Real-time Adaptive Tolling Scheme for Optimized Social Welfare in Traffic Networks

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ABSTRACT

Connected and autonomous vehicle technology has advanced rapidly in recent years. These technologies create possibilities for advanced AI-based traffic management techniques. Developing such techniques is an important challenge and opportunity for the AI community as it requires synergy between experts in game theory, multiagent systems, behavioral science, and flow optimization. This paper takes a step in this direction by considering traffic flow optimization through setting and broadcasting of dynamic and adaptive tolls. Previous tolling schemes either were not adaptive in real-time, not scalable to large networks, or did not optimize traffic flow over an entire network. Moreover, previous schemes made strong assumptions on observable demands, road capacities and users homogeneity. This paper introduces Δ-*tolling*, a novel tolling scheme that is adaptive in real-time and able to scale to large networks. We provide theoretical evidence showing that under certain assumptions Δ-*tolling* is equal to *Marginal-Cost Tolling*, which provably leads to system-optimal, and empirical evidence showing that Δ-*tolling* increases social welfare (by up to 33%) in two traffic simulators with markedly different modeling assumptions.

CCS Concepts

•**Computing methodologies** $\rightarrow$ *Multi-agent planning;*

Keywords

Autonomous Intersection Management, Autonomous vehicles, Multiagent systems

INTRODUCTION

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Communication and computation capabilities are becoming increasingly common on board vehicles. Such capabilities present opportunities for developing safer, cleaner and more efficient road networks. One way of increasing road efficiency is to incentivize vehicles to travel via less congested routes.

It has been known for nearly a century that drivers seeking to minimize their private travel times need not minimize the total level of congestion. In other words, self-interested drivers may reach a user equilibrium that is not optimal from a system perspective. On the other hand, disincentivizing vehicles to traverse certain links (using tolls for instance) can lead to system optimum [34, 5, 11].

This paper discusses the concept of *micro-tolling*, defined as the ability to set individualized and dynamic toll values for each link within a road network. Specifically, this paper defines the *micro-toll traffic optimization* (MTTO) problem where, given current, observable traffic conditions (traffic volume, travel speed, travel time etc.), the goal is to set a dynamic toll value to each link such that the user equilibrium aligns with the system optimum. The focus of this paper is on the optimization problem associated with toll-setting, and not on technical or policy issues associated with implementing micro-tolling. Our aim is merely to evaluate the potential performance benefits should the policy and human factors issues that are outside the scope of this paper be overcome.

This paper introduces a novel micro-tolling scheme denoted Δ-*tolling*. Δ-*tolling* assigns a toll to each link proportional to the difference between its current travel time and its free-flow travel time (denoted Δ). The constant of proportionality (denoted β) requires tuning. Since Δ changes according to observed traffic, Δ-*tolling* is adaptive to traffic changes in real-time. Since computing the toll value is done locally for each link, Δ-*tolling* is tractable for large networks.

This paper conjectures that Δ-*tolling* leads to optimal system performance. Two types of supporting evidence are provided for this claim. From a theoretical standpoint, we show that under additional assumptions Δ-*tolling* is equivalent to *marginal-cost tolling*, which provably yields optimal system performance. From an empirical standpoint, using two different traffic simulation models, we show that Δ-*tolling* leads to a significant improvement in system performance, up to 33% and 32% improvement in social welfare and average travel time respectively. As the annual cost of traffic congestion in the United States alone is $160 billion [30], even small reductions in travel time can have dramatic benefits.

To the best of our knowledge Δ-*tolling* is the first tolling scheme that is adaptive in real-time, able to scale to large networks, does not assume users homogeneity, and enhances system performance. Moreover, given appropriate communication capabilities Δ-*tolling* is practical to implement in real-life.

MOTIVATION

Self-interested agents (For the rest of this paper we relate to vehicles as agents) choose routes that maximize their own utility, regardless of their impact on *social welfare* (the total utility over all agents). This section presents examples of such scenarios which provide motivation for this work.

For the rest of this paper we assume that an agent's utility is the negative of travel cost, defined as the sum of travel time (converted to cost units through an agent-specific **value of time**, VOT) and money spent (tolls). Each agent seeks a route leading from its current location to its desired destination. In the **user equilibrium** (UE) state, agents' route choices are such that no single agent can improve his or her utility from a unilateral change in route, and in the **system optimum** (SO) state, agents' route choices are such that social welfare is maximized. Note that if VOT is homogeneous over all agents, maximizing social welfare is equivalent to minimizing average travel time. For the following example we assume homogeneous VOT.

Figure 1 contrasts the UE and SO states (this example was first presented by Pigou ([34])). The example problem consists of: one origin (O), one destination (D_1) (for now ignore destination D_2), a low capacity link representing a side-road shortcut (dotted line) and an infinite-capacity link representing a highway (solid line). In contrast to the highway, the shortcut is susceptible to congestion due to its low capacity.

A fixed number of agents are heading from O to D_1. Assume that units are chosen so that the travel time on the shortcut equals the fraction of the overall traffic using that road (e.g., if half of the agents travel via the shortcut, the travel time on it will be 0.5), and so that all drivers have a VOT of 1. Since the highway has infinite capacity, its travel time is 1 regardless of the amount of agents using it.

