedy-CC, $\ell_1/\ell_{\min}$-Monroe, and Greedy-Monroe fail consistency.*

We now consider homogeneity. Naturally, committee scoring rules are homogeneous because consistency implies homogeneity. For other rules, the situation is more complex.

THEOREM 15. *Both $\ell_{\min}$-CC and Greedy-CC satisfy homogeneity.*

Intuitively, these rules are homogeneous since they treat voters with the same preference orders identically. Interestingly, neither of the variants of Monroe's rule is homogeneous.

PROPOSITION 16. *ℓ_1-Monroe, $\ell_{\min}$-Monroe, and Greedy-Monroe are not homogeneous.*

PROOF. Consider an election with candidate set $C = \{a, b, c, d\}$ and three voters with preference orders *abdc*, *abdc*, and *cbda*. For ℓ_1-Monroe, Greedy-Monroe and $\ell_{\min}$-Monroe, the unique winning committee of size 2 is $\{a, c\}$. However, for $2E$ the winner sets for these rules include $\{a, b\}$. $\square$

On the positive side, if the number of voters is divisible by the size of the committee, then ℓ_1-Monroe and $\ell_{\min}$-Monroe are homogeneous. In essence, this means that variants of Monroe fail homogeneity due to rounding problems in the Monroe criterion. One solution would be to clone each voter k times when seeking a committee of size k. We do not consider this modification of Monroe's rule here, but it would be interesting to see how the satisfaction of a committee elected in this way compares to that elected without cloning.

THEOREM 17. *Both ℓ_1-Monroe and $\ell_{\min}$-Monroe satisfy homogeneity, provided that the number of voters in the election is divisible by the size of the committee to be selected.*

PROOF. Let $\mathcal{R} \in \{\ell_1\text{-Monroe}, \ell_{\min}\text{-Monroe}\}$, pick an election $E = (C, V)$, and let k be a positive integer that divides $\|V\|$. We will show that $\mathcal{R}(E, k) = \mathcal{R}(tE, k)$ for each $t > 0$.

Let W be a committee that wins in tE. We refer to the members of W as the *winners*. Let $\Phi\colon V \to C$ be an assignment of candidates to voters witnessing that $W \in \mathcal{R}(tE, k)$. By Monroe's criterion, for each $w \in W$ we have $\|\Phi^{-1}(w)\| = \frac{nt}{k}$. We now proceed as follows. First, we show how to transform Φ into an assignment Φ' such that (a) under Φ' each winner represents exactly $\frac{n}{k}$ voters in each copy of V and (b) the satisfaction of the voters under Φ' is the same as under Φ. We then prove that Φ' can be further transformed into Φ'' that uses the same assignment for each copy of V.

Let $n = \|V\|$. Let $V_1, \ldots, V_t$ be t copies of V so that $tV = V_1 + \ldots + V_t$; we assume that within each V_i, $i \in [t]$, voters are listed in the same order. For each $i \in [t]$, $\ell \in [n]$, we write $v_{i,\ell}$ to denote the ℓ-th voter in V_i. For each subsequence $V' = (v_{1,i_1}, \ldots, v_{1,i_p})$ of voters from V_1, we define the *closure* of V' to be $\overline{V'} = \{v_{1,i_1}, \ldots v_{1,i_p}, v_{2,i_1}, \ldots v_{2,i_p}, \ldots, v_{t,i_1}, \ldots, v_{t,i_p}\}$. For each sequence U of voters and each subset W' of the winners, we define $U(W')$ to be the subsequence of the voters from U that are represented by the members of W' under Φ; let $\texttt{repr}(U)$ denote the set of winners that represent the voters from U.

