Action models for knowledge and awareness

Hans van Ditmarsch
Logic
University of Seville, Spain
hvd@us.es

Tim French
Computer Science and Software Engineering
University of Western Australia
tim@csse.uwa.edu.au

Fernando R. Velázquez-Quesada
Logic
University of Seville, Spain
FRVelazquezQuesada@us.es

ABSTRACT

We consider semantic structures and logics that differentiate between being uncertain about a proposition, being unaware of a proposition, becoming aware of a proposition and getting to know the truth value of a proposition. We give a unified setting to model all this variety of static and dynamic aspects of awareness and knowledge, without any constraints on the modal properties of knowledge (or belief — such as introspection) or on the interaction between awareness and knowledge (such as awareness introspection). Our primitive epistemic operator is called *speculative knowledge*. This is different from the better known *implicit knowledge*, now definable, which plays a more restricted role. Some dynamic semantic primitives that are elegantly definable in our setting are the actions of 'becoming aware of a propositional variable', 'implicit knowledge', 'addressing a novel issue in an announcement', and also more complex ways in which an agent can become aware of a novel issue by way of increasing the complexity of the epistemic model.

Categories and Subject Descriptors

I.2.4 [**Artificial Intelligence**]: Knowledge Representation Formalisms and Methods—*Modal Logic*

General Terms

Theory

Keywords

Modal logic, Epistemic logic, Awareness, Dynamics

1. INTRODUCTION

We consider a framework that differentiate between (i) agents being uncertain about the value of a proposition, (ii) agents being unaware of a proposition, (iii) agents becoming aware of propositions, and (iv) agents being informed of the truth of propositions of which they were already aware.

EXAMPLE 1. *Alfred likes football. He supports the English national football team and he is aware that yesterday there was a match between England and The Netherlands, but he does not know which team won. He does not like other sports, so he is unaware that the English national rugby team played yesterday too. When looking online for the football match's result, Alfred sees a web page with header* "The English team faced a complicated rugby match yesterday", *hence becoming aware of that match (without getting to know who won). He keeps looking for the score of the football match and finally finds it: England 2 - The Netherlands 1.*

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In this paper we give a unified setting to model all this variety of static and dynamic aspects of awareness and knowledge, without any constraints on the modal properties of knowledge (or belief - such as introspection) or on the interaction between awareness and knowledge (such as awareness introspection). Our work is rooted in: the tradition of epistemic logic [11] and in particular multi-agent epistemic logic [13, 4]; in various works on the interaction of awareness and knowledge [3, 14, 15, 9] — including a relation to recent works like [10, 7, 8]; and in modal logical research in propositional quantification, starting in the 1970s with [5] and followed up by work on bisimulation quantifiers [24, 12, 6].

Works treating awareness either follow a *semantically* flavoured approach, where awareness concerns propositional variables in the valuation [15, 9], or a more *syntactically* flavoured approach, where awareness concerns all formulas of the language in a given set, in order to model 'limited rationality' of agents [3, 19]. Our proposal falls straight into the semantic corner: within the limits of their awareness, agents are fully rational. Our proposal extends the work of [20, 21] — these works treat the static interaction of knowledge and awareness but not its dynamics, and in particular not the wide variety of dynamics in action models.

2. STRUCTURES

Our semantic model augments standard epistemic (Kripke) models with a parameter to define the notion of awareness.

DEFINITION 1 (EPISTEMIC AWARENESS MODEL). *Given a countable set of atomic propositions P and a finite set of agents N, where these sets are disjoint, an* epistemic awareness model *is a tuple $M = (S, R, \mathcal{A}, V)$ where*

- S *is the* domain*: a non-empty set of (propositional)* states *also called* worlds *and also denoted by $\mathcal{D}(M)$;*
- $R : N \to \mathcal{P}(S \times S)$ *is an* accessibility function *assigning to each agent $i \in N$ a binary accessibility relation;*
- $\mathcal{A} : N \to S \to \mathcal{P}(P)$ *is an* awareness function *returning the set of atomic propositions agent $i \in N$ is aware of at state $s \in S$ (agent i's awareness state at s);*

- $V : P \to \mathcal{P}(S)$ *is a* valuation function *indicating, for each atomic proposition $p \in P$, the set of states $V(p)$ in which the proposition is true.*

We will write R_i for $R(i)$ and $\mathcal{A}_i$ for $\mathcal{A}(i)$. A pair (M, s) with M an epistemic awareness model and s a state in $\mathcal{D}(M)$, the evaluation state, is an epistemic awareness state.

In epistemic awareness models, awareness is specified by the awareness function. Our notion of awareness is given in terms of a set of atomic propositions, different from the models of general awareness of [3] in which awareness is given in terms of an arbitrary set of formulas.

Just like with epistemic models, we can impose requirements on epistemic awareness models. The standard ones are properties of the accessibility relations, like reflexivity, seriality or transitivity. We do not discuss closure properties of the awareness set $\mathcal{A}_i(s)$ as done in [3] because $\mathcal{A}_i(s)$ is a set of atoms rather than an arbitrary set of formulas. There are also properties that relate the accessibility relations with the awareness function. One interesting example is the property of *awareness introspection* [9], which holds when awareness sets are preserved by the accessibility relation: $p \in \mathcal{A}_i(s)$ implies $p \in \mathcal{A}_i(t)$ for every state t such that $(s, t) \in R_i$. As interesting as such models can be, we make no commitment to any particular property, focussing on the most general class of epistemic awareness models.

A notion of *bisimulation* [16] between epistemic awareness models can be obtained by extending the standard definition with a clause that asks for the awareness function to assign, for every agent, the same set of atomic propositions in bisimilar states. The bisimulation requirements can also be restricted to a subset of atomic propositions; this makes sense in our setting because agents may not be aware of every atom. But we go one step further: agents may be aware of different atomic propositions in different states, so in order to indicate when two epistemic awareness models are indistinguishable from the perspective of an agent (or a set of them), we ask for an additional restriction. The result is called *awareness bisimulation*.