Define v_s (and $v_h = 1 - v_s$) as the fraction of the overall traffic traveling via the shortcut (highway). The travel time (t) on the shortcut (highway) is a function of v_s (or v_h for the highway) and is defined by $t(v_s) = v_s$ (and $t_{D_1}(v_h) = 1$ for the highway). The proportional travel time (travel time multiplied by the fraction of traffic) on the shortcut is v_s^2 (and $1 \cdot v_h = 1 - v_s$ for the highway).

In the SO state, the average travel time (the negative of social welfare here), $f(v_s) = v_s^2 + (1 - v_s)$, is minimized. Using the derivative of the travel time function, $f'(v_s) = 2v_s - 1$, we find the system optimum to be at $2v_s - 1 = 0 \Rightarrow v_s = 0.5$. In the UE state, agents will choose to go via the shortcut (traveling for v_s hours) as long as it is faster than traveling via the highway (until its travel time reaches 1). As a result, in this example, $v_s = 1$ in the UE while $v_s = 0.5$ in the SO. Setting a toll of 0.5 on the shortcut will shift the UE state to one where $v_s + 0.5 = 1 \Rightarrow v_s = 0.5 \Rightarrow$ UE=SO.

Adaptive Tolling

Building on Pigou's ([34]) example, we give an example where aligning the UE with the SO requires tolls that dynamically change with traffic demands. Consider a second destination (D_2) in our example road network (Figure 1). Similar to destination D_1, the travel time form O to D_2 via the shortcut (dotted line) is equal to the fraction of the overall traffic traveling via the shortcut (heading both to D_1 and D_2). In contrast to destination D_1, the travel time on the highway (solid line) to destination D_2 is only 0.5 ($t_{D_2}(v_h) = 0.5$).

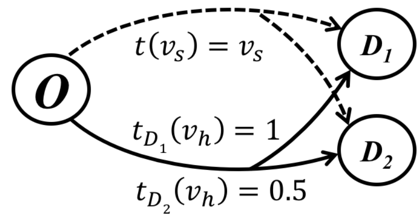

Figure 1: An example road network. One origin (O), two destinations (D_1, D_2), a low capacity link (dotted line) and an infinite capacity link (solid line).

Let z be the fraction of agents heading to destination D_1 ($1 - z$ heading to D_2). In this example, aligning SO and UE requires assigning the shortcut a dynamic toll that is a function of z. A full proof and analysis of this scenario is provided in the Appendix.

With the need for adaptive tolling in mind, the next section defines the *micro-toll traffic optimization* problem.

PROBLEM DEFINITION

We define the micro-toll traffic optimization (MTTO) problem as follows:

Input:

- Road network as a directed graph: $G(V, E)$.
- Free flow travel time on each link.
- Mean VOT across all agents traversing the network.[1] The VOT distribution is not assumed to be known.
- Time-varying traffic conditions on each link e, which are assumed to be visible at any time, including the number of agents on the link, traffic flow speed, and travel time.

Output: For each link in the network at any given time, a non-negative toll value.
Objective: Optimize social welfare.

Assumptions and Desiderata

A tolling scheme for solving MTTO should satisfy the following requirements:

- Adapt to traffic conditions in real-time. Traffic demands and road capacities are dynamically changing throughout the day. The system should adapt and react to such changes.
- Scale to large, city size networks. The system should require computational effort that is linear in the network's size.
- Robust to heterogeneous VOT. The system should not assume that all agents have uniform VOT.

[1]Many approaches exist for estimating mean VOT [6, 20, 26] without requiring individual drivers to truthfully report this information.

Based on today's communication capabilities on board connected vehicles[2] we make the following assumptions:

- The location (traveled links) of each agent is visible to the toll manager, for billing purposes.
- Toll values are visible to the agents.
- Knowing the toll values and their own VOT, agents are able to optimize their route.

Note that the above assumptions are intended for connected vehicles, but may still hold even for traditional vehicles. In fact, the technology on today's smart-phones can satisfy these assumptions. Current micro-tolling implementations are based on *Electronic road pricing* (ERP) systems that use an array of sensors and electronic signage for billing and reporting toll values. There are several ERP implementations currently being used (e.g., in Hong Kong [18] and Singapore [37]). The main drawback of ERP systems is that they are expensive to implement and thus usually cover a relatively small area within a city. Currently the toll assignment scheme of such systems is fixed according to time-of-day and is not adaptive. As a result, current ERP systems might benefit from the work presented in this paper.

PREVIOUS WORK

Road pricing has received considerable attention due to its potential to reduce congestion, and the economic fairness of charging users for the delays they cause to other travelers. It has long been established that in a static equilibrium setting, marginal tolls can eliminate the inefficiency associated with selfish routing [35, 5]. A detailed history along with practical aspects of congestion pricing can be found in [2]. However, such steady state conditions rarely exist in practice. Changes in supply, demand and other driver characteristics such as bounded rationality and value of time result in traffic that is dynamic both day-to-day and within-day. To control congestion in the presence of these factors, researchers have proposed a wide range of tolling models, based on different representations of traffic flow and different assumptions on the source of variability.

