A Better Maximization Procedure For Online Distributed Constraint Optimization

(Extended Abstract)

Yoonheui Kim
University of Massachusetts at Amherst, MA
01003, USA
ykim@cs.umass.edu

Victor Lesser
University of Massachusetts at Amherst, MA
01003, USA
lesser@cs.umass.edu

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems, Coherence and coordination*

General Terms

Performance, Algorithms

Keywords

Distributed Problem Solving, DCOP, Max-Sum

1. INTRODUCTION

Many message passing algorithms on graphical models include maximization operations on sums of local node function and message values from neighbors. In recent work by McAuly et al, faster maximization computation was achieved in a static environment by offline presorting of the values of local functions. However, this efficiency is only guaranteed in special cases when constraint nodes receive messages involving fewer variables than the local function. In this paper, we generalize the approach to be applicable to more general settings where offline presorting of constraint functions is not realistic and messages may involve as many variables as the constraint function. We further improve the approach in two ways, first by creating different value sets with sum values from the previous cycle and the changes in message values from the current cycle, and second by conditionally applying the technique based on a correlation measure. These new approaches with no preprocesing step obtain the expected computational complexity with an exponent of 1.5 of the possible values per node except the initial cycle which requires 2. We demonstrate the effectiveness of this approach in a distributed optimization problem involving the coordination and scheduling of radars.

2. FAST BELIEF PROPAGATION

The Fast Belief Propagation (FBP) [4] scheme was developed for a (undirected) graphical model, where the maximum a posteriori inference is done by finding the values of variables that maximize the sum of the node and edge potentials. In a pairwise graphical model, computing a message $m_{A\rightarrow B}$ between two neighboring cliques $A = (\mathbf{i}, \mathbf{j})$ and $B = (\mathbf{i}, \mathbf{k})$ is equivalent in complexity to solving

$$m_{A\rightarrow B}(y_i) = \Psi_i(y_i) + \max_{y_j}[\underbrace{\Psi_j(y_j)}_{\mathbf{v_a}} + \underbrace{\Phi_{i,j}(y_i, y_j)}_{\mathbf{v_b}}], \quad (1)$$

where $\Psi_i(y_i)$ is the sum of $\Phi(y_i|x_i)$ and any first-order messages over y_i, that is, the sum of the values only related to y_i given the observation x_i (similarly for $\Psi_j(y_j)$).

Let the list of values of v_a be L_A and that of v_b be L_B. Assuming the list L_B, which is of length N, contains the values of v_b already sorted offline for each value of y_i, and assuming the length of the list of L_A is much smaller than N where sorting of L_A is much smaller than $O(N)$, we can use the FBP technique to find the maximum sum of L_A and L_B with expected time complexity $O(\sqrt{N})$ given binary constraints and order statistics independence of variables.

3. FAST BELIEF PROPAGATION ON GENERAL GRAPHS

We relax the assumption that the constraint function is given offline by partially sorting the value list online which contains constraint function values for every variables' configuration. Also, the guarantee of FBP on performing in $O(\sqrt{N})$ is restricted to pairwise factor graphs using two lists and we maintain the benefit of $O(\sqrt{N})$ on n-ary factors by introducing the message list that we construct by merging incoming messages. Additionally we relax the assumption that there are fewer variables associated with messages than the number of variables in a constraint function by partially generating the message list, i.e. the same number of sorted items in value lists.

G-FBP: Algorithm for applying FBP with partial lists

We have modified the FBP technique, which we call G-FBP, so that it can be applied to partially sorted lists where the ranks of some items in unsorted part are not known. Once the bounding items, which has higher ranks than the item with maximum value are found, unmatched items are computed directly from constraint function and received messages. Let the number of sorted items $K\sqrt{N}$ with list of length N,

THEOREM 1. *The expected time complexity of $O(\sqrt{N})$ holds with partial lists when $(1 - \frac{K}{\sqrt{N}})^{K\sqrt{N}} < \frac{K}{\sqrt{N}}$.*

Appears in: *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2012)*, Conitzer, Winikoff, Padgham, and van der Hoek (eds.), 4-8 June 2012, Valencia, Spain.

Message-Passing with G-FBP technique

The technique works with any message-passing based optimization algorithm.

1. [Value List Construction] In cycle 1,
   1.1 work as in the original message passing algorithm which computes all possible variable nodes' value configuration $\mathbf{x}$ and store the values $F(\mathbf{x})$.
   1.2 Send messages for all x_i of neighbor i.
   1.3 For the lists created in step 1.1, select the top $K\sqrt{N}$ values from each list and order the items and save this as $v_b(x_i)$.
2. [Message List Construction] Sort the received messages from each neighbor. Combine the $K\sqrt{N}$ items by adding the sorted messages from the largest items.
3. [Finding Maximum] Find maximum using value list v_b and message list v_a for all x_i for all neighbors i using G-FBP technique
4. send the messages using the computed maximum value.
5. repeat step 2-4.

4. INDEPENDENCE ASSUMPTION AND CORRELATION MEASURE

With two negatively correlated lists, it is likely that the G-FBP scheme fails to find the maximum item within limited number of items therefore increasing the time complexity of the algorithm. The correlation of two lists are domain-dependent [4] and, from our observations, it also varies for each constraint function and received messages on each cycle. We extend G-FBP technique to ensure the independence of two lists.