DEFINITION 2 (AWARENESS BISIMULATION). *Let $M = (S, R, \mathcal{A}, V)$ and $M' = (S', R', \mathcal{A}', V')$ be two epistemic awareness models. For any $Q \subseteq P$, a relation $\mathfrak{R}[Q] \subseteq (S \times S')$ is called a Q-*awareness bisimulation between M and M' *if, for every $(s, s') \in \mathfrak{R}[Q]$:*

- **atoms:** *for all $p \in Q$, $s \in V(p)$ iff $s' \in V'(p)$;*
- **aware:** *for all $i \in N$, $Q \cap \mathcal{A}_i(s) = Q \cap \mathcal{A}'_i(s')$;*
- **forth:** *for all $i \in N$, if $t \in S$ and $R_i(s, t)$ then there is a $t' \in S'$ such that $R'_i(s', t')$ and $(t, t') \in \mathfrak{R}[Q \cap \mathcal{A}_i(s)]$;*
- **back:** *for all $i \in N$, if $t' \in S'$ and $R'_i(s', t')$ then there is a $t \in S$ such that $R_i(s, t)$ and $(t, t') \in \mathfrak{R}[Q \cap \mathcal{A}'_i(s')]$.*

*We say that (M, s) and (M', s') are Q-*awareness-bisimilar *(notation: $(M, s) \leftrightarroweq^{Q} (M', s')$) if there is a Q-awareness bisimulation between M and M' that contains (s, s').*

The **aware** clause is the additional 'atomic' requirement, given the nature of our models. The further requirement that distinguishes awareness bisimulation from a restricted bisimulation appears in the **forth** and **back** clauses: instead of being $\mathfrak{R}[Q]$-bisimilar, states t and t' need to be just $\mathfrak{R}[Q \cap \mathcal{A}_i(s)]$-bisimilar and $\mathfrak{R}[Q \cap \mathcal{A}'_i(s')]$-bisimilar, respectively — note that by the **aware** clause, $Q \cap \mathcal{A}_i(s)$ and $Q \cap \mathcal{A}'_i(s')$ are the same. The motivation is very simple: two states are Q-awareness-bisimilar for an agent i if they appear Q-identical to her. Since she does not need to be aware of every atom, the states just have to be identical up to those atoms of Q *the agent is aware of.* Then, for the **atoms** clause, we just need to check that both states coincide in the truth values of atoms in Q. Moreover, in the **forth** clause, the bisimulation for state t is further restricted to the propositions visible for agent i in s, the i-predecessor of t; similarly for **back**. This ensures us that only atoms the agent is aware of at the current state will matter when looking for a difference in accessible worlds. This chaining requirement was present in epistemic awareness structures since its inception in [3].

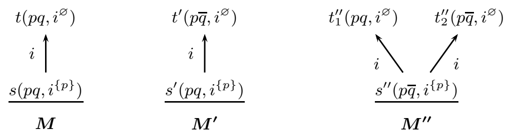
Awareness bisimulation gives us a form of *observational equivalence* among epistemic awareness models. If an agent i is in state s, then her perspective is that of $\mathcal{A}_i(s)$-awareness-bisimilarity: she cannot distinguish the current model from those that are in its $\mathfrak{R}[\mathcal{A}_i(s)]$ equivalence class. This can be generalized for a set of agents N; their perspective is that of $\bigcup_{i \in N} \mathcal{A}_i(s)$-awareness-bisimilarity, so two epistemic awareness states (M, s) and (M', s') are *observationally equivalent* for the agents in N iff no one can distinguish them, that is, iff they are $\mathcal{A}_i(s)$-awareness bisimilar for all $i \in N$:

$$(M, s) \leftrightarroweq^{\bigcup_{i \in N} \mathcal{A}_i(s)} (M', s') \ .$$

If every agent is aware of every atom at every state, we get standard (restricted) bisimulation: for agents with full awareness we go back to the standard multi-agent epistemic situation, where awareness plays no role.

EXAMPLE 2. *The diagram below shows three epistemic awareness states, (M, s), (M', s') and (M'', s''). In it, each state shows its name, the truth value it assigns to atoms (the overline indicates falsity) and the awareness set for agent i in the format $i^{\mathcal{A}_i(s)}$; the evaluation states are underlined.*

The states (M, s), (M', s') and (M'', s'') are $\{p\}$-awareness bisimilar (e.g., $\{(s, s''), (t, t''_1), (t, t''_2)\}$ is a $\{p\}$-awareness bisimulation between the first and the third). This is because not only the s-states (s, s' and s'') coincide in the truth value of and in agent i's awareness of every atom in $\{p\}$ (clauses **atoms** *and* **aware** *of the definition — note how the truth value of q is irrelevant), but also because every t-state in each model is $\{p\}$-awareness bisimilar to every t-state in the others (clauses* **back** *and* **forth**, *given that $\{p\} \cap \mathcal{A}_i(s) = \{p\}$). (Indeed, the awareness of i is $\varnothing$ in all four t-states; if it had been $\{p\}$ in all four it would have worked as well.)*



3. ACTION MODELS

Epistemic awareness models allow us to represent the information of agents who may be uncertain of the truth value of atomic propositions and may be even unaware of some of them. But, of course, the information of such agents can change via different informational acts. The general structure that we introduce now, epistemic awareness action models, allow us to represent, as far as we know, *any* conceivable form of awareness change or of knowledge change.

Definition 3 (Epistemic awareness action model). *Let P and N be sets of atomic propositions and agents, respectively, with properties as before. An* epistemic awareness **action** model *is a tuple* $\mathsf{M} = (\mathsf{S}, \mathsf{R}, \mathcal{A}, \mathsf{pre}, \mathsf{post})$ *where*

- S *is a non-empty domain: a set of* actions *also denoted by* $\mathcal{D}(\mathsf{M})$*;*
- $\mathsf{R} : N \to \mathcal{P}(\mathsf{S}\times\mathsf{S})$ *is an* accessibility function*, assigning to each agent* $i \in N$ *an accessibility relation* $\mathsf{R}(i)$*;*
- $\mathcal{A} : \{+,-\} \to N \to \mathsf{S} \to \mathcal{P}(P)$ *is an* awareness **change** function*, indicating the* disjoint sets *of atoms each agent* $i \in N$ *will become aware (+) and unaware of (-) after the execution of* $\mathsf{s} \in \mathsf{S}$*;*
- $\mathsf{pre} : \mathsf{S} \to \mathcal{L}$ *is a* precondition function *that specifies, for each action* $\mathsf{s} \in \mathsf{S}$*, the requirement for its execution;*
- $\mathsf{post} : \mathsf{S} \to P \to \mathcal{L}$ *is a* postcondition function *specifying, for each action in* $\mathsf{s} \in \mathsf{S}$*, how the truth value of each atomic proposition* $p \in P$ *will change.*

A pair (M, s) *with* M *an epistemic awareness action model and* s *an action in* $\mathcal{D}(\mathsf{M})$ *is an* epistemic awareness action.

The language $\mathcal{L}$ in terms of which we specify the preconditions and postconditions is a fixed parameter of this definition. In Section 4 we give an integrated approach for the syntax and semantics of a logical language with epistemic awareness action models, wherein $\mathcal{L}$ is not a fixed parameter. As before, we write R_i for $\mathsf{R}(i)$; also, we write $\mathcal{A}_i^+$ for $\mathcal{A}(+)(i)$ and $\mathcal{A}_i^-$ for $\mathcal{A}(-)(i)$.

We can now indicate how an epistemic awareness action model modifies an epistemic awareness model. The following definition is essentially the *product update* of [1] with an additional clause that deals with awareness.