To the best of our knowledge, no previously suggested tolling scheme is equipped to solve the MTTO problem, due to one or more of the following reasons:

- **Assumption that demand is known or fixed**:[3] [43, 19, 31]
- **Assumption that road capacity is known or fixed**:[4] [19, 28, 44]
- **Assumption that VOT is homogeneous**:[5] [45, 48]
- **Assumption that traffic follows a specific model**[6]: [43, 19, 28, 15]
- **Intractability for large networks**: [42, 28]
- **Not adaptive in real time**: [5, 41]
- **Optimization of a single corridor, not the full network**: [47, 16]

Next, we present Δ-*tolling*, a novel tolling scheme that satisfies all the MTTO desiderata without making any of the above assumptions.

Algorithm 1: Updating tolls according to Δ-*tolling*.

```
1 for each link e ∈ E do
2     Δ-tolling_e = β(t_e − T_e)
3     τ_e = R(Δ-tolling_e) + (1 − R)τ_e
```

DELTA-TOLLING

Given a MTTO problem instance, we use t_e to denote the current travel time on link $e \in E$, the constant T_e to denote its free flow travel time, and τ_e to denote its current toll value. For each link, Δ-*tolling* assigns τ_e to be the difference between the current flow time (t_e) and the free flow time (T_e) multiplied by a parameter (β), that is, $\tau_e = \beta(t_e - T_e)$. The Δ-*tolling* procedure is given in Algorithm 1. The parameter R is described later; for now, assume $R = 1$.

Since τ_e, as calculated by Δ-*tolling*, depends on t_e, Δ-*tolling* is adaptive to traffic conditions in real-time; as the travel time on the link grows so does the toll value for that link. Calculating τ_e using Δ-*tolling* requires a constant amount of time and can be done independently for each link. As a result, the complexity of computing tolls for an entire network is $\Theta(|E|)$ in a centralized manner or $\Theta(1)$ when distributed. Hence Δ-*tolling* can easily scale to large networks. Moreover, Δ-*tolling* assumes full knowledge of only one variable (current travel time t_e) and one constant (free flow travel time T_e), and both measurements are assumed feasible according to our problem definition. Finally, we present two propositions:

1. Under certain assumptions on the demand and traffic model, Δ-*tolling* optimizes social welfare.
2. Even without these assumptions, applying Δ-*tolling* results in a significant improvement in social welfare.

It is important to note that Δ-*tolling* does not requires or assumes that agents commit to an initially chosen route. Agents may re-optimize and change their route at any point.

The rest of this paper elaborates on and proves Proposition 1, and provides an empirical argument in support of Proposition 2.

MARGINAL COST TOLLING

It is known that charging each agent an amount equivalent to the cost it inflicts on all other agents, also known as *marginal-cost tolling* (MCT), results in optimal social welfare [34, 5, 11] (assuming nonatomic flows). For instance, if an agent increases the travel

[2] Connected vehicles are vehicles that can communicate with the following entities: other vehicles (vehicle-to-vehicle), roadside infrastructure (vehicle-to-infrastructure), and the internet.

[3] Knowing the demands require all users to report their origin, destination, and departure time. Even though there is a technological capability for doing so, it is not reasonable to assume that users will cooperatively share this information.

[4] It is still not possible to accurately estimate the capacity due to heterogenitey in driver behavior and vehicle composition (see Figure 1 in [13]). Furthermore, capacity of a roadway is not always fixed. Incidents, weather, moving bottlenecks etc. can dynamically change the capacity.

[5] Different drivers and vehicles differ from each other on many aspects [1] assuming otherwise is not realistic.

[6] Given demands, capacities and toll values, a traffic model returns the expected travel times. Examples of two traffic models (mesoscopic and macroscopic) can be found in the "Empirical Study" section. Since real traffic does not accurately follow any known model, a model dependent scheme is not practical for real-life.

time of v agents by two minutes, that agent should be charged a toll equal to the sum of the v agents' value of a two-minute delay.

Applying a MCT scheme is not feasible in practice since it requires knowing in advance the marginal delay that each agent will impose on all others. This, in turn, requires exact knowledge of future demand and roadway capacity conditions, as well as counterfactual knowledge of the network states without each driver.

Given the infeasibility of applying MCT, models for approximating MCT were suggested along the years. Such models commonly assume a *volume delay function* [39]. Given the volume (v_e) on a link, e, this function returns the travel time (t_e). Assuming that the volume delay function is accurate, the derivative of the function at $v_e = i$ returns the marginal impact of an additional vehicle using that link. Since this delay affects all i agents that are traveling the link, the MCT for the i^{th} agent equals $\frac{dt_e}{dv_e = i}(i)$.

