# Working Together: Committee Selection and the Supermodular Degree

# (Extended Abstract)

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## ABSTRACT

We introduce a voting rule for committee selection that captures positive correlation (synergy) between candidates. We argue that positive correlation can naturally happen in common scenarios that are related to committee selection. For example, in the movies selection problem, where prospective travelers are requested to choose the movies that will be available on their flight, it is reasonable to assume that they will tend to prefer voting for a movie in a series, only if they can watch also the former movies in that series. In elections to the parliament, it can be that two candidates are working extremely well together, so voters will benefit from being represented by both of them together.

In our model, the preferences of the candidates are represented by set functions, and we would like to maximize the total satisfaction of the voters. We show that although computing the best solution is $\mathcal{NP}$-hard, there exists an approximation algorithm with approximation guarantees that deteriorate gracefully with the amount of synergy between the candidates, as measured by an extension of the supermodular degree [Feige and Izsak, ITCS 2013] that we introduce – the joint supermodular degree.

## 1. INTRODUCTION

Consider the following scenario (see, e.g., [3, 9]). An airline is willing to increase the satisfaction of the travelers by letting them choose the set of movies that will be available on their flight. It is decided to store on the airplane some fixed number $k$ of movies. The airline surveys the preferences of the prospective passengers of the flight, and is willing to make the best decision given their preferences. Two questions arise. First, how should the preferences of the prospective travelers be modeled? Second, given the preferences of the travelers, how should the set of movies be chosen? This problem of choosing some fixed number of candidates to the satisfaction of the voters is a fundamental problem. Generally speaking, in the $k$-COMMITEE SELECTION problem, we have a set $V$ of $n$ voters and a set $C$ of $m$ candidates, and we would like to select $k$ candidates out of the $m$, such that the voters will be most satisfied. The answers to the two questions above vary in the literature (see, e.g., [1, 2, 3, 9]).

To the best of our knowledge, none of the models previously studied capture positive correlation (i.e. synergy) between specific candidates. Such positive correlation can happen in various cases: from two candidates to the parliament that are working great together (see Woolley et al. [10] for a research about collective intelligence[1]), to a series of movies that people tend to prefer watching the latter parts only after watching the former parts. In this paper we suggest a voting rule that captures positive correlation between specific candidates. Our answers to the two questions above are: (1) The preferences of each of the candidates are modeled by a non-decreasing monotone set function from subsets of candidates to non-negative real numbers. (2) A set of $k$ candidates that maximizes the sum of values of the voters is elected.

We study applications for the proposed model. In Section 3, we demonstrate how preference elicitation can be practically done. In Section 4, we study the computability of our voting rule. On the bright side, we show that although computing the optimum is, generally, $\mathcal{NP}$-hard, one can approximate the optimum with a guarantee that depends on the amount of synergy between different candidates. To measure the amount of synergy between candidates, we extend the supermodular degree [4], by introducing the joint supermodular degree (see Section 2.1). This enables us to use existing algorithms for set functions that were designed for the supermodular degree, in order to get approximation algorithms for our voting rule, both for offline and online settings. On the flip side, we show that the same results cannot be achieved for the supermodular degree.

## 2. MEASURING SYNERGY

The definitions below are taken from the works [4, 5]. Let $C$ be a set of items (e.g. candidates in election, movies to watch on an airplane) and let $f : 2^C \to \mathbb{R}^+$ be a set function (e.g. of preferences of one of the voters). The following definition is standard.

DEFINITION 1. *Let $c \in C$. The marginal set function $f_c : 2^{C\setminus\{c\}} \to \mathbb{R}^+$ is a function mapping every subset $S \subseteq C \setminus \{c\}$*

[1]They show that there is a measure for the collective intelligence of a group of people that is different from the intelligence quantities of different people in the group.

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*to the marginal value of c given S:* $f_c(S) \stackrel{def}{=} f(S \cup \{c\}) - f(S)$. *We denote the marginal value* $f_c(S)$ *by* $f(c \mid S)$.

The following has been introduced by Feige and Izsak [4].

Definition 2. *Let* $c \in C$. *The* **supermodular dependency set** *of c by f is the set of all items* $c' \in C$ *such that there exists* $S \subseteq C \setminus \{c, c'\}$ *such that* $f(c \mid S \cup \{c'\}) > f(c \mid S)$. *We denote the supermodular dependency set of c by* $\mathcal{D}^+_f(c)$. *The* **supermodular degree** *of f is defined as* $\mathcal{D}^+_f \stackrel{def}{=} \max_{c \in C} |\mathcal{D}^+_f(c)|$. *We sometimes omit f, when it is clear from the context.*

## 2.1 The joint supermodular degree

We introduce the following natural extensions of the definitions of Feige and Izsak [4] to a collection of set functions.

