The Complexity of Bribery and Control in Group Identification

Gábor Erdélyi
University of Siegen
erdelyi@wiwi.uni-siegen.de

Christian Reger
University of Siegen
reger@wiwi.uni-siegen.de

Yongjie Yang
Saarland University
yyongjie@mmci.uni-saarland.de

ABSTRACT

The goal of this paper is to analyze the complexity of bribery and destructive control in the framework of group identification. Group identification applies to situations where a group of individuals try to determine who among them are socially qualified for a given task. We consider consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule.

Keywords

computational social choice; algorithms; group identification; bribery; control; complexity

1. INTRODUCTION

There are many real-world situations, where a group of agents try to find a subgroup best qualified for a given task. For example, faculties elect faculty members for hiring committees best qualified in a special field, or a set of agents need to complete a task, but only some of them are capable doing it. In such settings multi-winner voting rules come to use.

We focus on the setting, where (1) the set of voters and the set of candidates coincide (from now on we call them *individuals*) and (2) the elected subgroup has no fixed size. This setting is called *group identification*. Which subgroup is actually elected, depends on each individual's valuation of both himself and all other agents. In our model, we are given an individual set N and each individual $a \in N$ either qualifies or disqualifies each individual $a' \in N$ (including a). As an example, regard an institution where a certain task must be assigned to some of its members. Each group member has individual abilities and estimates for all members (including themselves) if they are suitable for the task or not. Each individual votes honestly in our setting. Depending on each agent's valuations, a *social rule* is applied determining a subset of N which we refer to as *socially qualified* individuals. Note that the set of socially qualified individuals can be any subset of N.

In this work, we will focus on different ways how an external agent can influence the set of socially qualified individuals either by altering some individuals' valuations (bribery) or by adding, deleting or partitioning the set of individuals (control).

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1.1 Related Work

Our work fits in the line of research on the computational aspects of strategic behavior in elections initiated by a series of papers by Bartholdi et al. [2, 3, 4], proposing that computational hardness offers a (worst-case) protection against manipulative attacks. We refer to the paper by Faliszewski, Hemaspaandra, and Hemaspaandra introducing bribery to elections [12]. Electoral control was introduced in the constructive variant in the seminal paper by Bartholdi, Tovey, and Trick [4] and in the destructive variant by Hemaspaandra, Hemaspaandra, and Rothe [17]. A variant of destructive bribery has been introduced in the context of margin of victory, however, this model is not directly related to our models [26]. These problems have also been studied in the context of judgment aggregation [5], however, this is a very different setting compared to ours.

Settings where not just a winner, but a small set of candidates is selected from the set of candidates is called a multi-winner election. Elkind et al. proposed and studied the properties of multi-winner voting rules based on single-winner scoring rules [9]. There are many different multi-winner voting rules based on single-winner voting rules in the literature that we do not use in our work [10, 13, 15, 23]. Meir et al. initiated the complexity theoretic analysis of strategic behavior in multi-winner elections, where they have investigated the complexity of manipulation and control under some prominent voting rules [20]. Obraztsova, Zick, and Elkind investigated the complexity of manipulation in multi-winner scoring rules with a special focus on tie-breaking [22]. Furthermore, it is worth mentioning the work by Aziz et al. on the computational aspects of best responses in multi-winner approval voting [1].

In contrast, we are considering group identification, which differs from bribery and control in voting theory in several issues. In our model the sets of voters and candidates coincide and are called *individual set*. Each individual thus votes on every other individual and himself. Furthermore, in our setting agents are neither strategical nor selfish. Finally, in contrast to voting rules, in group identification we do not want to select winners, but provide quantitative criteria to decide if an individual is appropriate for a given task or not. Note that these criteria are fixed (e.g., for consent rules, two parameters uniquely determine these criteria) and can produce arbitrary numbers of socially qualified individuals while in multi-winner elections we are interested in electing a committee of fixed size. Group identification, in particular the liberal rule, the consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule, have been introduced and further studied from an economic point of view in [18, 7, 8, 21, 24]. In contrast, we are investigating the computational aspects of strategic behavior in group identification. Constructive control by adding, deleting and partitioning individuals for group identification have been first introduced and studied by Yang and Dimitrov [28]. Here, we are extending their work to destructive control and to constructive and destructive bribery. To the best of our knowledge, bribery has not been studied for group identification up to now.

All social rules in this paper are approval-based in a sense that each individual approves (qualifies) or disapproves (disqualifies) every other individual. As opposed to approval voting [6], we do not count the number of approvals and let the individuals with the highest number of approvals win, but apply a social rule given the approval assignment. The last related work to mention is the work by Kilgour, Brams, and Sanver on electing representative committees via approval ballotting [19]. Again, they are looking for a fixed committee size. Furthermore, they are not considering the complexity of manipulative actions in their model.

Organization.

In Section 2, we will give a survey about the basics in group identification and the social rules we will consider in this paper. In Section 3, we will introduce the underlying problems. Section 4 is about constructive bribery whereas Section 5 deals with destructive misuses on groups. Section 6 concludes the paper.

2. PRELIMINARIES

Let $N := \{a_1, \dots, a_n\}$ be a set of $n \in \mathbb{N}$ *individuals*. Let $\mathcal{P}(N)$ be the collection of all nonempty subsets of N. For our purposes, we will mostly use a and a' for individuals in N throughout this paper. We will sometimes say *agents* instead of individuals. A *profile* over N is a function $\varphi : N \times N \to \{0, 1\}$. We say that individual $a \in N$ *qualifies* $a' \in N$ if $\varphi(a, a') = 1$ and *disqualifies* a' if $\varphi(a, a') = 0$. The mapping φ induces a matrix $(\varphi) \in \{0, 1\}^{n \times n}$ where $\varphi_{ij} := \varphi(a_i, a_j)$. A *social rule* is defined as a function $f : (\varphi, N) \to \mathcal{P}(N)$, i.e., it selects some individuals in N that are said to be *socially qualified* with respect to f and φ. Note that we can restrict f and φ to each subset $T \subseteq N$ by replacing N by T in the definition of a social rule. In the following, we will define the social rules considered in this paper. We can roughly divide them into *consent rules* and *procedural rules*. The former class is specified by two parameters and is directly applied to an instance (N, φ). The latter rules iteratively amplify the set of socially qualified individuals and stop as soon as the socially qualified individuals do not change anymore.