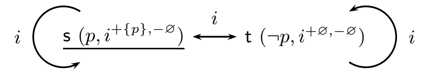
Definition 4 (Action model execution). *Let* $M = (S, R, \mathcal{A}, V)$ *and* $\mathsf{M} = (\mathsf{S}, \mathsf{R}, \mathcal{A}, \mathsf{pre}, \mathsf{post})$ *be an epistemic awareness model and an epistemic awareness action model, respectively. The epistemic awareness model* $M \otimes \mathsf{M} = (S', R', \mathcal{A}', V')$ *– the result of executing* M *in* M *– is defined as follows:*

$$\begin{aligned} S' &:= \{(s, \mathsf{s}) \mid (M, s) \models \mathsf{pre}(\mathsf{s})\} \\ R'_i &:= \{((s, \mathsf{s}), (s', \mathsf{s}')) \mid (s, s') \in R_i \text{ and } (\mathsf{s}, \mathsf{s}') \in \mathsf{R}_i\} \\ \mathcal{A}'_i(s, \mathsf{s}) &:= (\mathcal{A}_i(s) \cup \mathcal{A}_i^+(\mathsf{s})) \setminus \mathcal{A}_i^-(\mathsf{s}) \\ V'(p) &:= \{(s, \mathsf{s}) \mid (M, s) \models \mathsf{post}(\mathsf{s}, p)\} \end{aligned}$$

The new set of states is given by the restricted Cartesian product of S and S: a pair (s, s) will be a state in the new model iff s satisfies s's precondition in M. Since the precondition is given as a formula of a language $\mathcal{L}$, we assume a satisfiability relation $\models$ that indicates whether a formula of $\mathcal{L}$ evaluates to true or false in an epistemic awareness state. For the accessibility relation of the new model, we simply combine the accessibility relation of the 'static' and the 'action' model: a state (s', s') is R'_i-accessible from state (s, s) iff s' is R_i-accessible from s, and s' is R_i-accessible from s. For the awareness function of each agent i in each state (s, s), we add the atoms in $\mathcal{A}_i^+(\mathsf{s})$ and remove the atoms in $\mathcal{A}_i^-(\mathsf{s})$ (in whatever order—we require these sets to be disjoint). Finally, for the valuation, an atomic proposition p is true at state (s, s) iff s satisfies $\mathsf{post}(\mathsf{s}, p)$ in M.

The epistemic awareness *state* that results from executing (M, s) in (M, s) is given by$(M \otimes \mathsf{M}, (s, \mathsf{s}))$ whenever $(M, s) \models \mathsf{pre}(\mathsf{s})$.

Example 3. *Below is the diagram of an epistemic awareness action model. Each one of the actions indicates also its precondition and its awareness change function (the latter with the format* $i^{+\mathcal{A}_i^+, -\mathcal{A}_i^-}$ *for every agent* i*). Here the postcondition function is trivial:* $\mathsf{post}(\mathsf{s})(p) = \mathsf{post}(\mathsf{t})(p) = p$*.*



The only difference between actions s *and* t *is the precondition and the fact that* s *adds* p *to agent* i*'s awareness. This epistemic awareness action model can be seen as a form of 'conditionally becoming aware', where the agent becomes aware of* p *in the states in which* p *holds, and keeps her old awareness in states in which* p *fails (see Definition 13). We will see more examples of epistemic awareness action models and their execution in Section 8.*

4. LANGUAGE

Section 2 presented a semantic structure, epistemic awareness model, for representing the information of agents that do not need to be aware of all the relevant atoms. Then Section 3 introduced another structure, epistemic awareness action model, that allows us to represent diverse actions that can change the agents' information. However, a parameter in these action models was a logical language.

We can also do this 'all at once': an inductively defined language, with an appropriately compositional semantics, wherein a countable set of 'action model shapes' features as a parameter in the language. This language allows us to describe epistemic awareness models and how they change after an epistemic awareness action model is applied.

Definition 5 (Language). *Given sets of atomic propositions* P *and agents* N *as before, the language* $\mathcal{L}$ *of the* logic of knowledge and awareness change *is given by*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i^S \varphi \mid A_i \varphi \mid [\mathsf{M}, \mathsf{s}]\varphi$$

where $i \in N$*,* $p \in P$ *and* (M, s) *is a epistemic awareness action satisfying that:*

(a) its domain is finite;

(b) the postcondition function changes the valuation of only a finite number of atomic propositions.

(c) the awareness function returns two finite sets of atomic propositions;

The language $\mathcal{L}$ extends multi-agent epistemic logic with two operators. The first, $A_i\varphi$, expresses that agent i *is* aware of φ; the second, $[\mathsf{M}, \mathsf{s}]\varphi$, stands for *"after (every) execution of the epistemic awareness action* (M, s)*,* φ *is the case"*. Implication $\to$, disjunction $\vee$, and equivalence $\leftrightarrow$ are defined by abbreviation as usual, and $L_i^S\varphi$ is defined as $\neg K_i^S \neg\varphi$. Note that $\top$ is explicitly a primitive in the language; we do not define it with an abbreviation of the form $p \vee \neg p$ for some atom p because we want every agent to be aware of $\top$ even if they are unaware of every atomic proposition.

The epistemic operator K_i^S is non-standard. It stands for agent i's *speculative knowledge* (a notion called implicit knowledge in [20]), and its semantic interpretation will be introduced in the next section. Explicit knowledge is defined by $K_i^E\varphi$ iff $K_i^S\varphi \wedge A_i\varphi$ (cf. [3]).

The case $[\mathsf{M}, \mathsf{s}]\varphi$ of the inductive language definition is indeed a proper induction, because it can be seen as an operation on the set of all preconditions $\mathsf{pre}(\mathsf{t})$ of actions in the action model, and the formula φ. In the definition, all these are supposed to be of type formula and lower in the inductive hierarchy. The restrictions on epistemic awareness actions in the language are imposed so that the inductively defined language $\mathcal{L}$ is well-defined—the class of epistemic awareness actions needs to be enumerable, and (as an independent requirement) the number of arguments in the inductive construct $[\mathsf{M}, \mathsf{s}]\varphi$ needs to be finite. For this we need all three finiteness requirements. For (a) this was known since [22]; for (b) this was known since dynamic epistemic logics modelling factual change, namely [18]; we can make our logic tick by the novel requirement (c). If we merely wished a semantic treatment of epistemic awareness actions, the finiteness requirements would not be needed.

As mentioned, our notion of awareness is semantic: an agent is aware of a formula if she is aware of the set of atomic propositions in that formula. For this, we need to define the *free variables* of a formula.