Definition 3. *Let* $f_1, \ldots, f_t$ *be set functions for some* $t \in \mathbb{N}$ *and let* $c \in C$. *The* **joint supermodular dependency set** *of c by* $f_1, \ldots, f_t$ *is* $\bigcup_{i=1}^{t} \mathcal{D}^+_{f_i}(c)$. *The* **joint supermodular degree** *of* $f_1, \ldots, f_t$ *is the maximum cardinality among the cardinalities of joint dependency sets of items of C by* $f_1, \ldots, f_t$.

The main property of the joint supermodular degree that we use is that the sum function of functions with joint supermodular degree $d$ has supermodular degree of at most $d$.

We think that this definition is natural for voting rules, since it means that positive correlation between the candidates can be modeled, when it is inherent to the candidates themselves (rather than to the perspective of the voters about them). For example, if two candidates are known to work very well together, a voter has the possibility to give them together a value that is higher than the sum of their individual values. If two movies are part of a series, then a voter can express her preference to watch both of them.

# 3. PREFERENCE ELICITATION

Consider the movies selection example. When a prospective passenger is asked to express her preferences about possible movies, it seems unreasonable to require her to specify her values for all the exponentially many possibilities. We briefly demonstrate a simple user interface to elicit users' preferences in that case, while enabling them to benefit from the possibility of expressing positive correlations.

The user interface will be as follows. Each of the prospective passengers will be able to give a value for each of the possible movies (these are the values of the singleton subsets). In addition, the prospective passengers will be able to add for each of the movies other values – the marginal values of a movie, with respect to a subset of its joint supermodular dependency set. In order to select such a subset of the movies, a list of the movies in the joint supermodular dependency set will be presented, and a passenger will be able to select the relevant movies (e.g. by checking them by a 'V'). In order to enforce the preference functions of the prospective passengers to be well defined (i.e. a single value for each of the subsets), we will let the prospective passengers check by a 'V' only the movies that were former to a movie in a series.

To see the power of combining supermodular dependencies with submodular behaviour, note that we can also ask each passenger how many movies she would like to watch in her flight (with a maximum that depends on the duration of the flight), and then calculate as her preference, the best subset of that number of movies, from any input subset of movies.

# 4. COMPUTATIONAL RESULTS

We show that there exists an approximation algorithm with approximation guarantee that is linear in the joint supermodular degree of the preference functions of the voters. For submodular set functions, the approximation guarantee coincides with the optimal approximation guarantee for submodular set functions of the algorithm of Fisher, Nemhauser and Wolsey [7] that is used by Skowron, Faliszewski and Lang [9].

Theorem 1. *When the joint supermodular degree of the preferences functions of the voters is d, the k-committee selection problem admits an approximation algorithm with an approximation guarantee of* $(1 - e^{-1/(d+1)}) \geq 1/(d+2)$.

Note that the above result captures the example of movies selection. Note also that the proof of the above result applies to the case of committee selection subject to a *general* matroid constraint (cardinality constraint is a special case of a matroid constraint), but with an approximation guarantee of $1/(d+2)$, by using the respective algorithm of Feldman and Izsak [5].

Moreover, one can use the algorithms of Feldman and Izsak [6], in order to get an online (secretary like) version of Theorem 1, when the candidates arrive one by one in an online fashion, and we need to decide on the spot, irrevocably, whether to elect a candidate or not, based on the preferences of the voters (for exact details of the model, see [6]). As an example, consider hiring a team to a project, where each of the candidates meets with a few interviewers. Then, an optimal team of candidates should be hired, according to the preferences of the interviewers.

By using the algorithm in [6] for a cardinality constraint, one gets competitive ratios polynomial in the joint supermodular degree (see also Oren and Lucier [8]).

The full version of the paper contains the proof of Theorem 1, as well as a hardness result for the case of non-bounded joint supermodular degree, even when the supermodular degree of each of the set functions is bounded by 1.

# 5. CONCLUSIONS

We suggest a new voting rule for committee selection that enables the voters to express positive correlation between the candidates. We also introduce the joint supermodular degree that enables us to use existing computational results for the supermodular degree, and get efficient approximation algorithms for our voting rule. We see our work as a proof of concept, and hope that it will lead to further study of committee selection with positive correlation between the candidates.

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