- **Consent Rule** ($f^{(s,t)}$): Each consent rule is specified by two parameters $s, t \in \mathbb{N}$ such that for an individual set N and individual $a \in N$:
  - If $\varphi(a, a) = 1$, then $a \in f^{(s,t)}(\varphi, N)$ if and only if $|\{a' \in N : \varphi(a', a) = 1\}| \geq s$.
  - If $\varphi(a, a) = 0$, then $a \notin f^{(s,t)}(\varphi, N)$ if and only if $|\{a' \in N : \varphi(a', a) = 0\}| \geq t$.

  The two parameters s and t are called *consent quotas*. For $s = t = 1$, we obtain the **Liberal Rule** f^L where an individual is socially qualified if and only if he qualifies himself, i.e., we have $f^L(\varphi, N) = \{a \in N : \varphi(a, a) = 1\}$. The Liberal Rule is the only consent rule with the property that it only depends on a's self-evaluation and is independent on the others' valuations whether a is socially qualified or not. Note that for our complexity analysis, s and t are always constant and do particularly not depend on the number of individuals.

- **Consensus-Start-Respecting Rule** (f^{CSR}). This rule is defined recursively by first determining a starting set of socially qualified individuals and then iteratively extending the set of socially qualified individuals until there is no change anymore. Formally, f^{CSR} is defined as follows. Let $K_0^C(\varphi, N)$ be the set of initially qualified alternatives defined as:

  $$K_0^C(\varphi, N) := \{a \in N : \forall a' \in N : \varphi(a', a) = 1\}.$$

  Then we successively compute for nonnegative integers i:

  $$K_i^C(\varphi, N) =$$
  $$\{a \in N : \exists a' \in K_{i-1}^C(\varphi, N) : \varphi(a', a) = 1\} \cup K_{i-1}^C(\varphi, N).$$

  We obtain $f^{CSR}(\varphi, N) = K_i^C(\varphi, N)$ for some i with $K_i^C(\varphi, N) = K_{i+1}^C(\varphi, N)$.

- **Liberal-Start-Respecting Rule** (f^{LSR}). Again, we have a social rule iteratively defined as follows:

  $$K_0^L(\varphi, N) := \{a \in N : \varphi(a, a) = 1\}.$$

  The remaining iterations are computed as for f^{CSR}, i.e., via

  $$K_i^L(\varphi, N) = \{a \in N : \exists a' \in K_{i-1}^L(\varphi, N) : \varphi(a', a) = 1\}$$
  $$\cup K_{i-1}^L(\varphi, N) \ (i \in \mathbb{N}).$$

  Likewise, we obtain $f^{LSR}(\varphi, N) = K_i^L(\varphi, N)$ for some i with $K_i^L(\varphi, N) = K_{i+1}^L(\varphi, N)$.

Note that the starting set of f^{LSR} includes all individuals that qualify themselves whereas f^{CSR} contains only individuals qualified by each individual. In particular, we have $K_0^L(\varphi, N) \supseteq K_0^C(\varphi, N)$ for an instance (φ, N). When N and φ are clear from the context, we will write K_0^L and K_0^C instead of $K_0^L(\varphi, N)$ and $K_0^C(\varphi, N)$, respectively.

3. PROBLEM SETTINGS

In this section we introduce the formal definitions of the problems considered in this paper. In *control* an external agent—called the chair—makes some structural changes to the election in order to tamper with the outcome of it. Constructive control by adding, deleting, and partitioning individuals in the context of group identification was introduced and studied in [28]. We will extend their work by defining the destructive versions of these problems. In the destructive version of adding individuals the chair's goal is to socially disqualify a distinguished set of agents by adding some individuals to the election.

Destructive Group Control by Adding Individuals	
Given:	A 6-tuple $(f, N, \varphi, S, T, \ell)$ of a social rule f, a set N of individuals, a profile φ over N, two nonempty subsets S and T, such that $S \subseteq T \subseteq N$ and $S \cap f(\varphi, T) \neq \emptyset$, and a positive integer ℓ.
Question:	Is there a subset $U \subseteq N \setminus T$ such that $\|U\| \leq \ell$ and $S \cap f(\varphi, T \cup U) = \emptyset$?

We will write f-DGCAI, for short. In the destructive version of deleting individuals the chair is removing some individuals from the election in order to socially disqualify a distinguished set of agents. Note that the chair is not allowed to remove agents from the distinguished set.

Destructive Group Control by Deleting Individuals	
Given:	A 5-tuple (f, N, φ, S, ℓ) of a social rule f, a set N of individuals, a profile φ over N, a nonempty subset $S \subseteq N$ such that $S \cap f(\varphi, N) \neq \emptyset$, and a positive integer ℓ.
Question:	Is there a subset $U \subseteq N \setminus S$ such that $\|U\| \leq \ell$ and $S \cap f(\varphi, N \setminus U) = \emptyset$?

We will write f-DGCDI, for short. Our final destructive control model is partitioning the set of individuals.