Δ-*tolling* Equals MCT

Any proof regarding Δ-*tolling* requires assuming a flow model. For our analysis we assume the following commonly accepted flow model:

ASSUMPTION 1. *The delay on each link is expressed by the BPR volume delay function,* $t_e(v_e) = T_e(1 + \alpha(\frac{v_e}{c_e})^\beta)$ *where* c_e *is a scale parameter related to the capacity of link* e *and* $\alpha,\ \beta$ *are constants (this is the same* β *used by* Δ*-tolling). The values of* $\alpha,\ \beta$ *is identical across all edges.*

The BPR volume delay function is widely accepted in the literature and has been validated in field tests [39, 21].

LEMMA 1. *Under the above assumption, the tolls computed by* Δ*-tolling are equal to MCT.*

Proof: We express the BPR volume delay function as:
(1) $t_e(v_e) = T_e + av_e{}^\beta$ where $a = T_e \frac{\alpha}{c_e{}^\beta}$.
MCT is defined as the derivative of the delay function ($\frac{dt_e(v_e)}{dv_e}$) multiplied by the traffic volume (v_e). So we get:
(2) $MCT_e = v_e \frac{dt_e(v_e)}{dv_e} = v_e(\beta a v_e{}^{\beta-1}) = \beta a v_e{}^\beta = \beta(T_e + av_e{}^\beta - T_e)$.
Combining (1) and (2) we get: $MCT_e = \beta(t_e - T_e) = \Delta\text{-}tolling_e$.
□

OSCILLATION EFFECT AND TOLL SPIKES

Under Assumption 1, Δ-*tolling* calculates the marginal impact of an agent according to current traffic conditions. In other words, Δ-*tolling* as calculated at a given time step, i, and for a given link, e, equals MCT for an agent entering e at time i. By contrast, Δ-*tolling* (as calculated at time step i) might be different than MCT for an agent entering the link at a later time step ($> i$). In a road network, agents often plan a full origin to destination route prior to embarking. The route chosen by an agent is optimized according to current traffic conditions and tolls. This might lead to an oscillation effect as demonstrated in the following example.

Time step	Shortcut			Highway		
	v	t	τ	v	t	τ
0	0	0	0	0	1	0
1	1	1	1	0	1	0
2	0	0	0	1	1	0
3	1	1	1	0	1	0

Assume that time is discretized into time steps and that an agent planing its route at time step i will traverse the system at time step $i+1$. Reconsider our example network from Figure 1 with a single destination, D_1 (disregard D_2). Assume that Δ-*tolling* is used and that at $i = 0$ the network is empty and tolls are 0 for both links. Table 3 specifies the flow volume (v), travel time (t) and toll value (τ) for both the highway and shortcut at each time step. At $i = 0$ the travel time equals 0 on the shortcut and 1 on the highway. As a result, all agents planning at $i = 0$ will choose the shortcut causing it to congest, travel time would increase to 1 and so will the toll affiliated with it. All agents planning at $i = 1$ would thus choose to travel via the highway. As a result, the shortcut will remain empty at $i = 2$ and its travel time and toll will reduce to 0. This phenomenon, where all agents choose to go via the shortcut in one time step and the highway in the next, will repeat itself, resulting in sub optimal system performance.

Moreover, if the BPR function isn't assumed, tolls may unpredictably spike due to traffic irregularities. For instance, whenever a traffic light turns red, traffic stops and the toll set by Δ-*tolling* spikes. On the other hand, when the light turns green traffic flow resumes and the toll drops.

Avoiding Oscillation and Spikes

Given an oracle that can predict future traffic conditions oscillation could easily be avoided. In such a scenario the anticipated toll for a future time step i would be calculated according to the predicted conditions at step i and a stable UE would be reached. Unfortunately, predicting traffic congestion is currently infeasible [29], let alone predicting congestion when factoring in tolls.

To deal with oscillation and spikes, under the assumption that predicting congestion is not possible, we introduce the responsiveness parameter R. This parameter defines an exponential smoothing of the toll values over time, where $\tau_e(i) = R \cdot \Delta\text{-}tolling_e(i) + (1 - R)\tau_e(i-1)$ (Line 3 in Algorithm 1), $\tau_e(i)$ is the toll value assigned to link e at time step i, and $\Delta\text{-}tolling_e(i)$ is Δ-*tolling* as calculated for link e at time step i. Define $\tau_e(-1) = 0$ for all links. Note that this variant (Δ-*tolling* with R) requires no extra communication or computation capabilities. When $R = 1$ the system responds immediately to changes in traffic but is susceptible to oscillation and spikes. On the other hand, as $R \to 0$ the tolls are stable, but also are unresponsive to changes in traffic conditions. For any extreme case a low enough R value would eliminate oscillation and spikes but at the cost of responsiveness.

EMPIRICAL STUDY

Assumption 1 is amenable to mathematical analysis, but abstracts many details of traffic behavior which can be significant in practice, including the effects of travel time variations due to traffic lights (as mentioned above). Therefore, we complement our theoretical analysis by evaluating Δ-*tolling* using established traffic simulators.[7] Existing traffic simulators can be divided into three main classes:

Macroscopic models — These models use volume delay functions to model congestion, as described above. On the one hand, macroscopic models do not model the evolution of traffic over time. On the other hand, they have nice mathematical properties that admit efficient algorithms and provable convergence for large networks. Such models are still used by many planning organizations [7].