Definition 6 (Free variables). *The free propositional variables of $\varphi \in \mathcal{L}$ are defined inductively in the following way: $v(\top) := \varnothing$; $v(p) := \{p\}$; $v(\neg\varphi) := v(\varphi)$; $v(\varphi \wedge \psi) := v(\varphi) \cup v(\psi)$; $v(K_i^S\varphi) := v(\varphi)$, $v(A_i\varphi) := v(\varphi)$, $v([\mathsf{M}, \mathsf{s}]\varphi) := \bigcup_{\mathsf{t} \in \mathcal{D}(\mathsf{M})} v(\mathsf{pre}(\mathsf{t})) \cup \bigcup_{\mathsf{t} \in \mathcal{D}(\mathsf{M}), p \text{ changes}} v(\mathsf{post}(\mathsf{t})(p)) \cup v(\varphi)$ where 'p changes' means that $p \in \mathcal{A}_i^+(\mathsf{t})$ or $p \in \mathcal{A}_i^-(\mathsf{t})$ for some agent i.*[1]

Concerning $v([\mathsf{M}, \mathsf{s}]\varphi)$, recall that the modality $[\mathsf{M}, \mathsf{s}]$ represents an inductive case of the language with the preconditions $\mathsf{pre}(\mathsf{t})$, postconditions $\mathsf{post}(\mathsf{t})(p)$ and the formula φ as arguments.

5. SEMANTICS

Having defined the structures and the language to describe them, we now define the semantic interpretation.

Definition 7 (Semantics). *Let $M = (S, R, \mathcal{A}, V)$ and $s \in \mathcal{D}(M)$ be given. The semantics for $\top$, atoms, negation and conjunction is as usual. For the rest,*

$$
\begin{array}{lll}
(M, s) \models K_i^S\varphi & \textit{iff} & \forall (s,t) \in R_i \textit{ and } \forall (M', t') \underline{\leftrightarrow}^{\mathcal{A}_i(s)} (M, t), \\
& & (M', t') \models \varphi \\
(M, s) \models A_i\varphi & \textit{iff} & v(\varphi) \subseteq \mathcal{A}_i(s) \\
(M, s) \models [\mathsf{M}, \mathsf{s}]\varphi & \textit{iff} & (M, s) \models \mathsf{pre}(\mathsf{s}) \Rightarrow (M \otimes \mathsf{M}, (s, \mathsf{s})) \models \varphi
\end{array}
$$

The set of validities of the language $\mathcal{L}$ is called the logic L.

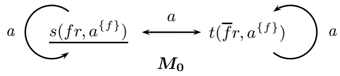
An agent speculatively knows φ when φ remains true in all accessible states for *every* possible interpretation of all propositions she is unaware of. We achieve this by composing the agent's accessibility relation with bisimulation restricted to propositions the agent is aware. This speculative knowledge is not implicit knowledge K^I in the Fagin et al. [3] sense where this corresponds to mere modal accessibility

$$(M, s) \models K_i^I\varphi \quad \text{iff} \quad \forall (s,t) \in R_i, (M, t) \models \varphi$$

[1]The clause for $[\mathsf{M}, \mathsf{s}]\varphi$ makes the semantics of $A_i\varphi$ (below) work but is too restrictive given our intuitions of awareness. For example, given that $[p := q]p \leftrightarrow q$, we want that $v([p := q]p) = v(q) = q$, but our definition gives $\{p, q\}$. This will be further investigated.

Example 4. *Consider the epistemic awareness states of Example 2. In all three cases agent i knows p explicitly but she does not know q explicitly at s because the accessible t is p-awareness bisimilar to (e.g.) t' where q is false. If she becomes aware of q in state s, then she will know q explicitly: any state $\{p, q\}$-awareness bisimilar to t* must *satisfy q.*

Example 5. *Here is an epistemic awareness state for Alfred's situation before his online search (Example 1); we will use formulas of $\mathcal{L}$ to show that it represents the situation faithfully. We will use f (r) to indicate that England won the football (rugby) match.*



$\boldsymbol{M_0}$

At (M_0, s), Alfred (a) is aware of the football match (f) but unaware of the rugby match (r) because A_af and $\neg A_ar$ hold (f is in $\mathcal{A}_a(s)$ but r is not). Moreover, Alfred does not have any speculative (and hence does not have any explicit) knowledge about who won any of the matches since neither of the following formulas hold at (M_0, s): K_a^Sf, $K_a^S\neg f$, K_a^Sr, $K_a^S\neg r$. This is because for $f, \neg f, r$ and $\neg r$ we can find a state u reachable from s (e.g., t for the first formula; s for the second, third and fourth) and an epistemic awareness state (M', u') that is $\{f\}$-awareness-bisimilar to (M_0, u) and in which the given formula fails. For the first formula, any state $\{f\}$-awareness-bisimilar to (M_0, t) should satisfy $\neg f$; analogously for the second. For the third and fourth, a state $\{f\}$-awareness-bisimilar to (M_0, s) can assign any truth value to r.

Our main result is that in epistemic awareness bisimilar states the agents have the same explicit knowledge; this justifies a more complex form of bisimulation and the notion of speculative knowledge that is more complex than 'standard' (implicit) knowledge. Although there is fairly direct proof of this (consisting of *that* case of the following inductive proof), we present it in the context of a more general, staged, result. Below, we will use the following valid observation. Let $v(\varphi) = Q' \subseteq Q$ and take an epistemic awareness state (M, s): φ is true in all states (M', s') that are Q-bisimilar to (M, s) iff φ is true in all states (M'', s'') that are Q'-bisimilar to (M, s). From left to right the statement holds because we look at less atoms; from right to left it holds because the extra atoms do not appear in φ. In other words, variation in variables *not* occurring in φ does not affect its value. For $Q \setminus Q' = \{p\}$ this corresponds to the validity $\varphi \leftrightarrow \forall p\varphi$ whenever $p \notin v(\varphi)$, in bisimulation quantified logic.

Theorem 8. *If $(M, s) \underline{\leftrightarrow}^Q (M', s')$ and $v(\varphi) \subseteq Q$, then $(M, s) \models \varphi$ iff $(M', s') \models \varphi$.*

Proof. To be more precise, the statement we prove is "Let $\varphi \in \mathcal{L}$. Then for every epistemic awareness states (M, s) and (M', s'), if $(M, s) \underline{\leftrightarrow}^Q (M', s')$ and $v(\varphi) \subseteq Q$, then $(M, s) \models \varphi$ iff $(M', s') \models \varphi$." The proof is by induction on φ, and the cases of interest are $K_i^S\varphi$, $A_i\varphi$, and $[\mathsf{M}, \mathsf{s}]\varphi$.

- **Base case p:** From the **atoms** clause of bisimulation, the fact that $p \in Q$, and $(M, s) \underline{\leftrightarrow}^Q (M', s')$, it follows that $(M, s) \models p$ iff $(M', s') \models p$.

- **Inductive case $\neg\varphi$:** Showing that $(M, s) \models \neg\varphi$ iff $(M', s') \models \neg\varphi$ is equivalent to showing that $(M, s) \not\models \varphi$ iff $(M', s') \not\models \varphi$; swapping the order delivers $(M, s) \models \varphi$ iff $(M', s') \models \varphi$ which follows by induction.

