Distance-Based Rules for Weighted Judgment Aggregation

(Extended Abstract)

Marija Slavkovik
University of Luxembourg
Computer Science and Communication
marija.slavkovik@uni.lu

Wojciech Jamroga
University of Luxembourg
Computer Science and Communication
wojtek.jamroga@uni.lu

ABSTRACT

Cooperating agents need to reach group decisions on several logically related issues. These decision-making problems are studied in social choice theory by the discipline of judgment aggregation. Judgment aggregation produces group decisions by aggregating individual answers to binary questions, however existing aggregation rules are defined for a very restricted setting, insufficient for aggregating opinions in a computer science contexts. We propose a family of distance-based judgment aggregation rules and study their properties.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent systems; I.2.4 [**Knowledge representation formalisms and methods**]

General Terms

Theory

Keywords

judgment aggregation, distance-based merging

1. BACKGROUND AND MOTIVATION

Social choice develops and analyzes methods for reaching group decisions by aggregating individual information. Judgment aggregation in particular explores how the truth-values, called judgments, that individuals assign to logically connected issues can be aggregated into a consistent set of truth values [7]. Judgment aggregation problems occur in computer science contexts, *e.g.,* [1], as well is in society in committee decision making contexts, such as juries and expert panels. There is a notable difference between problems in the two contexts. In society problems it can always be assumed that each individual is capable of making and stating an opinion on each issue, namely that the sets of individual judgments are *complete*. It is further assumed that each individual is equally competent to give an informed opinion on each issue, when compared to the other individuals, i.e., the judgments are non-weighted. These two assumptions cannot be plausibly made in computer science settings. For instance, a robot that does not have a microphone cannot produce opinions regarding sound issues. Moreover, one facial recognition program can be much better than another.

A judgment aggregation rule is a function that assigns a consistent set of truth values to a collection of individually assigned truth values. Like for other social choice methods, it is impossible to construct a judgment aggregation rule that satisfies a minimal set of desirable criteria [7]. There is a small number of rules developed in the judgment aggregation literature all of which are only applicable to complete judgment sets, see [6] for an overview.

Here, we develop and study the properties of a family of judgment aggregation rules for aggregating incomplete sets of judgments in the presence of weights. The family is constructed by generalizing the distance-based merging procedure of [3], inspired by belief merging [5]. Although multi-valued rules are considered in belief merging, weights associated with pairs of agents and issues have not been considered. Furthermore, the properties considered in belief merging are not identical to the properties of interest in judgment aggregation which we define and analyze here, with the exception of the unanimity property.

2. RULES AND PROPERITES

Let $\mathcal{L}_3$ be a ternary propositional logic, and $\models_3$ the entailment operator for this logic. A judgment aggregation problem is specified by: a set of agent names N, a consistent set of formulas $\mathcal{A} \subseteq \mathcal{L}_3$ called the agenda, a set of formulas $\mathcal{R} \subseteq \mathcal{L}_3$ called constraints and a set of truth values $T = \{0, \frac{1}{2}, 1\}$. We use the value $\frac{1}{2}$ to represent the case when no judgment has been assigned to an issue.

A judgment for $a \in \mathcal{A}$ is an assignment of a truth value from T. The collection of judgments assigned to each $a \in \mathcal{A}$ is called a judgment sequence A. We use A_j to denote the judgment in the sequence regarding $a_j \in \mathcal{A}$. Let n be the cardinality of N and m the cardinality of $\mathcal{A}$. A *profile of judgments* is a $n \times m$ matrix π with elements $\pi(i, j) \in T$.

A judgment sequence A is consistent if and only if it is a truth assignment such that $\mathcal{A} \cup \mathcal{R} \not\models_3 \perp$. A judgment sequence is complete if and only if for each $a \in \mathcal{A}$ either $a \in \hat{A}$ or $\neg a \in \hat{A}$. For each $\mathcal{A}, \mathcal{R}$ and $\models_3$ we can construct the set of all consistent judgment sequences $\Phi(\mathcal{A}, \mathcal{R}, \models_3)$. We write only Φ when the arguments are clear from the context. In defining the judgment aggregation problem we do not limit ourselves to a particular ternary logic that has to be used to represent the judgments. In theory, any ternary logic can be used. The choice of logic specifies the domain and co-domain of the judgment aggregation rule.