Mesoscopic models — These model the evolution of traffic according to the kinematic wave theory of traffic flow [36, 27], and are tractable for solving dynamic traffic assignment on large networks yet improve on macroscopic models by including dynamic

[7]Field testing Δ-*tolling* with real-life traffic is infeasible at this stage due to technological and regulatory limitations.

travel times and queue spillback. They are well-established [7] and have been calibrated to match observed travel times and link counts for city networks.

Microscopic models — These are the most detailed models [14, 4, 12]. They simulate the exact state of each vehicle (position, heading, speed) and system (traffic lights, dynamic obstacles, road conditions) in real-time. The computational requirements of these models typically limit them to detailed study of relatively small road networks. Moreover, calibrating these models is challenging and requires distributional knowledge of vehicle characteristics (acceleration and braking rates), driver characteristics (reaction time, aggression), and system infrastructure (details of signal timing, including actuation).

This section presents results from implementing Δ-*tolling* on both macroscopic (demonstrating convergence to SO, cf. Proposition 1), and mesoscopic models demonstrating effectiveness on city networks (Proposition 2).

As part of this study Δ-*tolling* was also implemented and tested on the AIM micro-simulator [12]. Smaller road networks were considered due to the simulator's limitations. Nevertheless, general trends similar to those obtained by the mesoscopic model were observed. We omit these results as they give no further insights. A detailed report on the experimental setting and results obtained by the AIM simulator can be found in [38].

Note that traffic-related studies in engineering venues commonly present experimental results using only a single traffic model. Our empirical study covering three different models (macroscopic, mesoscopic, and microscopic) demonstrates that Δ-*tolling* is effective regardless of the underlying traffic model.

Scenarios

Each traffic scenario is defined by the following inputs:

1. Road network, specifying: links' length, links' capacity, traffic lights timing and speed limits.
2. Trips table with each trip specifying: origin, destination, departure time, type of vehicle.

For this study two scenarios were used:

Sioux Falls scenario [23] — this scenario is widely used in the transportation research literature [3], and consists of 76 directed links, 24 nodes (intersections) and 360,600 trips spanning 24 hours. The full scenario is accessible online
(https://github.com/bstabler/TransportationNetworks)

Downtown Austin scenario [25] — this network consists of 1247 directed links, 546 nodes and 62,836 trips spanning 2 hours during the morning peak. The exact trips and signals timing were measured and provided by capital area metropolitan area (CAMPO) in respond to our request.

The road networks of both scenarios are depicted in Figure 2 (I). All models used in our empirical study assume that the departure times and origin-destination pairs are strict. Developing new models to study the effect of tolls on delaying departure times or changing the destinations is left for future work.

Macroscopic Model

The macroscopic models calculate the UE in a given scenario using Algorithm B [10]. At each time step the UE is calculated according to given demands, link capacities, network topology and toll values. For both scenarios the model assumed that travel times follow the BPR function with the commonly used parameters $\beta_m = 4$, $\alpha = 0.15$. We use β_m to denote the β value used by the model which may be different the the value used by Δ-*tolling*. Toll values for a given time step are updated according to traffic patterns observed in the previous time step. In the experiments presented in this paper, the algorithm was terminated when the difference between total system travel time over successive iterations was less than 0.1 minutes. Homogeneous VOT was assumed thus we report average travel time instead of social welfare for these experiments.

As was mentioned above, macro-models are useful for proving theoretical attributes regarding UE and SO (under simplified assumptions[8]). We use such a model to give supporting evidence for the following conjecture (supporting Proposition 1):

CONJECTURE 1. *Under static traffic conditions (demands, link capacities, network topology) and assuming UE emerges at each iteration (i),* Δ*-tolling would converge to SO when setting* $R = 1/i$ *(equivalent to setting the average* Δ*-tolling observed over all iterations so far).*

Note that conjecture 1 differs from Lemma 1 by taking into account the time leg between planning and acting. According to the macroscopic model, for instance, an agent that plans its route at time step i (according to tolls and travel times observed at time i) will traverse the system at time step $i+1$. This conjecture resembles the Method of Successive Averages (MSA) for MCT convergence [46], however, the MSA algorithm modifies link volumes in each iteration. In Delta-toll, link volumes are a function of tolls and the updated link volumes don't necessarily fit the MSA update rule.

Table 1 presents average travel times for our two scenarios (for now ignore the "Meso" lines). In line with conjecture 1, we observed that for $\beta = 4$, Δ-*tolling* converged to the provable system optimum after 11 and 27 iterations for the Sioux Falls and Austin scenarios respectively.[9]

Mesoscopic Simulator

Mesoscopic flow models are typically used in dynamic traffic assignment [7]. Dynamic traffic assignment iterates between finding shortest paths, assigning vehicles, and evaluating travel times through simulation, to find a route assignment near dynamic user equilibrium [25]. Mesoscopic models can be used to perform many simulations of city network traffic in a reasonable time. Most mesoscopic models use the kinematic wave theory of traffic flow, which models traffic as a compressible fluid [27, 36]. The kinematic wave theory models several important aspects of traffic behavior including the formation and dissipation of congestion waves over time due to bottlenecks. The kinematic wave model involves a system of partial differential equations which are solved numerically given initial and boundary conditions. One common solution method is

[8]Macroscopic models assume that the link delay function truly reflects all delays imposed on traffic including inter-link effects (intersections).