Destructive Group Control by Partition of Individuals	
Given:	A 4-tuple (f, N, φ, S) of a social rule f, a set N of individuals, a profile φ over N, a nonempty subset $S \subseteq N$ such that $S \cap f(\varphi, N) \neq \emptyset$.
Question:	Is there a subset $U \subseteq N$ such that $S \cap f(\varphi, V) = \emptyset$ with $V = f(\varphi, U) \cup f(\varphi, N \setminus U)$?

We will write f-DGCPI, for short. In contrast to control, in *bribery*, an external agent is allowed to change some individuals' opinions over N in a way he desires.

Constructive Group Bribery (CGB)	
Given:	A 5-tuple (f, N, φ, S, ℓ) of a social rule f, a set N of individuals, a profile φ over N, a nonempty subset $S \subseteq N$ with $S \not\subseteq f(\varphi, N)$, and a positive integer ℓ.
Question:	Is there a way to change at most ℓ rows of the matrix φ such that $S \subseteq f(\varphi', N)$ where $\varphi' \in \{0,1\}^{n \times n}$ is the resulting new profile?

We will write f-CGB, for short. By replacing $S \not\subseteq f(\varphi, N)$ by $S \cap f(\varphi, N) \neq \emptyset$ and $S \subseteq f(\varphi', N)$ by $S \cap f(\varphi', N) = \emptyset$, we obtain the definition of Destructive Group Bribery (f-DGB, for short) which asks whether the briber can change ℓ individuals' valuations in a way that all individuals in subgroup S are socially unqualified after the bribery.

We say that a social rule f is *immune* to a constructive (destructive) problem defined above if it is impossible to make every individual in S socially qualified (not socially qualified) by performing the corresponding manipulative action, i.e., the problem has only NO-instances. Otherwise, the social rule f is said to be *susceptible* to the problem.

Note that in case of NP-completeness results we will only show NP-hardness. It is easy to see that all our problems are in NP.

4. CONSTRUCTIVE GROUP BRIBERY

Throughout this section, we let (f, N, φ, S, ℓ) denote a bribery instance where N is the set of individuals, φ the profile, $S \subseteq N$ the set of distinguished individuals the briber wants to make socially qualified, and ℓ be a nonnegative integer representing the maximum number of individuals the briber may bribe. φ' denotes the profile after the bribery. Our results on constructive group bribery are summarized in Table 1. Our first result indicates that bribery in consent rules is easy given $t = 1$.

Theorem 4.1. *$f^{(s,1)}$-CGB is in P for all constants s.*

Proof. Given $(f^{(s,1)}, N, \varphi, S, \ell)$, first we compute $S_0 := \{a \in S : \varphi(a, a) = 0\}$ and $S_1 := S \setminus S_0$. Our algorithm checks the following cases:

- $\ell < |S_0|$. Then our instance is a NO instance as there is an $a \in S$ disqualifying himself after the bribery and consequently $a \notin f(\varphi', N)$ after the bribery.
- $\ell \geq |S_0|$. In this case, the briber bribes all agents in S_0 and makes all of them qualify all agents (at least they all must qualify all agents in S). For $s = 1$, our instance is a YES instance because each agent in S qualifies himself and this is sufficient for S being socially qualified. For $s > 1$, we must ensure that each agent qualifying himself has s qualifications in total. As there are still $\ell - |S_0|$ bribes left and $S = S_1$ currently holds (i.e., each agent in S qualifies himself), we may w.l.o.g. restrict ourselves to a (transformed) bribery problem with $S_0 = \emptyset$ (where all already bribed agents qualify all agents). There are still two cases left to differentiate between:
  - $\ell \geq s$. Then the briber bribes arbitrary s individuals and makes them all qualify all individuals. Thus – due to our assumption $\varphi(a, a) = 1\ \forall a \in S$ – it holds $S \subseteq f(\varphi', N)$ after the bribery as each $a \in S$ has s qualifications in total.
  - $\ell < s$. Now ℓ is bounded from above by the constant s. It suffices to check for all $\binom{|N|}{\ell}$ ways to bribe ℓ agents (there are $O(|N|^{s-1})$ such possibilities to check) if all individuals in S are qualified provided that the briber makes each bribed individual qualify all individuals (in S).

❑

In contrast, we cannot preserve easiness by fixing $s = 1$ and permitting t to be an arbitrary nonnegative integer. We will show hardness via reduction from the NP-complete problem Restricted Exact Cover By 3-Sets [16].

Restricted Exact Cover By 3-Sets (RX3C)	
Given:	A set $B = \{b_1, \ldots, b_{3m}\}$ and a collection $\mathcal{S} = \{S_1, \ldots, S_n\}$ of 3-element subsets of B such that each $b_j \in B$ occurs in exactly three subsets $S_i \in \mathcal{S}$.
Question:	Does $\mathcal{S}$ contain an exact cover for B (i.e., a subcollection $\mathcal{S}' \subseteq \mathcal{S}$ such that every element of B occurs in exactly one member of $\mathcal{S}'$)?

Theorem 4.2. *$f^{(s,t)}$-CGB is NP-complete for each $s \geq 1$ and $t \geq 3$.*

f	$f^{(s,1)}$	$f^{(s,2)}$	$f^{(s,t)}$, where $s \geq 1$ and $t \geq 3$	f^{CSR}	f^{LSR}
f-CGB	P (Theorem 4.1)		NPC (Theorem 4.2)	P (Theorem 4.3)	

Table 1: Results for constructive group bribery. Here, "P" stands for "polynomial-time solvable" and "NPC" for "NP-complete".