An aggregation function is a function $\odot : (\mathbb{R}^+)^n \mapsto \mathbb{R}^+$

Appears in: *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2012)*, Conitzer, Winikoff, Padgham, and van der Hoek (eds.), 4-8 June 2012, Valencia, Spain.

which is non-decreasing (if $x \leq y$ then $\odot(x_1, \ldots, x, \ldots, x_n) \leq \odot(x_1, \ldots, y, \ldots, x_n)$) and satisfies the boundary condition: the infimum of $\odot(\mathbf{x})$ is 0 [4, p.3]. An aggregation function is: *symmetric* iff $\odot(\mathbf{x}) = \odot([\mathbf{x}]_\sigma)$ for every $\mathbf{x} \in (\mathbb{R}^+)^n$ and permutation σ [4, p.22]; *associative* iff $\odot(x) = x$ for all $x \in \mathbb{R}^+$ and $\odot(\mathbf{x}, \odot(\mathbf{x}'), \mathbf{x}'') = \odot(\mathbf{x}, \mathbf{x}', \mathbf{x}'')$ for all $\mathbf{x}, \mathbf{x}', \mathbf{x}'' \in \bigcup_{n \in \mathbb{N}^0} (\mathbb{R}^+)^n$ [4, p.22].The most commonly used aggregation functions are the $\sum$, $\prod$, max and min. All of these functions are all symmetric and associative.

A function $\delta : \Phi \times \Phi \to \mathbb{R}^+$ is called a distance if, for all $x, y, z \in \Phi$, there holds: $\delta(x, y) \geq 0$ (non-negativity), $\delta(x, y) = \delta(y, x)$ (symmetry) and $\delta(x, x) = 0$ (reflexivity). A distance δ is called a *metric* on Φ if, for all $x, y, z \in \Phi$, there holds:$\delta(x, y) = 0$ if and only if $x = y$ (identity of indiscernible) and $\delta(x, y) \leq \delta(x, z) + \delta(z, y)$ (triangle inequality) [2, p.3-4].

A collection of judgment weights W is also an $n \times m$ matrix whose elements $w(i, j) \in \mathbb{R}^+$, $\mathbb{R}^+$ being the interval of reals $[0, +\infty)$. If no weights are given, then $W = U$, where U is such that for each i and j, $w(i, j) = 1$. If only the weights associated with an agent are given then for each i, $w(i, 1) = w(i, 2) = \cdots = w(i, m)$; this kind of weights have been usually considered in belief merging. If only the weights associated with the relevance of each agenda issue are given then for each j, $w(1, j) = w(2, j) = \cdots = w(n, j)$. If agent i's judgment on issue a_j is useless, then $w(i, j)$ should be set to 0.

We can now define the weighted distance-based judgment aggregation rule Δ as follows.

Definition 1. *Let $\odot$ be an aggregation function, $\circledast$ a symmetric aggregation function, and δ a distance metric. The distance-based aggregator $\Delta^{\delta,\circledast,\odot}$ is a weighted judgment aggregation rule specified as*

$$\Delta^{\delta,\circledast,\odot}(\pi, W) = \arg\min_{A \in \Phi} \odot \left(\circledast \left(w(i, j) \cdot \delta(A_j, \pi(i, j)) \right)_{j=1}^{m} \right)_{i=1}^{n}.$$

The well known distances: Hamming distance d_H, Taxicab distance d_T, and Drastic distance d_D can be defined as: $d_H(A, A') = \sum_i^m \delta_H(A_i, A'_i)$, $d_T(A, A') = \sum_i^m \delta_T(A_i, A'_i)$, $d_D(A, A') = max(\delta_H(A_i, A'_i)$, where $\delta_T(A, A') = |A_i - A'_i|$ while $\delta_H = 0$ when $A(a_i) = A'_i$ and $\delta_H = 1$ when $A_i \neq A'_i$.

A judgment aggregation rule for a particular aggregation problem in multi-agent systems can be selected by looking at the properties which that rule satisfies. Typically in social choice theory one does not study how to select a rule, but which properties can be satisfied at the same time by a rule. We show which types of $\odot, \circledast$ and δ guarantee that the resulting rule would satisfy a social-theoretic property and study the properties of some specific $\odot, \circledast$ and δ.

We introduce two auxiliary concepts and then define the social-theoretic properties for $\Delta^{\delta,\circledast,\odot}$ desirable for virtually any judgment aggregation context.

Let $M_{n\times m}$ and $M'_{n\times m}$ be matrices. M' is a σ-permutation of M, if it is obtained by permuting the rows of M using a permutation σ.