[9]No statistical significance tests are provided for the Macroscopic results since this model is deterministic.

Scenario	No tolls	$\beta = 1$	$\beta = 2$	$\beta = 4$	$\beta = 8$
Sioux Falls					
Macro	20.74	20.09	19.98	**19.95**	19.96
Meso	24.74	20.28	20.08	**19.92**	20.26
Downtown Austin					
Macro	21.92	21.81	21.79	**21.78**	21.79
Meso	20.67	16.06	**15.64**	15.82	17.39

Table 1: Average travel time (Minutes) for different β values (as defined for Δ-*tolling*) using Macroscopic and Mesoscopic ($R = 10^{-4}$) models.

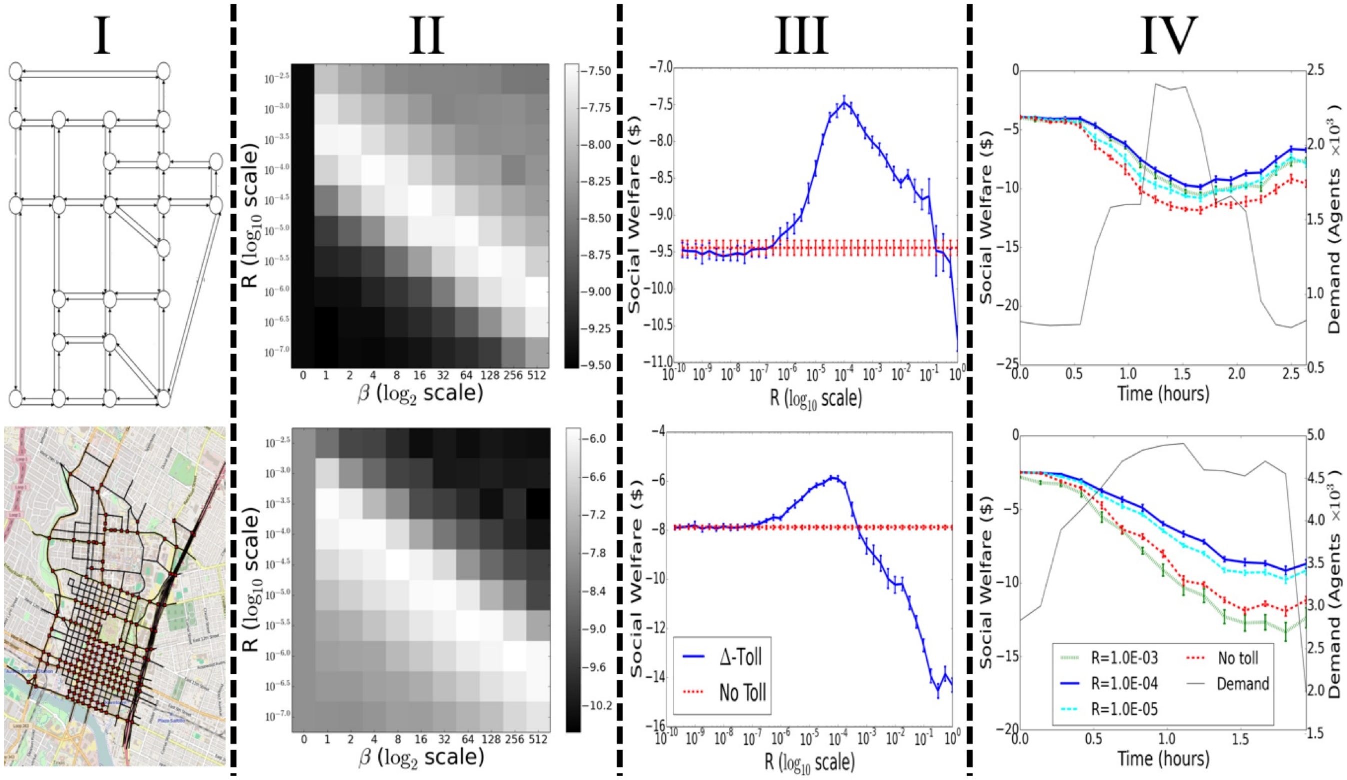

Figure 2: Mesoscopic results for the Sioux Falls (top row) and Austin (bottom row) scenarios. I: Sioux Falls network (all intersections are traffic lights), Austin network (traffic lights are marked with a red square, unmarked intersections use stop/yield signs). II: Heatmap presenting social welfare for β vs R. Darker color represents worse social welfare. $\beta = 0$ represents no tolls. III: R vs social welfare (constant β value = 4) each data point is an average over 10 full scenario runs. IV: Time vs social welfare ($\beta = 4$) each data point is an average over the past 500 simulated seconds.

the cell transmission model (CTM) [8, 9], which is a Godunov scheme [17] for the kinematic wave theory. The CTM can be used with a variety of intersection models [40], including traffic signals and autonomous reservation schemes [24]. Using such intersection models, CTM, unlike the macro model, takes into account inter-link effects, making CTM more realistic on the one hand but intractable to compute the SO on the other. We use such a model (CTM) to give supporting evidence for Proposition 2.