Proof. We will prove our theorem for $s = 1$ and $t = 3$ and argue later how to adjust the proof for higher values s and t. Given an RX3C instance by a pair $(B, \mathcal{S})$ defined as in the RX3C definition, we will construct the following $f^{(1,3)}$-CGB instance $(f^{(1,3)}, N, \varphi, S, \ell)$ with distinguished subgroup $S = B$, bribery limit $\ell = n - m = 2m$ and individuals $N = B \cup \mathcal{S}$. The profile φ is defined as follows:

- Each $b_j \in B$ disqualifies himself and qualifies all other individuals.
- S_i disqualifies b_j if and only if $b_j \in S_i$.

Each element in B has four disqualifications in total. In order to be socially qualified, an individual in B must either qualify himself or have at most two disqualifications after the bribery, i.e., (at least) two disqualifications must fall away by the bribery. Obviously, a bribed individual qualifies all individuals including himself (at least the individuals in S). We claim that there is an exact cover of B if and only if there exists a successful bribery.

($\Rightarrow$:) Assume that $\mathcal{S}' \subseteq \mathcal{S}$ is an exact cover of B. Then the briber bribes all individuals in $\mathcal{S} \setminus \mathcal{S}'$ such that they qualify all individuals. Thus one disqualification remains for each b and consequently each b has 2 disqualifications in the end and so $S \subseteq f^{(1,3)}(\varphi', N)$.

($\Leftarrow$:) Now assume that there is a successful bribery. First we show that the briber could not bribe any individuals in B. Second, we prove that a successful bribery in $\mathcal{S}$ requires the existence of an exact cover of B.

Suppose that the briber bribes α individuals in B and $2m - \alpha$ individuals in $\mathcal{S}$. Clearly, as mentioned above, each bribed individual qualifies all individuals after the bribery. The b_j bribed are socially qualified (as they qualify themselves after the bribery) and can be ignored for the remaining bribes. To successfully bribe, the briber must take away at least two disapprovals from each remaining $b \in B$, i.e., $2(3m - \alpha)$ disapprovals in total. As there are only $2m - \alpha$ bribes left and three disapprovals are met by each bribe (i.e., $3(2m - \alpha)$ in total), a necessary condition for a successful bribery is that $2(3m - \alpha) \leq 3(2m - \alpha)$, this implies that $\alpha \leq 0$ or—as α is non-negative—$\alpha = 0$, i.e., we can deduce that the briber bribes only individuals in $\mathcal{S}$.

As he must take away at least two disapprovals from all $3m$ individuals in B, and $2m$ bribes take away $2m \cdot 3 = 6m$ disapprovals in total, this implies that each $b \in B$ loses exactly two disapprovals and this in turn means that exactly one disapproval remains in the unbribed individuals in $\mathcal{S}$ for each $b \in B$. Thus the individuals in $\mathcal{S}$ not bribed form an exact cover of B.

For $t \geq 4$, we set $\ell = 2m + (t-3)$ and add $t-3$ dummy individuals disqualifying each individual. These dummy individuals are bribed with the highest priority. The remainder of the proof remains identical.

For $s \geq 2$, the same reasoning as for $s = 1$ can be used: s as a parameter is only relevant when $b \in B$ is qualified after the bribery. As bribing b and making him qualify himself is too costly given our construction (i.e., bribing only one $b \in B$ makes it impossible that all other individuals in B lose their disqualifications), for $s \geq 2$, our focus lies on making each element in B falling below the threshold t, too. ❑

In contrast, constructive group bribery is easy for procedural rules.

THEOREM 4.3. *Both* f^{LSR}-CGB *and* f^{CSR}-CGB *are solvable in polynomial time.*

Proof. For f^{LSR}, our instance is always a YES instance: The briber simply bribes an arbitrary agent $\hat{a}$ and makes $\hat{a}$ qualify himself and all individuals in S. This ensures that $\hat{a}$ is in the starting set and each individual $a \in S$ is socially qualified, as a is qualified by $\hat{a}$, i.e., we have $K_0^L(\varphi', N) \supseteq \{\hat{a}\}$, $K_1^L(\varphi', N) \supseteq \{\hat{a}\} \cup S$.

Given f^{CSR}, we first check if $K_0^C \neq \emptyset$ (which requires $O(|N|^2)$ time). If yes, we bribe any $a \in K_0^C$ and make a qualify all agents including himself. This makes all individuals (and in particular the agents in S) socially qualified. If no, we distinguish the following cases:

- Let $y(a)$ be the number of individuals a' with $\varphi(a', a) = 1$. If $\max_{a \in N} y(a) + \ell < |N|$, there is no way to make any $a \in S$ socially qualified as even the individual with the highest number of qualifications in the original election cannot reach K_0^C after the bribery (even if all bribed agents disqualify this individual before and qualify him after the bribery). As $K_0^C = \emptyset$ after the bribery, there is no way for the briber to reach his goal since due to the definition of the f^{CSR} rule, $K_0^C = K_i^C = \emptyset$ for all positive integers i holds.
- For $\max_{a \in N} y(a) + \ell > |N|$, the briber chooses an individual $\hat{a}$ with $y(\hat{a}) = \max_{a \in N} y(a)$, bribes $\hat{a}$ and all remaining individuals initially not qualifying $\hat{a}$ and makes each bribed individual qualify w.l.o.g. all individuals in N. This makes all individuals in S socially qualified.
- If $\max_{a \in N} y(a) + \ell = |N|$ (i.e., an individual with the maximum number of qualifications can barely reach the starting set K_0^C after the bribery), we check for each $\hat{a}$ maximizing this expression (there may be more than one maximum individual) if making $\hat{a}$ reach K_0^C implies that all individuals in S belong to some K_i^C ($i \in \mathbb{N}_0$). The briber makes each bribed individual qualify all individuals. As the individuals to bribe are fixed, our problem reduces to evaluating the f^{CSR} rule. This subcase requires at most $O(|N|^4)$ time: We bribe the remaining agents initially not qualifying a (quadratic time) by filling the corresponding rows of the matrix φ with ones, then we check by computing the socially qualified individuals for f^{CSR} if all individuals in S are socially qualified (f^{CSR} can clearly be valuated in cubic time in $|N|$).