We consider the function m, a simple majority function which, for a given $a \in \mathcal{A}$, returns the judgment for a supported by a strict majority of agents, taking into account also the weights. Namely, for $\pi \in T^{n\times m}$, $W \in (\mathbb{R}^+)^{n\times m}$ and $V \in T^n$, let $N_v \subseteq N$ be the set of agents i for which, for a given $a_j \in \mathcal{A}$, $\pi(i, j) = v$. For $T = \{0, \frac{1}{2}, 1\}$, the function $m : T^n \times (\mathbb{R}^+)^{n\times m} \mapsto T$ is $m(\pi\nabla j, W) = v$ when $\sum_{i \in N_v} w(i, j) > \sum_{i \in N \setminus N_v} w(i, j)$ and $\frac{1}{2}$ otherwise. $Maj(\pi, W)$ is the sequence obtained by applying m to each pair of columns of π and W.

Since all the properties in the judgment aggregation literature, with the exception of [6] have been defined for a different function type than our Δ we need to construct corresponding definitions of the most interesting properties.

Definition 2. *Let Δ be a distance-based judgment aggregation rule specified by $\delta, \circledast, \odot$. Δ satisfies* **anonymity** *iff $\Delta(\pi, W) = \Delta([\pi]_\sigma, [W]_\sigma)$ for every $\pi \in T^{n\times m}$, every $W \in (\mathbb{R}^+)^{n\times m}$, and every permutation σ. Δ satisfies* **unanimity** *iff for every $W \in (\mathbb{R}^+)^{n\times m}$ and every $\pi \in T^{n\times m}$ such that $\pi_1 = \cdots = \pi_n = A$, $\Delta(\pi, W) = \{A\}$. Δ satisfies* **majority-preserving** *iff for every $\pi \in T^{n\times m}$ and every $W \in (\mathbb{R}^+)^{n\times m}$, $Maj(\pi, W) \in \Phi$ implies that $Maj(\pi, W) \in \Delta(\pi, W)$. Δ satisfies* **separability** *iff for every two profiles $\pi^1_{[n_1\times m]}$ and $\pi^2_{[n_2\times m]}$ (and corresponding $W^1_{[n_1\times m]}$, $W^2_{[n_2\times m]}$), the $[A \in \Delta(\pi^1, W^1)$ and $A \in \Delta(\pi^2, W^2)]$ implies that $A \in \Delta(\pi, W)$. Matrices π and W are the concatenations of π^1, π^2 and W^1, W^2 correspondingly.*

Proposition 2.1. *If $\odot$ and $\circledast$ are symmetric and associative then $\Delta^{\delta,\odot,\circledast}$ satisfies anonymity, unanimity and separability. $\Delta^{d_T,\Sigma}$ is majority-preserving.*

The proof of anonymity follows from the definition of symmetry. The proof for unanimity follows from the boundary condition of an aggregation function and the reflexivity property of δ. The proof of separability follows from the definition of associativity and the non-decreasing property of the $\odot$ function.

The property of majority-preservation is not satisfied by almost all distance-based rules. To prove that $\Delta^{d_T,\Sigma}$, one needs to swap the order of the sums in the rule definition. The property follows from the triangular inequality property of the metric. $\Delta^{d_H,\Sigma}$, $\Delta^{d_T,max}$, or even $\Delta^{d_D,max}$ are not majority-preserving. We can construct a function $d_e(A, A') = \Pi_{j=1}^m \delta_e(A(j), A'(j))$, where $\delta_e(x, y) = k^{\delta_H(x,y)}$, for which $\Delta^{d_e,\Pi}$ is a majority-preserving rule. However, $\Delta^{d_e,\Pi}(\pi, W) = \Delta^{d_h,\Sigma}(\pi, W)$ for each π, W. It is our conjecture that $\Delta^{d_T,\Sigma}$ is the only majority-preserving operator.

3. REFERENCES

[1] G. Boella, G. Pigozzi, M. Slavkovik, and L. van der Torre. A satisficing agreements model. In *Proceedings of COIN@WI-IAT*, 2011.

[2] M.M. Deza and E. Deza. *Encyclopedia of Distances.* Springer, 2009.

[3] U. Endriss, U. Grandi, and D. Porello. Complexity of winner determination and strategic manipulation in judgment aggregation. In *Proceedings of the 3rd COMSOC*, 2010.

[4] M. Grabisch, J-L. Marichal, R. Mesiar, and E. Pap. *Aggregation Functions.* Cambridge University Press, 2009.

[5] S. Konieczny and R. Pino-Pérez. Logic based merging. *Journal of Philosophical Logic*, 40(2):239–270, 2011.

[6] J. Lang, G. Pigozzi, M. Slavkovik, and L. van der Torre. Judgment aggregation rules based on minimization. In *TARK*, pages 238–246, 2011.

[7] L. List and B. Polak. Introduction to judgment aggregation. *Journal of Economic Theory*, 145(2):441 – 466, 2010.