We now show how to transform Φ so that it assigns each winner to exactly $\frac{n}{k}$ voters from V_1. Let W_+ be the set of winners who, under Φ, represent more than $\frac{n}{k}$ different voters from V_1. Similarly, let W_0 and W_- be the sets of winners who, respectively, represent exactly $\frac{n}{k}$ and less than $\frac{n}{k}$ different voters from V_1. Naturally, if $W_+ = \emptyset$ we can take $\Phi^{(1)} = \Phi$. Thus, assume that $W_+ \neq \emptyset$. We define a swap operation as follows: for two voters, $v_{1,\ell}$ and $v_{i,\ell}$, $i \in [t]$, $\ell \in [n]$, $\texttt{swap}(v_{1,\ell}, v_{i,\ell})$ modifies Φ by assigning the representative of $v_{i,\ell}$ to $v_{1,\ell}$ and vice versa. We claim that Algorithm 1 finds a winner $w_+ \in W_+$, a winner w_- in W_-, and a sequence of swaps after which (a) the number of voters in V_1 represented by w_+ decreases by one, (b) the number of voters in V_1 represented by w_- increases by one, and (c) for each winner w in $W \setminus \{w_+, w_-\}$, the number of voters in V_1 represented by w does not change. Clearly, swap operations do not break the Monroe condition and do not change the satisfaction of the voters from the assignment. Thus, if our claim is correct, after executing Algorithm 1 sufficiently many times (and recomputing the sets W_+, W_-, and W_0 before each run), we transform Φ so that each winner in W represents exactly $\frac{n}{k}$ voters in V_1. (We omit a detailed analysis of Algorithm 1 due to space constraints.) $\square$

Algorithm 1: The algorithm performing the swaps used in the proof of Proposition 17

$W' \leftarrow W_+$;
Considered $\leftarrow W_+$;
Subs $\leftarrow []$;
$w \leftarrow -1$;
while *true* **do**
    $U \leftarrow \overline{V_1(W')}$;
    Subs.push_back(U) ;
    if $\texttt{repr}(U) \cap W_- \neq \emptyset$ **then**
        $w \leftarrow$ any winner from $(\texttt{repr}(U) \cap W_-)$;
        break;
    else
        $W' \leftarrow \texttt{repr}(U) \setminus$ *Considered* ;
        Considered $\leftarrow$ *Considered* $\cup W'$;
while *Subs.nonempty()* **do**
    $U \leftarrow$ *Subs*.pop() ;
    $v_{i,\ell} \leftarrow$ any voter from U, with $i > 1$, represented by w ;
    $w \leftarrow$ representative of $v_{1,\ell}$;
    $\texttt{swap}(v_{i,\ell}, v_{1,\ell})$;

PROPOSITION 18. *Greedy-Monroe fails homogeneity even if the size of the committee divides the number of voters.*

The above proposition relies heavitly on parallel-universes tie-breaking. It is possible to refine the intermediate tie-breaking procedure of Greedy-Monroe so that it becomes homogeneous when k divides $\|V\|$. We omit the details here.

9. CONCLUSIONS

We have put forward a framework for studying multiwinner rules and considered a number of their properties. We believe that our results give a better understanding of applicability of various multiwinner rules to particular tasks. For example, we see that best-k rules are well-suited for picking a group of finalists in a competition, whereas rules based on the Monroe criterion (ℓ_1-Monroe, $\ell_{\min}$-Monroe, and Greedy-Monroe), as well as STV, seem to be more appropriate for applications that require proportional representation (e.g., parliamentary elections). In this context, Greedy-Monroe is particularly interesting. It was derived as an approximation algorithm for ℓ_1-Monroe [22], but it has more appealing properties than the original rule. We believe that Greedy-Monroe should be taken as a full-fledged voting rule.

Our results for ℓ_1-CC and $\ell_{\min}$-CC are similar to those for Monroe, but intuitively these rules are better suited for applications such as movie selection (see the introduction) than, say, parliamentary elections. The reason is that they may assign very different numbers of voters to each winning candidate (naturally, one could imagine rules for parliamentary elections where voters would be represented by more than a single person—and thus different winning candidates might represent different numbers of voters—but ℓ_1-CC and $\ell_{\min}$-CC do not operate on such basis). It is disappointing that Greedy-CC, which was designed as an approximation algorithm for ℓ_1-CC, does not seem to perform very well.

Finally, it is interesting that SNTV (which, in essence, is the multiwinner variant of the Plurality rule) satisfies all the properties that we defined (though it only satisfies unanimity in the weak sense). Yet one should not forget that SNTV shares the negative features of the Plurality rule (it ignores most of the information regarding voters' preferences).

Acknowledgements. We would like to thank the reviewers for an extremely useful set of comments. This work was supported in part by NCN grants 2012/06/M/ST1/00358 and 2011/03/B/ST6/01393, and by the Polish Ministry of Science and Higher Education (under AGH University Grant 11.11.230.015 (statutory project)).