❑

Note that the complexity of $f^{(s,2)}$-CGB is still open. One can easily verify that the problem is easy unless $s < \ell < |S_0|$ with $S_0 = \{a \in S : \varphi(a,a) = 0\}$, i.e., to show hardness one needed to regard this open subcase.

5. DESTRUCTIVE GROUP ACTIONS

In this section, we are dealing with destructive influences on group identification, i.e., the briber or the chair seeks to prevent all individuals in a certain group $S \subseteq N$ from being socially qualified. Our results are summarized in Table 2. The first result gives a connection between the constructive and destructive variants of our problems for consent rules. We point out that Theorems 5.1 and 5.2 only hold for constant s and t.

THEOREM 5.1. *$f^{(s,t)}(\varphi, N) = N \setminus f^{(t,s)}(-\varphi, N)$, where $-\varphi$ is obtained from φ by reversing the values (i.e., $\varphi(a,b) = 1$ if and only if $-\varphi(a,b) = 0$).*

Proof. If there is an $a \in N$ such that $\varphi(a,a) = 1$, then there are at least s individuals qualifying a with respect to φ. Hence, there are at least s individuals disqualifying a with respect to $-\varphi$. As $-\varphi(a,a) = 0$, $a \notin f^{(t,s)}(-\varphi, N)$. We can show in a similar manner that in the case of $\varphi(a,a) = 0$, it still holds that $a \notin f^{(t,s)}(-\varphi, N)$. Moreover, with similar argument, we can show that if $a \notin f^{(t,s)}(-\varphi, N)$, then $a \in f^{(s,t)}(\varphi, N)$. ❑

As a direct consequence of Theorem 5.1, we can reduce constructive group bribery/control problems to destructive group bribery/control problems. As an example, if the chair tries to make a socially qualified individual a by adding some individuals for the $f^{(10,4)}$ rule, he faces exactly the same problem as for the case where he wants to prevent a from being socially qualified for the $f^{(4,10)}$ rule and reversed φ function. Let AI, DI, and PI denote adding individuals, deleting individuals, and partitioning of individuals, respectively. In general, the following link between destructive and constructive group control/bribery holds.

THEOREM 5.2. *Suppose that $X \in \{AI, DI, PI\}$. Then*

$$\begin{pmatrix} f^{(s,t)}\text{-CGCX} \\ f^{(s,t)}\text{-CGB} \end{pmatrix} \textit{ is } \begin{pmatrix} \textit{in P} \\ \textit{NP-complete} \\ \textit{immune} \end{pmatrix} \textit{ if and only if}$$

$$\begin{pmatrix} f^{(t,s)}\text{-DGCX} \\ f^{(t,s)}\text{-DGB} \end{pmatrix} \textit{ is } \begin{pmatrix} \textit{in P} \\ \textit{NP-complete} \\ \textit{immune} \end{pmatrix}.$$

For the corresponding results, we refer to Table 2, the results are displayed in italic. Note that for $s = t$, the complexities for the constructive and destructive variants coincide. Theorems 5.1 and 5.2 do not apply to f^{CSR} and f^{LSR} as we will see in the following section.

5.1 Destructive Group Control

Our first result shows that adding individuals in the destructive case for the consensus-start-respecting rule is computationally hard. We will prove the theorem via reduction from the NP-complete problem EXACT COVER BY 3-SETS (X3C), a generalization of RX3C, where there is no restriction regarding in how many subsets each $b_j \in B$ occurs [14].

THEOREM 5.3. *f^{CSR}-DGCAI is NP-complete.*

Proof. Given an X3C instance $(B, \mathcal{S})$, we construct an instance $(f^{CSR}, N, \varphi, S, T, \ell)$ of f^{CSR}-DGCAI with $N = \mathcal{S} \cup B$, $T = S = B$, $\ell = m$ and φ defined as follows:

- For all $r, j \in \{1, \ldots, 3m\}$, let $\varphi(b_r, b_j) = 1$.
- For all $i, h \in \{1, \ldots, n\}$, let $\varphi(S_i, S_h) = 0$.
- If $b_j \in S_i$, for all $1 \le i \le n$ and $1 \le j \le 3m$, let $\varphi(S_i, b_j) = 0$, otherwise let $\varphi(S_i, b_j) = 1$.

The values $\varphi(b_j, S_i)$ may be arbitrary. Note that each $b \in B$ is qualified by each element in B. Thus all individuals in B are qualified at the beginning. We claim that an exact cover of B exists if and only if there is a successful control.

($\Rightarrow$:) Suppose that there is an exact cover for B, i.e., there is a subset $\mathcal{S}' \subseteq \mathcal{S}$ so that each $b \in B$ is in precisely one $S_i \in \mathcal{S}'$. By adding the individuals in $\mathcal{S}'$, we achieve $K_0^C = \emptyset$ and thus $f(\varphi, B \cup \mathcal{S}') = \emptyset$. Each added S_i disqualifies itself and is thus not qualified by each individual. The same holds for each $b \in B$ as we have added an S_i according to the exact cover with $\varphi(S_i, b) = 0$.

($\Leftarrow$:) Suppose that each individual $b \in B$ can be prevented from being socially qualified. As all individuals in B qualify themselves and $K_0^C = B$ holds in the original situation, we must add a subset S_i with $\varphi(S_i, b) = 0$ to throw b out of the starting set. As $3m$ individuals require at least one disqualification from added individuals, and m added individuals mean $3m$ disqualifications, this in turn implies that all $b \in B$ are disqualified by exactly one S_i added. Consequently the added S_i form an exact cover of B. ❑

The following theorem says that group control by adding individuals in f^{LSR} is never possible.