- **Inductive case $\varphi \wedge \psi$:** Let $(M,s) \underline{\leftrightarrow}^Q (M',s')$ and $v(\varphi \wedge \psi) \subseteq Q$ (so $v(\varphi), v(\psi) \subseteq Q$). Now $(M,s) \models \varphi \wedge \psi$ iff $(M,s) \models \varphi$ and $(M,s) \models \psi$, iff (using the induction hypothesis on φ and on ψ) $(M',s') \models \varphi$ and $(M',s') \models \psi$, i.e., $(M',s') \models \varphi \wedge \psi$.

- **Inductive case $K_i^S\varphi$:** Let $(M,s) \underline{\leftrightarrow}^Q (M',s')$ and $v(K_i^S\varphi) \subseteq Q$ (so $v(\varphi) \subseteq Q$ as well). Assume $(M,s) \models K_i^S\varphi$. Take some $t' \in R_i(s')$ and (N',u') such that $(M',t') \underline{\leftrightarrow}^{\mathcal{A}'_i(s')} (N',u')$; we will show that $(N',u') \models \varphi$. Because $v(\varphi) \subseteq Q$ and the observation above it is sufficient to prove this for an arbitrary (N',u') with $(M',t') \underline{\leftrightarrow}^{Q \cap \mathcal{A}'_i(s')} (N',u')$.

  From **back** it follows that there is a $t \in R_i(s)$ such that $(M,t) \underline{\leftrightarrow}^{Q \cap \mathcal{A}'_i(s')} (M',t')$. From $(M,t) \underline{\leftrightarrow}^{Q \cap \mathcal{A}'_i(s')} (M',t')$ and $(M',t') \underline{\leftrightarrow}^{Q \cap \mathcal{A}'_i(s')} (N',u')$ follows $(M,t) \underline{\leftrightarrow}^{Q \cap \mathcal{A}'_i(s')} (N',u')$ (bisimilarity is an equivalence relation).

  From that, the semantics of $K^S\varphi$ and again the observation that we may restrict $\mathcal{A}_i(s')$ bisimilarity to $Q \cap \mathcal{A}'_i(s')$ bisimilarity, it follows immediately that $(N',u') \models \varphi$.

  The other direction is similar. Note that, somewhat surprisingly, we have not used induction in this inductive case of the proof.

- **Inductive case $A_i\varphi$:** Let $(M,s) \underline{\leftrightarrow}^Q (M',s')$ and also $v(A_i\varphi) \subseteq Q$ (so $v(\varphi) \subseteq Q$). Now, $(M,s) \models A_i\varphi$ implies $v(\varphi) \subseteq \mathcal{A}_i(s)$, and since $v(\varphi) \subseteq Q$, we have $v(\varphi) \subseteq \mathcal{A}_i(s) \cap Q$. By the **aware** clause of bisimulation, $\mathcal{A}_i(s) \cap Q = \mathcal{A}'_i(s') \cap Q$, so $v(\varphi) \subseteq \mathcal{A}'_i(s') \cap Q$ and hence $v(\varphi) \subseteq \mathcal{A}'_i(s')$, which implies $(M',s') \models A_i\varphi$. The other direction is similar.

- **Inductive case $[\mathsf{M},\mathsf{s}]\varphi$:** Suppose $(M,s) \models [\mathsf{M},\mathsf{s}]\varphi$. Then $(M,s) \models \mathsf{pre}(\mathsf{s})$ implies $((M \otimes \mathsf{M}),(s,\mathsf{s})) \models \varphi$. By induction we have that $(M,s) \models \mathsf{pre}(\mathsf{s})$ iff $(M',s') \models \mathsf{pre}(\mathsf{s})$. The modal product construction in $(M \otimes \mathsf{M})$ is well-known to be bisimulation preserving (see e.g. the original publication [1]); an easily observable fact when one realizes that pairs in the new accessibility relation require the first argument to be in the accessibility relation in the original model (given $(t,t') \in \mathfrak{R}[Q]$, the induced bisimulation $\mathfrak{R}'[Q]$ on the product is defined as $((t,\mathsf{t}),(t',\mathsf{t})) \in \mathfrak{R}'[Q]$). Of course, for our present logic we also have to satisfy the requirement **aware**. In the model $(M \otimes \mathsf{M})$ the level of awareness $\mathcal{A}_i(t,\mathsf{t})$ is a function of the prior level of awareness $\mathcal{A}_i(t)$ in t and the deleted or added propositional variables $\mathcal{A}_i^+(\mathsf{t})$ and $\mathcal{A}_i^-(\mathsf{t})$. As the prior awareness $\mathcal{A}_i(t)$ is the same in any Q bisimilar state t', and the added or deleted atoms are also the same, the posterior awareness must therefore also be the same for any pairs (t,t) and (t',t) in the Q-bisimulation. Therefore, $((M \otimes \mathsf{M}),(s,\mathsf{s})) \underline{\leftrightarrow}^Q ((M' \otimes \mathsf{M}),(s',\mathsf{s}))$. Now using induction again, we conclude $((M' \otimes \mathsf{M}),(s',\mathsf{s})) \models \varphi$, and from that and $(M',s') \models \mathsf{pre}(\mathsf{s})$ we conclude $(M',s') \models [\mathsf{M},\mathsf{s}]\varphi$.

□

COROLLARY 9. *If $(M,s) \underline{\leftrightarrow}^P (M',s')$, then $(M,s) \models \varphi$ iff $(M',s') \models \varphi$.*

PROOF. *Apply Theorem 8 with $Q = P$.* □

COROLLARY 10. *Epistemic awareness bisimilar states coincide in K^E. For $i \in N$ and $\varphi \in \mathcal{L}$, if $(M,s) \models K_i^E\varphi$ and $(M,s) \underline{\leftrightarrow}^{\mathcal{A}^i(s)} (M',s')$ then $(M',s') \models K_i^E\varphi$.*

PROOF. *Apply Theorem 8 with $Q = \mathcal{A}_i(s)$, also using that $v(K_i^E\varphi) = v(\varphi) \subseteq \mathcal{A}_i(s)$.* □

This is a good moment to point out that if we define explicit knowledge in the 'standard' way, namely as awareness plus modal accessibility: $K_i^{EX}\varphi \leftrightarrow (K^I\varphi \wedge A_i\varphi)$ [3], then Corollary 10 fails. Epistemic awareness states that are bisimilar up to the awareness of a given agent do not need to provide the agent the same explicit knowledge in the awareness plus modal accessibility sense.