Correlation Measure

We modify the Spearman's rank correlation measure [5] to measure correlation among two partially sorted lists and conditionally apply G-FBP technique. Let x and y be two sorted lists where x_i and y_i are the ranks of the items with index i. m is the median rank. We use correlation measure:

$$\rho' = \frac{\sum_i (r_{x_i})(r_{y_i})}{\sqrt{\sum_i r_{x_i}^2 \sum_i r_{y_i}^2}} \tag{2}$$

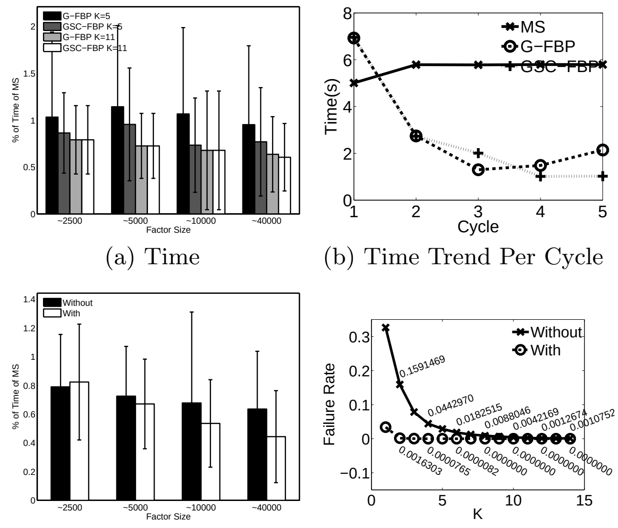
$$\text{where } r_k = \begin{cases} \frac{(N-K\sqrt{N})}{K\sqrt{N}}(k_i - m_k), & \text{if } k_i \text{ in sorted lists} \\ (\frac{(K\sqrt{N}+1+2K\sqrt{N})}{2} - m_k), & \text{if } k_i \text{ is not found} \end{cases}$$

and $m_k = K\sqrt{N} + 1/2$, imaginary median, as we consider the imaginary length of list $2K\sqrt{N}$.

GSC-FBP: Improving the Rank of Items in the list

We create two lists, sum list L_{sum} , the value of sum for variables' configuration in the previous round, and change list $L_{changes}$, the difference between previous and current round messages as the following equation 3 and replace value list and message list. As the algorithm proceeds, the list L_{change} becomes closer to the uniform distribution which makes it independent of L_{sum}. Let $r_{m\rightarrow n}$ be the message from function node m to variable node n.

$$\begin{aligned} & r_{m\rightarrow n}(x_n) \\ & = \max_{x_m \backslash n}(\Sigma_{n' \in N(m) \backslash n}(\underbrace{q_{n'\rightarrow m}(x_{n'})}_{\text{data dependent}} + \underbrace{f_m(x_m)}_{\text{data independent}})) \\ & = \max_{x_m \backslash n}(\Sigma_{n' \in N(m) \backslash n}(\underbrace{q'_{n'\rightarrow m}(x_{n'}) - q_{n'\rightarrow m}(x_{n'})}_{changes} \\ & + \underbrace{f_m^{previoussum}(x_m) + q_{n'\rightarrow m}(x_{n'})}_{sum}))) \end{aligned} \tag{3}$$



(a) Time (b) Time Trend Per Cycle

(c) Time (d) Failing Probability

Figure 1: (a) Computation Time Ratio against Max-Sum. (d) The Probability of Failing to Find the Maximum Item in Sorted Parts. With: G-FBP using correlation. Without: G-FBP not using correlation.

Round	4	5	6	7
G-FBP	0.0793	0.2612	0.2607	0.3543
GSC-FBP	0.0500	0.0450	0.0730	0.0208

Table 1: Relative position of Bounding Items

5. EXPERIMENTS

We compared the performance with the Max-Sum approximate distributed constraint algorithm [1] on the domain of real-time adaptive NetRad system [3]. See [2], for more details on the formulation of the distributed constraint optimization problem for this problem. We use an abstract simulator that involved 48 radars with a scenario of 96 phenomena with random locations, size, and type(density 2 constraint graphs).

6. REFERENCES

[1] A. Farinelli, A. Rogers, A. Petcu, and N. R. Jennings. Decentralised coordination of low-power embedded devices using the max-sum algorithm. In *AAMAS '08: Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems*, pages 639–646, Richland, SC, 2008. International Foundation for Autonomous Agents and Multiagent Systems.

[2] Y. Kim, M. Krainin, and V. Lesser. Effective Variants of the Max-Sum Algorithm for Radar Coordination and Scheduling. In *Proceedings of 2011 IEEE/WIC/ACM International Conference on Intelligent Agent Technology*, pages 357–364, Lyon, France, October 2011.

[3] J. F. Kurose et al. An End-User-Responsive Sensor Network Architecture for Hazardous Weather Detection, Prediction and Response. In *Proceedings of the Second Asian Internet Engineering Conference, AINTEC*, pages 1–15, 2006.

[4] J. J. McAuley and T. S. Caetano. Exploiting data-independence for fast belief-propagation. In J. Fürnkranz and T. Joachims, editors, *ICML*, pages 767–774. Omnipress, 2010.

[5] C. Spearman. The proof and measurement of association between two things. *The American Journal of Psychology*, 100(3/4):441–471, 1987.