THEOREM 5.4. *f^{LSR} is immune to destructive group control by adding individuals.*

Proof. First suppose that there is an $a \in S$ with $\varphi(a,a) = 1$. Then $a \in K_i^L$, for each nonnegative integer i, and this can obviously not be changed by adding additional individuals. Thus the only chance for the chair to reach his goal at all is when $\varphi(a,a) = 0$ holds for all $a \in S$ at the beginning. $a \in f^{LSR}(\varphi, N)$ implies that there are individuals $a_{i_0}, a_{i_1}, \ldots, a_{i_r}$ where r is a nonnegative integer, $a_{i_0} \in K_0^L$, $a_{i_j} \in N$ for $0 \le j \le r$, $\varphi(a_{i_j}, a_{i_{j+1}}) = 1$ for $0 \le j \le r-1$, and $\varphi(a_{i_r}, a) = 1$, i.e., there is a sequence of individuals qualifying the respective successors with starting point in K_0^L and endpoint in a. Such sequences exist for each $a \in S$ and do not change no matter which additional individuals are added. Thus we have immunity for this case. ❑

In contrast, destructive group control by deleting individuals in both f^{CSR} and f^{LSR} is solvable in polynomial time. In order to show membership in P, we give a reduction from our problems to the MINIMUM (u,u')-SEPARATOR problem, which is known to be in P [25].

For a digraph $G = (V(G), E(G))$ and $V' \subseteq V(G)$, let $\Gamma_G(V')$ be the set of all vertices reachable via edges from a vertex in V', i.e., for every $v \in \Gamma_G(V')$ there is a directed path from a vertex in V' to v in G. Notice that a loop on a vertex is also considered as a directed path. For two nonadjacent vertices $u, u' \in G$, an (u,u')-separator is a subset of vertices in $V(G) \setminus \{u, u'\}$ whose removal destroys all directed paths from u to u'. A minimum (u,u')-separator is

f	$f^{(s,t)}$							f^{LSR}	f^{CSR}
	$s=1$		$s=2$			$s\geq 3$			
	$t=1$	$t\geq 2$	$t=1$	$t=2$	$t\geq 3$	$t=1$	$t\geq 2$		
f- DGCAI	I	NPC	I	NPC	NPC	I	NPC	I (Theorem 5.4)	NPC (Theorem 5.3)
f- DGCDI	I	I	P	P	P	NPC	NPC	P (Theorem 5.5)	
f- DGCPI	I	I	NPC	NPC	NPC	NPC	NPC	P (Theorem 5.6)	
f- DGB	P	P				NPC	NPC	P (Theorem 5.8)	

Table 2: Results for destructive group control and destructive group bribery. Results for destructive group control for consent rules follow from Theorems 5.1 and 5.2, and the results in [28]. Results for destructive group bribery for consent rules follow from Theorem 5.1 and Table 1. In the table, "I" stands for "immune", "P" for "polynomial-time solvable" and "NPC" for "NP-complete".

a (u,u')-separator with minimum cardinality. For a subset $A \subseteq V$, merging vertices in A is the operation which (1) creates a new vertex denoted by $v(A)$; (2) for $v \in V \setminus A$ such that there is an edge from v to some vertex in A, create an edge from v to $v(A)$; (3) for $v \in V \setminus A$ such that there is an edge from some vertex in A to v, create an edge from $v(A)$ to v; and (4) remove all vertices in A and edges incident to them. With these preconsiderations, we can give the formal definition.

Minimum (u,u')-separator	
Given:	A digraph $G = (V(G), E(G))$, two designated vertices $u, u' \in V(G)$, and a non-negative integer ℓ.
Question:	Is it possible to find a (u,u')-separator with cardinality of at most $\ell \in \mathbb{N}_0$?

For each pair of vertices (u,u'), a minimum (u,u')-separator can be calculated in polynomial time [25].

Theorem 5.5. *Both f^{LSR}-DGCDI and f^{CSR}-DGCDI are in P.*

Proof. Let $(f^{LSR}, N, \varphi, S, \ell)$ be a given instance. We construct the following auxiliary digraph G. An individual $a \in N$ yields vertex $v(a)$. There is an edge from vertex $v(a)$ to vertex $v(a')$, $a, a' \in N$ if and only if a qualifies a'. Note that there may be loops, i.e., $\varphi(a,a) = 1$ yields a loop on vertex $v(a)$. Let V^{LSR} be the set of all vertices with loops on them and let $\Gamma_G(V^{LSR})$ be the set of all vertices reachable from a vertex in V^{LSR}. Then the socially qualified individuals with respect to f^{LSR} are the ones corresponding to vertices in $\Gamma_G(V^{LSR})$. Due to this observation, we can solve the instance as follows. If $v(a) \in V^{LSR}$ for $a \in S$, the given instance is a NO instance. Otherwise, merge all vertices corresponding to individuals in S into $v(S)$, create a new vertex w and an edge from w to each node in V^{LSR}. Let V' be a minimum $(w, v(S))$-separator. If $|V'| \leq \ell$, the given instance is a YES instance; otherwise, it is a NO instance. Clearly, this problem is in P as the computation of the help graph and all decisions require $O(|N|^2)$ time. Moreover, we know from [25] that computing a minimum separator can be done in polynomial time.