EXAMPLE 6. *Consider the following two models:*

$$\boldsymbol{M}\!: s(p, i^{\{p\}}) \xrightarrow{i} t(p, i^{\varnothing}) \xrightarrow{i} u(p, i^{\varnothing})$$

$$\boldsymbol{M'}\!: s'(p, i^{\{p\}}) \xrightarrow{i} t'(p, i^{\varnothing}) \xrightarrow{i} u'(\overline{p}, i^{\varnothing})$$

Model M has domain $\{s,t,u\}$, a single agent with accessibility relation $R = \{(s,t),(t,u)\}$, atom p true everywhere, and the agent is aware of p only in s. Model M' is like M except that p is false in u'. The only difference between M and M' is therefore p's truth value on the u's states. Observe how (M,u) and (M',u') are $\varnothing$-bisimilar; then, because of this and because the t's states coincide in p's truth value and in i's awareness of p, the epistemic awareness states (M,t) and (M',t') are $\{p\}$-awareness bisimilar. This and the fact that s and s' coincide p's truth value and in i's awareness of p makes the epistemic awareness states (M,s) and (M',s') $\{p\}$-awareness bisimilar too. Nevertheless, we can find formulas with atoms in $\{p\}$ that are known explicitly (in the awareness plus modal accessibility sense) at (M,s) but not at (M',s'). Formula K^Ip is an example, as $K^{EX}K^Ip$ holds at the first, but not at the second.

Of course this does not say that [3] is incorrect: their setting is for $KD45$ models and then our counterexample, which does not satisfy transitivity, does not work there.

6. PARTIAL AXIOMATIZATION

The logic L is partially axiomatized by the axioms and rules of Table 1. The complete axiomatization in [21] can be seen as the special case for the action model encoding 'all agents become aware of atom p in the entire model', see Definition 13, below. The complete axiomatization of standard action model logic [1] might be seen as the special case for actions that only change knowledge but not awareness, see Section 7, below.

The table starts with axioms and rules for the 'static' part of the language, that is, the one that does not involve either awareness operators or epistemic awareness action modalities [13]. Then we have axioms for speculative knowledge, describing how this operator carries features of bisimulation: if an agent is unaware of an atom, he does not refute any interpretation of that atom, nor does he refute the interpretation of any other agent's awareness of that atom [21].

The next part of the table characterizes the behaviour of the awareness operator. The first five axioms are all standard for atom-based awareness [3, 21]; for the last, the one concerning $A_i[\mathsf{M},\mathsf{s}]\varphi$, the right-hand side merely expresses that the agent has to be aware of all variables in all preconditions of actions in the action model, and also of the variables in φ.

The last part of the table contains reduction axioms for the epistemic awareness actions, relating the truth value of formulas *after* an action to the truth value of formulas *before* it. The one for $\top$ indicates that after any successful execution of (M, s), $\top$ is the case. That for atomic propositions is inherited from [18], and indicates that after an execution of (M, s) the atom p will be true iff, provided it satisfies s's precondition, the current state satisfies (s, p)'s postcondition. The axioms for $\neg$ and $\wedge$ are standard [1].

The axioms for awareness after an epistemic awareness action are novel; they take into account that the level of awareness may increase or decrease after action execution, which gives us two cases. If φ contains an atom the agent becomes unaware of (that is, an atom in $\mathcal{A}_i^-(\mathsf{s})$), then she will not be aware of φ after the epistemic awareness action. Otherwise, after the action the agent will become aware of φ iff she is already aware of φ or if she will be after becoming aware of the atoms in $\mathcal{A}_i^+(\mathsf{s})$, that is, iff she was aware of $\varphi[\top \backslash \mathcal{A}_i^+(\mathsf{s})]$, where $\psi[\chi \backslash \{p_1, \ldots, p_n\}]$ is the simultaneous substitution of χ for all occurrences of every $p_1, \ldots, p_n$ in ψ.

We have not found axioms for speculative knowledge after an epistemic awareness action. The form $[\mathsf{M}, \mathsf{s}]K_i^I \varphi \leftrightarrow (\mathsf{pre}(\mathsf{s}) \to \bigwedge_{(\mathsf{s},\mathsf{t}) \in \mathsf{R}_i} K_i^I [\mathsf{M}, \mathsf{t}]\varphi)$ for modal accessibility [1] does not hold for speculative knowledge $K_i^S \varphi$. If $[\mathsf{M}, \mathsf{t}]$ does not change awareness, the axiom still holds, but the problem with an axiom for K_i^S of that form is that the level of awareness for agent i before and after action execution may be different. Surely, if p is true in all accessible states, we want for i to know p explicitly after becoming aware of p (and if that is the *only* dynamics — action models with factual change might after all change the value of p). But we do not want for i to know that before becoming aware of p (neither explicitly, nor speculatively). Dually, if i knows p explicitly, then after becoming unaware of p (as a conscious abstraction action, so to speak), she should no longer explicitly know that. The axioms for the interaction of awareness dynamics and speculative knowledge in [21] suggest that the interaction axioms for speculative knowledge and epistemic awareness actions will *not* be *reduction* axioms, i.e., they will not be equivalences helping us to rewrite formulas into formulas without epistemic awareness actions. Therefore, we may also have to face expressivity questions.

It is worthwhile to note that, though an axiomatization would add value to our framework, it would be mainly for logicians. It is often said that the value of Hilbert-style complete axiomatizations is limited for the multi-agent system applications. Similar logics are undecidable, so it is unclear if effective procedures can be found.

7. TYPES OF EPISTEMIC AWARENESS ACTIONS

We can distinguish different types of epistemic awareness actions, in which we recognize a number of actions familiar from the literature, but also novel additions.

Knowledge change without awareness change To model pure knowledge (or belief) change, the awareness functions $\mathcal{A}^+$ and $\mathcal{A}^-$ should neither add nor delete propositional variables for any agent.

Definition 11 (No awareness change). *An epistemic awareness action model* M *is of type* 'no awareness change' *if for any agent* i *and action* $\mathsf{s} \in \mathcal{D}(\mathsf{M})$, $\mathcal{A}_i^+(\mathsf{s}) = \mathcal{A}_i^-(\mathsf{s}) = \varnothing$.

All propositional tautologies	From $\varphi \to \psi$ and φ infer ψ	
$K_i^S(\varphi \to \psi) \to (K_i^S \varphi \to K_i^S \psi)$	From φ infer $K_i^S \varphi$	
$\big((K_i^S(p \to \varphi) \vee K_i^S(\neg p \to \varphi)) \wedge \neg A_i p\big) \to K_i^S \varphi$		if $p \notin v(\varphi)$
$\big((K_i^S(A_j p \to \varphi) \vee K_i^S(\neg A_j p \to \varphi)) \wedge \neg A_i p\big) \to K_i^S \varphi$		if $p \notin v(\varphi)$
$A_i \top$	$A_i \neg \varphi \leftrightarrow A_i \varphi$	
$A_i(\varphi \wedge \psi) \leftrightarrow (A_i \varphi \wedge A_i \psi)$	$A_i K_i \varphi \leftrightarrow A_i \varphi$	
$A_i A_j \varphi \leftrightarrow A_i \varphi$		
$A_i [\mathsf{M}, \mathsf{s}]\varphi \leftrightarrow (\bigwedge_{\mathsf{t} \in \mathcal{D}(\mathsf{M})} A_i \mathsf{pre}(\mathsf{t}) \wedge A_i \varphi)$		
$[\mathsf{M}, \mathsf{s}]\top \leftrightarrow \top$		
$[\mathsf{M}, \mathsf{s}]p \leftrightarrow \big(\mathsf{pre}(\mathsf{s}) \to \mathsf{post}(\mathsf{s}, p)\big)$		
$[\mathsf{M}, \mathsf{s}]\neg\varphi \leftrightarrow \big(\mathsf{pre}(\mathsf{s}) \to \neg[\mathsf{M}, \mathsf{s}]\varphi\big)$		
$[\mathsf{M}, \mathsf{s}](\varphi \wedge \psi) \leftrightarrow \big([\mathsf{M}, \mathsf{s}]\varphi \wedge [\mathsf{M}, \mathsf{s}]\psi\big)$		
$[\mathsf{M}, \mathsf{s}]A_i \varphi \leftrightarrow \neg \mathsf{pre}(\mathsf{s})$		if $v(A_i \varphi) \cap \mathcal{A}_i^-(\mathsf{s}) \neq \varnothing$
$[\mathsf{M}, \mathsf{s}]A_i \varphi \leftrightarrow (\mathsf{pre}(\mathsf{s}) \to A_i \varphi[\top \backslash \mathcal{A}_i^+(\mathsf{s})])$		otherwise
From φ infer $[\mathsf{M}, \mathsf{s}]\varphi$		