Our experiments required two adaptations to the CTM model:

- Allowing agents to change routes adaptively in response to dynamic travel times and tolls. When arriving to a node (intersection) each agent calculates and follows its optimal route according to currently observed traffic conditions (tolls and travel times).
- When running CTM, agents seeking to enter a link might be blocked if that link is fully occupied and no free space exists. To avoid traffic gridlock (agents are blocking each other in a cycle), if an agent is blocked for more than 96 seconds it is assigned a different path to its destination while avoiding the blocked link (value of 96 seconds resulted in the best performance, with regards to social welfare, when no tolls are considered). If no such path exists its current path is not changed.
- Due to the dynamic nature of traffic and toll values, agents are required to constantly re-optimize their route and adjust towards the lowest cost path leading to their destination, where the cost of the path equals the travel time times VOT plus tolls.

For running the Sioux Falls scenario under CTM we used an adapted version suitable for a dynamic traffic assignment [24]. The adapted scenario consist of 28,835 trips over 3 hours and traffic signal timing based on Webster's formula [22]. The Austin scenario, on the other hand, is originally suitable for a dynamic traffic assignment and also includes real traffic light timing. The mesoscopic model for both Sioux Falls and downtown Austin use dynamic network loading and demands meaning that travel times and congestion vary with time.

Results are provided in Figure 2 as three plots for each of the two scenarios. Each data point represents an average of 10 full scenarios.[10] 95% confidence intervals are provided when applicable. Social welfare is reported as average utility instead of total utility for ease of presentation. For the purpose of social welfare analysis we assume that collected tolls are redistributed (as tax reductions for instance).

In general we observe that similar trends arise from both scenarios as we specify next. The first plot (II) is a heatmap showing how different combinations of the β and R parameters affect system performance (with regards to social welfare). We can observe that for any fixed value for one parameter there is a value for the

[10]In each scenario the VOT assigned to each agent was randomly chosen using a Dagum distribution with parameters $\hat{a} = 22,020.6$, $\hat{b} = 2.7926$, $\hat{c} = 0.2977$, which models personal incomes in USA [32].

second parameter that achieves near optimal performance. Hence, for the following experiments we set a constant $\beta = 4$ value. The second plot (III) shows the average social welfare over all agents for a given R value. High R values (close to 1) hurt overall performance (due to oscillation, as was explained above). Very low R values (close to 0) make no difference (tolls remain insignificant). An R value of about 10^{-4} yielded the best performance of 26% and 33% increase in social welfare for the Sioux Falls and Austin scenarios respectively. The third plot (IV) presents a breakdown of the average social welfare into time intervals. The demands (number of agents embarking on each time step) are also presented as a function of time. Results are presented for three representative R values (10^{-3}, 10^{-4}, 10^{-5}) as well as no tolls. We can see, again, that a $R = 10^{-4}$ yields best performance during peak hour as well as helping to delay the formation of congestion.

Table 1 also presents average travel times for the Meso-model. We observe a 24% and 32% reduction in average travel time for Sioux Falls and Austin respectively when Δ-*tolling* is applied. The observed reduction in travel time is proportionally smaller compared to the increase in social welfare due to heterogeneous VOT.

Note that the benefits of Δ-*tolling* observed under the macro-model are significantly lower compared to those observed by the meso-model. This is because the volume delay functions in the macro-model do not explicitly enforce capacities. The meso-model strictly prohibits flows from exceeding capacity which results in spillbacks. Spillbacks can dramatically increase travel time during periods when demand exceeds capacity.

CONCLUSIONS

This paper discusses the micro-toll traffic optimization (MTTO) problem where tolls should be assigned to every link in a road network. The objective in MTTO is that self-interested agents would reach a user equilibrium that is optimal from a system perspective.

A novel tolling scheme denoted Δ-*tolling* is presented and analyzed. We show that under simplifying assumptions, Δ-*tolling* is equivalent to marginal-cost tolling, which is known to yield optimal system performance. Empirical results are provided, supporting the claim that Δ-*tolling* results in system optimum under the simplifying assumptions of a macroscopic-model. Next, the effectiveness of Δ-*tolling* is presented when using a more realistic mesoscopic traffic simulator showing a 33% increase in social welfare when Δ-*tolling* is applied in downtown Austin during rush hour.

This paper looks into the future and assumes that all agents are connected and have full knowledge of tolls and travel times while acting fully rationally. Future work will deal with nowadays practical implementation issues such as partial compliance and rationality (mainly based on [33]) and solving MTTO using positive incentives (instead of tolls, for political reasons).