In the case of consensus-start-respecting rules, let $(f^{CSR}, N, \varphi, S, \ell)$ be a given instance. Again, we create an auxiliary digraph G identical to the graph in the first part of the proof. Let V^{CSR} be the set of all vertices with indegree $|N|$. Then, the socially qualified individuals with respect to f^{CSR} are the ones corresponding to vertices in $\Gamma_G(V^{CSR})$. Due to this observation, we can solve the instance as follows. If there is an $a \in S$ such that $v(a) \in V^{CSR}$, our instance is a NO instance. Otherwise we merge all vertices corresponding to individuals in S into $v(S)$, create a new vertex w and an edge from w to every vertex in V^{CSR}. Let V' be a minimum $(w, v(S))$-separator. If $|V'| > \ell$, the given instance is a NO instance. If $|V'| \leq \ell$ and $|V^{CSR}| > |V'|$, our instance is a YES instance. In fact, deleting the individuals according to vertices in V' makes all $a \in S$ socially disqualified. The reason is as follows. First, $V^{CSR} \setminus V' \neq \emptyset$. Assume for the sake of contradiction that after deleting all individuals standing for vertices in V', some $a \in S$ is still socially qualified. Due to the definition of the graph, the vertex $v(S)$ is still reachable from some vertices in a set U whose corresponding individuals are in the starting set of socially qualified individuals after deleting individuals corresponding to vertices in V'. Moreover, all individuals in $V^{CSR} \setminus V'$ qualify individuals corresponding to U. Thus there are edges from $V^{CSR} \setminus V'$ to U, and hence there are edges from w to $v(S)$ which is a contradiction. On the other hand, if $|V'| \leq \ell$ but $|V^{CSR}| \leq |V'|$, we distinguish two cases. If there is a minimum $(w, v(S))$-separator V' such that $V^{CSR} \setminus V' \neq \emptyset$, the given instance is a YES instance, as the individuals corresponding to V' is a solution. If V^{CSR} is the unique minimum $(w, v(S))$-separator, we delete all individuals according to vertices in V^{CSR}, reset $\ell := \ell - |V^{CSR}|$ and repeat the algorithm with the new instance after the deletion of the individuals. If after several repetitions only individuals in S are left, our instance is a NO instance. ❑

We now turn to the partition of individuals cases.

Theorem 5.6. *f^{LSR}-DGCPI is in P.*

Proof. We let (f^{LSR}, N, φ, S) be a f^{LSR}-DGCPI instance. First, in case there is an $a \in S$ with $\varphi(a,a) = 1$, our instance is a NO instance: $a \in \bigcup_{i \in \mathbb{N}_0} K_i^L$ and this holds for each subset of N containing a. Thus a survives – qualifying himself – both the preliminary and the final round and is thus socially qualified being in the starting set independent on the other individuals' decisions. Thence the only case worth studying is $\varphi(a,a) = 0$ for all $a \in S$. In this case, our instance is a YES instance: The partition (U, W) of N with $U = S$ and $W = N \setminus S$ ensures that neither in S survives the preliminary round (due to $K_0(\varphi, S) = \emptyset$) and thus no individual from S is socially qualified. As our problem merely reduces to the question whether there exists an $a \in S$ such that $\varphi(a,a) = 1$ (which can be checked in linear time in $|N|$), it is apparently in P. ❑

The problem f^{CSR}-DGCPI is still open, however, we can show easiness under the restriction $\varphi = \varphi^T$ (i.e., a qualifies a' if and only if a' qualifies a).

THEOREM 5.7. *f^{CSR}-DGCPI is in P if $\varphi = \varphi^T$ holds.*

Proof. We are given an instance (f^{CSR}, N, φ, S) with $\varphi = \varphi^T$. Our algorithm checks the following cases:

- If for all $a \in N$ there is an $a' \in N$ such that $\varphi(a', a) = 0$, then $K_0^C = \emptyset$ and there are no socially qualified individuals. The trivial partition $U := N$ and $W := N \setminus U = \emptyset$ does as desired.
- $K_0^C \cap S \neq \emptyset$. Then $a \in K_0^C \cap S$ is in the starting set for each subset of N, reaches the final round and is hence socially qualified. Thus we reject in this case.
- $K_0^C \cap S = \emptyset$, $K_0^C \neq \emptyset$. Thus there is some $a_0 \in K_0^C \setminus S$, i.e., a_0 is qualified by each individual in N. This implies that a_0 is in the starting set for each subgroup $N' \subseteq N$. Due to $\varphi = \varphi^T$, we have $\varphi(a_0, a) = 1$ for each $a \in N$ and especially each $a \in S$. One can easily verify that each $a_0 \in K_0^C$ must be in a partition set different from each individual in S. Otherwise a_0—socially qualified in his partition set—ties all S individuals belonging to the same partition set to the final round which makes them socially qualified, too (as a_0 is in the starting set in the final round, too). Consequently we w.l.o.g. set $S \subseteq U$ and $K_0^C \subseteq W$. Now we check if there is some $a \in S$ qualified by each $a' \in S$. If no, we may set $U = S$ and $W = N \setminus S$ and—as no S individual survives the prelim—are done and accept. If yes, we do the following. Check if there is currently some $a \in N \setminus (U \cup W)$ being qualified by each individual in $N \setminus W$. Such individuals are approved by each subgroup of $N \setminus W$ (and thus by each individual in U including all $a \in S$), reach the final round and—as φ is symmetric—each other element in U including S, too. Thus the chair must successively set these individuals into W. In each iteration, we therefore check if there is some agent not yet assigned to U or W which is qualified by each individual not in the present set W. If yes, we fix this element in W and proceed with the next iteration. If no, we are done and put all still unassigned individuals (if any) in U. For these individuals, we know that they are not qualified by all individuals in $N \setminus W = U$. Thus they all do not belong to the starting set in U. If there is a chance at all to get each $a \in S$ out of the starting set in U (and thus make all individuals in S socially disqualified), this procedure is the best strategy for the chair as he adds as many individuals from $N \setminus S$ as possible to the partition set U (i.e., the chance that for each $a \in S$ an agent disqualifying a belongs to the same partition set is thus maximized). After putting each individual in N either in U or W, it hence suffices to check if the starting set in U is empty or not (note that at most elements in S can belong to the starting set in U after applying this algorithm). If yes, our instance is a YES instance. If no, we are dealing with a NO instance as all agents but S are set into W, and hence some $a \in S$ and (due to the symmetry of φ) all individuals in S reach the final round and are socially qualified (as some $a_0 \in K_0^C$ survives the partition set W, reaches the final round and in turn makes all agents in S socially qualified in the final round). Note that this algorithm requires $O(|N|^3)$ time as we regard $O(|N|)$ pivotal individuals in $N \setminus (K_0^C \cup S)$ one by one and verify in each step if all remaining $O(|N|)$ individuals not yet regarded are qualified by all $O(|N|)$ agents in $N \setminus W$.