Table 1: Axiom system

This way, we recapture all 'standard' action models à la [1], modulo the additional information in structures on the static awareness of agents.

Awareness change without knowledge change To model pure awareness change, no agent should learn any non-trivial formula. This is guaranteed if in the action model, in any equivalence class for any agent, the disjunction of the preconditions of these actions are equivalent to the triviality.

Definition 12 (No knowledge change). *An epistemic awareness action model* M *is of type* 'no knowledge change' *if for any agent* $i \in N$, *and for all* $\mathsf{s} \in \mathcal{D}(\mathsf{M})$

$$\bigvee_{\mathsf{t} \in \mathsf{s} R_i} \mathsf{pre}(\mathsf{t}) \leftrightarrow \top$$

As awareness change is central to this contribution, let us investigate even more specific ways of becoming aware without knowledge change.

Definition 13 (Conditionally becoming aware). *The epistemic awareness action* $(\mathsf{A}_i^{+p}(\varphi), \mathsf{t})$, *wherein agent* i *becomes aware of* p *in the states satisfying* φ, *has two actions indistinguishable from one another,* t *and* f. *The precondition function is given by* $\mathsf{pre}(\mathsf{t}) = \varphi$ *and* $\mathsf{pre}(\mathsf{f}) = \neg\varphi$, *with* post *the trivial assignment for both actions:* $\mathsf{post}(\mathsf{t})(q) = \mathsf{post}(\mathsf{f})(q) = q$ *for every* $q \in P$ *(including* p*). For the awareness change function only* p *is added, only in* t, *and no atom is removed:* $\mathcal{A}_i^+(\mathsf{t}) = \{p\}$, $\mathcal{A}_i^+(\mathsf{f}) = \mathcal{A}_i^-(\mathsf{t}) = \mathcal{A}_i^-(\mathsf{f}) = \varnothing$.

Other epistemic awareness actions, like $(\mathsf{A}_i^{+Q}(\varphi), \mathsf{t})$, an action wherein i becomes aware of all variables in $Q \subseteq P$, or $(\mathsf{A}_i^{+p}(\top), \mathsf{t})$, the one in which i becomes aware of p in *all* states of the epistemic awareness model, can be defined in a similar way. For the latter we can also use a one-action epistemic awareness action model, because the other action now has a precondition that is never satisfied ($\neg\top$).

Another interesting case is the one in which *every* agent becomes aware of p. We can model this by either the composition $\mathsf{A}_{i_1}^{+p}(\varphi); \ldots; \mathsf{A}_{i_n}^{+p}(\varphi)$ when we have $N = \{i_1, \ldots, i_n\}$, or, more elegantly, as an epistemic awareness action $(\mathsf{A}_N^{+p}(\varphi), \mathsf{t})$ wherein $\mathcal{A}_i^+(\mathsf{t}) = \{p\}$ for *every* agent $i \in N$. Then, $\mathsf{A}_N^{+p}(\top)$ represents an action wherein every agent becomes aware of p in the entire model. (This is the special case mentioned at the start of Section 6.)

On finite epistemic awareness models, where each state s has a *distinguishing formula* δ_s only true in (the bisimulation class of) that state [2, 17], we can define an epistemic awareness action with which agent i becomes aware of p only in the actual state:

$$\mathsf{A}_i^{+p}(\delta_{\mathsf{t}}) \ .$$

With the action model for becoming aware, implicit knowledge K^I in the Fagin et al. modal accessibility sense [3] is now definable. (Otherwise, it is not.)

Definition 14 (Implicit knowledge).

$$K_i^I \varphi \quad \textit{iff} \quad [\mathsf{A}_i^{+v(\varphi)}(\top)] K_i^S \varphi \ .$$

If the agent is aware of all variables in φ, $A_i\varphi$ holds, so that speculative knowledge $K_i^S\varphi$ now entails explicit knowledge $K_i^E\varphi$. In other words, the definition spells out that an agent implicitly knows φ (in the sense that φ is true in all i-accessible states) iff after becoming aware of all the variables in φ, she explicitly knows φ.

We should point out however, that in our framework this definition is of limited use, as we also allow for more involved ways of becoming aware, wherein the value of atoms that the agent is unaware of may change (see the next section for a detailed example). In such cases, p may now be true in all accessible states, so you may implicitly know p but after the epistemic awareness action, you explicitly know that p is false, because its value changed in the execution of the action. The reason to permit such actions is to allow for the epistemic complexity of states to increase, thus reflecting the growing knowledge of agents about the atoms they are aware of. Before they were aware of these atoms, there was no need for such complexities and their values could therefore be considered 'don't care' values.

Addressing a novel issue A typical conversational act is an announcement wherein a novel issue is being addressed. Such announcements makes the addressed agents aware and knowledgeable at the same time. Such announcements have been differently modelled in [23].

Definition 15 (Addressing a novel issue). *The awareness announcement*

$$!_A\varphi$$

is the composition of the standard announcement $!\varphi$ *with the action* $\mathsf{A}_N^{+v(\varphi)}(\top)$ *that makes every agent aware all variables in* φ*, or, alternatively and equivalently, defined directly as the singleton epistemic awareness action consisting of domain* s*, accessible to all agents, with* $\mathsf{pre}(\mathsf{s}) = \varphi$*, trivial postcondition* $\mathsf{post}(\mathsf{s})(p) = p$ *for every* $p \in P$*, and such that* $\mathcal{A}_i^+(\mathsf{s}) = v(\varphi)$ *and* $\mathcal{A}_i^-(\mathsf{s}) = \varnothing$ *for all agents.*

In the definition above, in the variant defined by composition, it is imperative that $\mathsf{A}_N^{+v(\varphi)}(\top)$ is *after* the announcement, not before. After all, a truthful announcement to all could be

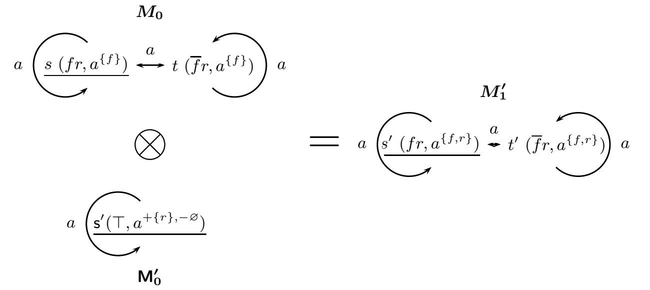
> "You are not aware of the fact that Valencia oranges mature in November!"