10. REFERENCES

[1] S. Barberà and D. Coelho. How to choose a non-controversial list with k names. *Social Choice and Welfare*, 31(1):79–96, 2008.

[2] N. Betzler, A. Slinko, and J. Uhlmann. On the computation of fully proportional representation. *Journal of AI Research*, 47:475–519, 2013.

[3] S. Brams and P. Fishburn. Voting procedures. In K. Arrow, A. Sen, and K. Suzumura, editors, *Handbook of Social Choice and Welfare, Volume 1*, pages 173–236. Elsevier, 2002.

[4] S. Brams, M. Kilgour, and R. Sanver. A minimax procedure for electing committees. *Public Choice*, 132(3-4):401–420, 2007.

[5] F. Brandt. Some remarks on Dodgson's voting rule. *Mathematical Logic Quarterly*, 55(4):460–463, 2009.

[6] I. Caragiannis, C. Kaklamanis, N. Karanikolas, and A. Procaccia. Socially desirable approximations for Dodgson's voting rule. In *Proceedings of ACM EC-10*, pages 253–262, June 2010.

[7] I. Caragiannis, D. Kalaitzis, and E. Markakis. Approximation algorithms and mechanism design for minimax approval voting. In *Proceedings of AAAI-10*, pages 737–742, July 2010.

[8] B. Chamberlin and P. Courant. Representative deliberations and representative decisions: Proportional representation and the Borda rule. *American Political Science Review*, 77(3):718–733, 1983.

[9] V. Conitzer, M. Rognlie, and L. Xia. Preference functions that score rankings and maximum likelihood estimation. In *Proceedings of IJCAI-09*, pages 109–115, July 2009.

[10] B. Debord. An axiomatic characterization of Borda's k-choice function. *Social Choice and Welfare*, 9(4):337–343, 1992.

[11] M. Dummett. *Voting Procedures*. Oxford University Press, 1984.

[12] C. Dwork, R. Kumar, M. Naor, and D. Sivakumar. Rank aggregation methods for the web. In *Proceedings of the 10th International World Wide Web Conference*, pages 613–622, March 2001.

[13] E. Elkind, J. Lang, and A. Saffidine. Choosing collectively optimal sets of alternatives based on the condorcet criterion. In *Proceedings of IJCAI-11*, pages 186–191, July 2011.

[14] P. Fishburn. An analysis of simple voting systems for electing committees. *SIAM Journal on Applied Mathematics*, 41(3):499–502, 1981.

[15] W. Gehrlein. The condorcet criterion and committee selection. *Mathematical Social Sciences*, 10(3):199–209, 1985.

[16] J. Kemeny. Mathematics without numbers. *Daedalus*, 88:577–591, 1959.

[17] T. Lu and C. Boutilier. Budgeted social choice: From consensus to personalized decision making. In *Proceedings of IJCAI-11*, pages 280–286, 2011.

[18] B. Monroe. Fully proportional representation. *American Political Science Review*, 89(4):925–940, 1995.

[19] A. Procaccia, J. Rosenschein, and A. Zohar. On the complexity of achieving proportional representation. *Social Choice and Welfare*, 30(3):353–362, 2008.

[20] T. Ratliff. Some startling inconsistencies when electing committees. *Social Choice and Welfare*, 21(3):433–454, 2003.

[21] R. Sanver and W. Zwicker. Monotonicity properties and their adaptation to irresolute social choice rules. *Social Choice and Welfare*, 39:371–398, 2012.

[22] P. Skowron, P. Faliszewski, and A. Slinko. Fully proportional representation as resource allocation: Approximability results. In *Proceedings of IJCAI-13*, pages 353–359, 2013.

[23] N. Tideman and D. Richardson. Better voting methods through technology: The refinement-manageability trade-off in the Single Transferable Vote. *Public Choice*, 103(1–2):13–34, 2000.

[24] D. Woodall. Properties of preferential election rules. *Voting Matters*, 3:Paper 4, 1994.

[25] H. Young. Social choice scoring functions. *SIAM Journal on Applied Mathematics*, 28(4):824–838, 1975.