Observe that the problem (especially the algorithm in the last subcase) takes no more than $O(|N|^3)$ time as the first two cases (mainly computing the starting set) are in $O(|N|^2)$ and the algorithm in the third case works in cubic time. ❑

The assumption $\varphi = \varphi^T$ makes sense in practice and induces several interesting substructures. Agents may build coalitions and hence there is mutual support (or rejection). Another, possibly more restrictive interpretation of a symmetric φ is that similar agents (or agents within a certain distance threshold) qualify each other.

5.2 Destructive Group Bribery

Due to Theorem 5.1, we directly obtain the complexities of group bribery in consent rules. We refer the reader to Table 2. Thus only the results for procedural rules are missing.

THEOREM 5.8. *f^{CSR}-DGB and f^{LSR}-DGB are in P.*

Proof. Destructive group bribery in f^{CSR} is always possible. The briber simply bribes an arbitrary individual and makes him disqualify each individual. By this, we obtain $K_0^C = \emptyset$.

For f^{LSR}, our input is an instance $(f^{LSR}, N, \varphi, S, \ell)$. We let $S_1 := \{a \in S : \varphi(a, a) = 1\}$ and $S_0 := S \setminus S_1 = \{a \in S : \varphi(a, a) = 0\}$. A necessary condition for the briber to reach his goal is obtaining $S_1 = \emptyset$ after the bribery. We may assume that each bribed individual disqualifies all individuals after the bribery in order to keep the number of qualifications as low as possible in the final election. Our algorithm checks the following cases:

- $|S_1| > \ell$. Then our instance is a NO instance as the briber cannot reach $K_0^L \cap S \neq \emptyset$ after the bribery.
- $|S_1| = \ell$. Then the briber bribes all S_1 individuals, each bribed individual disqualifies all individuals after the bribery, and it remains to validate if some $a \in S$ is socially qualified or not in the final election.
- $|S_1| < \ell$. The briber then bribes all individuals in S_1 (these bribes are fixed) and $\ell - |S_1|$ further individuals. Thus w.l.o.g. we regard an instance with $S_0 = S$. Similar to the proof of Theorem 5.5, we define the same help graph.

  It remains to argue that there is a minimum $(w, v(S))$-separator of size less or equal ℓ if and only if there is a successful bribery of at most ℓ agents. Bribing an individual and making him disqualify each individual can be interpreted as deleting this individual: Let $a \in N \setminus (K_0^L \cup S)$, then, after the bribery, all edges outgoing from $v(a)$ are deleted (as the briber makes a disqualify all individuals after the bribery). Although there may be some ingoing edges left, they are worthless as $v(a)$ is a dead end and no other node is reached by $v(a)$. Thus a can be treated as if deleted. Obviously the briber does not bribe agents in S in doubt as his aim is to destroy all ingoing edges with higher priority than edges between some element in S and other elements (possibly in S). The reason is that it does not matter if one or more individuals in S are reached, i.e., we want to cut off the "first" connection to agents in S.

The algorithm clearly takes polynomial time as all computations and decisions except the last subcase require $O(|N|^2)$ time and computing a minimum separator is in P, too. ❑

6. CONCLUSION

We have investigated the complexity of destructive group control and constructive and destructive group bribery in the context of group identification. Constructive group control was previously considered in the work by Yang and Dimitrov [28]. We have found a direct connection between destructive and constructive problems for consent rules and could thus derive all complexities for the destructive problems of consent rules. According to our results and the results obtained in [28], every consent rule $f^{(s,t)}$ with $s, t \geq 3$ resists all group control and bribery problems studied so far in the literature. In contrast, many group control and bribery problems for the liberal-start-respecting rule and consensus-start-respecting rule turned out to be polynomial-time solvable. Hence, our results suggest that consent rules outperform the liberal-start-respecting and consensus-start-respecting rules in terms of the resistance to strategic behavior.

We have not considered *manipulation* [2, 27] (i.e., some strategic individuals report insincere preferences in order to make a desired group of individuals socially qualified). Given constructive group manipulation, the best a manipulator can do is qualifying all individuals. Likewise, the manipulators disqualify all agents in the destructive version. These problems all are in P.

For future research, we refer to the open problems in this paper. As an example, we mention constructive group bribery in $f^{(s,2)}$ respectively the "mirrored" problem in destructive group bribery. One could further extend the model to other social rules or other strategical influences on groups such as runoff partition of individuals or other control models [11].

Another direction for future research would be to consider some variants of the group bribery problems studied in this paper. For instance, the briber has to pay 1 dollar for an individual to change his opinion over an individual, and the briber wants to pay in total k dollars in order to reach his goal. In addition, it is natural to assume in some setting that all individuals do not want to deviate much from their true opinions. In this setting, a bribed individual only flips at most t of his qualifications or disqualifications over the individuals, where t is a small number.

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