This is true at the moment of the announcement, but after that no longer: all have now become aware. We have just discovered the *unsuccessful awareness update*, an announcement to i of the archetypical form $!_A(p \wedge \neg A_i p)$.

8. DETAILED EXAMPLE

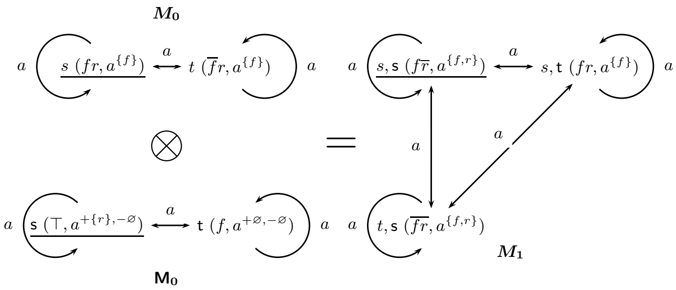
We have shown how Alfred's initial situation (Example 1) can be represented with an epistemic awareness model and described with the language $\mathcal{L}$ (Example 5). Now we will show how his online search and other actions can be represented with action models, and described with modalities expressing the actions' effect.

By reading *"The English team faced a complicated rugby match yesterday"*, Alfred becomes aware of the rugby match. One could think that this act is represented by an epistemic awareness action with a single reflexive action s' and awareness change, precondition and postcondition functions given by $\mathcal{A}_a^+(\mathsf{s}') := \{r\}$ and $\mathcal{A}_a^-(\mathsf{s}') := \varnothing$, $\mathsf{pre}(\mathsf{s}') := \top$ and $\mathsf{post}(\mathsf{s}')(p) := p$ for every atom p, respectively, but this is not the case.



Observe how, in the resulting epistemic awareness state, (M'_1, s'), Alfred is indeed aware of the rugby match ($A_a r$ holds) but he also knows speculatively that England won it ($K_a^S r$). (Equivalently, $[\mathsf{M}'_0, \mathsf{s}'](A_a r \wedge K_a^S r)$ holds at (M_0, s).) This is because every epistemic awareness state that is $\{f, r\}$-awareness bisimilar to either (M'_1, s') or else (M'_1, t') must satisfy r.

The act of becoming aware of r without getting to know its truth value is represented by the epistemic awareness action $(\mathsf{M}_0, \mathsf{s})$ below, with postconditions given by $\mathsf{post}(\mathsf{s})(f) = \mathsf{post}(\mathsf{t})(f) = f$, $\mathsf{post}(\mathsf{s})(r) = \neg\top$ and $\mathsf{post}(\mathsf{t})(r) = \top$.



Now in the resulting epistemic awareness state, (M_1, s, s), Alfred is aware of the rugby match ($A_a r$) without knowing speculatively who won it ($\neg K_a^S r \wedge \neg K_a^S \neg r$). (Equivalently, $[\mathsf{M}_0, \mathsf{s}](A_a r \wedge \neg K_a^S r \wedge \neg K_a^S \neg r)$ holds at (M_0, s).) This is because state (t, s) is reachable from (s, s) and we can find an epistemic awareness state that is $\{f, r\}$-awareness bisimilar to (M_1, t, s) and in which r fails (so $\neg K_a^S r$ holds), and because state (s, t) is reachable from (s, s) and we can find an epistemic awareness state that is $\{f, r\}$-awareness bisimilar to (M_1, s, t) and in which $\neg r$ fails (so $\neg K^S_a \neg r$ holds). Note how $(\mathsf{M}_0, \mathsf{s})$ is an action of type 'no knowledge change' (Definition 12) because the disjunction of the preconditions of all the actions, $\top \vee f$, is equivalent to $\top$.

Since Alfred's main concern is football, he keeps looking until he finds out that England defeated The Netherlands. In this act there is no change in awareness; this is a simple announcement of f in the classical sense.

$$
\begin{tikzcd}
\boldsymbol{M_1} \otimes & \underline{\mathsf{s}(f, a^{+\varnothing,-\varnothing})} \arrow[loop above, "a"] & = & \underline{s, \mathsf{s}\ (f\overline{r}, a^{\{f,r\}})} \arrow[loop above, "a"] \arrow[r, leftrightarrow, "a"] & s, \mathsf{t}\ (fr, a^{\{f\}}) \arrow[loop above, "a"]
\end{tikzcd}
$$

$\mathsf{M_1}$ $\qquad\qquad$ $\boldsymbol{M_2}$

Now Alfred knows explicitly (i.e., is aware of and knows speculatively) that England won the football match. This is expressed equivalently by the following facts: $K^E f$ holds at (M_2, s, s), $[\mathsf{M}_1, \mathsf{s}]K^E f$ holds at (M_1, s, s) and $[\mathsf{M}_0, \mathsf{s}][\mathsf{M}_1, \mathsf{s}]K^E f$ holds at (M_0, s). Note how the epistemic awareness action $(\mathsf{M}_1, \mathsf{s})$ is of type 'no awareness change' (Definition 11).

9. CONCLUSION, FURTHER RESEARCH

We presented a unified setting to model all static and dynamic aspects of awareness and knowledge, without any constraints on the modal properties of knowledge or on the interaction between awareness and knowledge. For this, we needed a primitive epistemic operator called *speculative knowledge*, that is different from *implicit knowledge*. Common awareness dynamics is elegantly definable in our setting, e.g.: 'an agent becoming aware of a propositional variable', 'implicit knowledge', 'addressing a novel issue in an announcement', and also more complex ways in which an agent can become aware of a novel issue by way of increasing the complexity of the epistemic model.

A complete axiomatization is still lacking, as are more detailed investigations of special modal classes, such as $S5$ and $KD45$ and how this influences the axiomatization, and compares to other approaches of awareness change that restrict themselves to such classes. Another future direction is to go from bisimulation to simulation: we expect that within reasonable restrictions (e.g., finite models) execution of an epistemic awareness action models is an *awareness simulation* of the initial epistemic awareness state, and vice versa. This would open the window to more succinct axiom systems and complexity results, and provide corroboration that our language is a suitable and adequate formalization for *any conceivable change of awareness or information.*

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