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APPENDIX:

Example Where Adaptive Tolling Is Required

Building on Pigou's ([34]) example, we give an example where aligning the UE with the SO requires tolls that dynamically change with traffic demands. Consider a second destination (D_2) in our example road network (Figure 1). Similar to destination D_1, the travel time form O to D_2 via the shortcut (dotted line) is equal to the fraction of the overall traffic traveling via the shortcut (heading both to D_1 and D_2). In contrast to destination D_1, the travel time on the highway (solid line) to destination D_2 is only 0.5 ($t_{D_2}(v_h) = 0.5$).

Let z be the fraction of agents heading to destination D_1 ($1 - z$ heading to D_2). Next we prove that, in this example, aligning SO and UE requires assigning the shortcut a dynamic toll that is a function of z. We begin with a supporting Lemma.

LEMMA 2. *At equilibrium, there cannot both be agents heading to D_1 via the highway and heading to D_2 via the shortcut.*

Proof: At equilibrium, a vehicle heading to D_1 will consider traversing the highway only if traveling the shortcut takes at least one unit of time. On the other hand, a vehicle heading to D_2 will consider traversing the shortcut only if traveling the shortcut takes no more than 0.5. □

Given Lemma 2, there are three complementary cases that may apply:

1. All agents taking the highway are heading to D_2 (v_h in total) and some agents taking the shortcut are also heading to D_2. This case requires that the fraction of agents heading to D_1 is smaller than the fraction taking the shortcut ($z < v_s$).

2. All agents heading to D_2 are taking the highway ($1-z$ in total) and some agents heading to D_1 are also taking the highway ($(v_h) - (1 - z) = z - v_s$ in total). This case requires that the fraction of agents heading to D_1 is greater than the fraction taking the shortcut ($z > v_s$).

3. All agents heading to D_2 are taking the highway and all agents heading to D_1 are taking the shortcut. This case requires $z = v_s$.

The proportional travel time (travel time multiplied by the fraction of traffic) for case 1 is v_s^2 for the shortcut and $0.5(v_h)$ for the highway. The average travel time in this case is $f_1(v_s) = v_s^2 + 0.5(1 - v_s)$. As a result, $f_1'(v_s) = 2v_s - 0.5$ and $v_s = 0.25$ in the SO. In order for the SO and UE to align, a toll of 0.25 should be applied on the shortcut. This case applies when $z < v_s \Rightarrow z < 0.25$.

The proportional travel time for case 2 is v_s^2 for the shortcut and $0.5(1 - z) + 1(z - v_s)$ for the highway. The average travel time is $f_2(v_s) = v_s^2 + 0.5(1 - z) + 1(z - v_s)$. $f_2'(v_s) = 2v_s - 1$ and $v_s = 0.5$ in the SO. This case applies when $z > v_s \Rightarrow z > 0.5$. Finally, the proportional travel time for case 3 (where $z = v_s$) is z^2 for the shortcut and $0.5(1 - z)$ for the highway. The average travel time is $f_3(z) = z^2 + 0.5(1 - z)$.

LEMMA 3. *For $0.25 \le z \le 0.5$ case 3 represents the SO.*

Proof: If case 3 is not the SO than directing some of the flow heading to D_1 via the highway **xor** directing some of the flow heading to D_2 via the shortcut can decrease average travel time.[11] We treat each of these two cases separately:

[11] It is never beneficial to direct vehicles heading to D_1 via the highway while simultaneously directing vehicles heading to D_2 via the shortcut as proven in Lemma 2.

1. **Directing $\delta > 0$ flow heading to D_1 via the highway:** In this case, the average travel time equals $(z - \delta)^2 + 0.5(1 - z) + 1(\delta) = z^2 + 0.5(1 - z) - 2\delta z + \delta^2 + \delta = f_3(z) - 2\delta z + \delta^2 + \delta$. Reducing average travel time (compared to $f_3(z)$) requires $-2\delta z + \delta^2 + \delta < 0 \Rightarrow \delta < 2z - 1$. There is no $\delta > 0$ that satisfies this requirement when $z < 0.5$.

2. **Directing $\delta > 0$ flow heading to D_2 via the shortcut:** In this case, the average travel time equals $(z + \delta)^2 + 0.5(1 - z - \delta) = z^2 + 0.5(1 - z) + 2\delta z + \delta^2 - 0.5\delta = f_3(z) + 2\delta z + \delta^2 - 0.5\delta$. Reducing average travel time (compared to $f_3(z)$) requires $2\delta z + \delta^2 - 0.5\delta < 0 \Rightarrow \delta < 0.5 - 2z$. There is no $\delta > 0$ that satisfies this requirement when $z > 0.25$.

□

According to Lemma 3, case 3 is SO for $0.25 \leq z \leq 0.5$. For case 3 to apply, traveling the shortcut should cost more than 0.5 and less than 1 (travel time and toll combined). Since the travel time on the shortcut equals $p = z$ and $0.25 \leq z \leq 0.5$, any toll from the range $(0.25, 0.5)$ would lead to the SO and UE to align.

In this example problem, achieving SO=UE requires assigning the shortcut a dynamic toll that is a function of z: For $z < 0.25$ the toll should be 0.25, for $0.25 \leq z \leq 0.5$, any toll value from the range $(0.25, 0.5)$ and for $z > 0.5$ the toll should be 0.5.

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