Solid Semantics and Extension Aggregation Using Quota Rules under Integrity Constraints

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Introduction

In this paper, we propose solid admissibility that is a strengthened version of Dung's admissibility [4] to obtain the most acceptable set of arguments. Besides, other solid extensions based on solid admissibility are defined. Such extensions not only include all defenders of its elements but also exclude all arguments indirectly attacked and indirectly defended by some argument(s). We also aggregate solid extensions by using approaches from judgment aggregation. Especially, although no quota rule preserves Dung's admissibility for any argumentation framework [2], we show that there exist quota rules preserve solid admissibility for any argumentation framework.

Basic Definitions

Argumentation Framework. An *argumentation framework AF* is a pair $\langle \textit{Arg}, \rightharpoonup \rangle$, where *Arg* is a finite and non-empty set of arguments, and $\rightharpoonup$ is a binary relation on *Arg*. For any $A, B \in \textit{Arg}$, $A \rightharpoonup B$ (or A *attacks* B) denotes that $(A, B) \in \rightharpoonup$.

Indirect attack and defense. An argument A *indirectly defends* an argument B iff there exists a finite sequence $A_0, \dots, A_{2n}$ such that (*i*) $B = A_0$ and $A = A_{2n}$, and (*ii*) for each i, $0 \leqslant i < 2n$, $A_{i+1} \rightharpoonup A_i$. An argument A is *controversial* w.r.t. an argument B iff A indirectly attacks and indirectly defends B. Note that direct attackers (resp. defenders) are also indirect attackers (resp. defenders).

Defense Function and Neutrality Function. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. The *defense function* $d\colon 2^{\textit{Arg}} \to 2^{\textit{Arg}}$ of *AF* is defined as:

$$d(\Delta) = \{C \in \textit{Arg} \mid \Delta \text{ defends } C\}. \tag{1}$$

The *neutrality function* $n\colon 2^{\textit{Arg}} \to 2^{\textit{Arg}}$ of *AF* is defined as:

$$n(\Delta) = \{B \in \textit{Arg} \mid \text{NOT } \Delta \rightharpoonup B\}. \tag{2}$$

Dung's Semantics. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. For any $\Delta \subseteq \textit{Arg}$, (i) Δ is a *conflict-free* extension iff $\Delta \subseteq n(\Delta)$; (ii) Δ is a *self-defending* extension iff $\Delta \subseteq d(\Delta)$; (iii) Δ is an *admissible* extension iff $\Delta \subseteq n(\Delta)$ and $\Delta \subseteq d(\Delta)$; (iv) Δ is a *complete* extension iff $\Delta \subseteq n(\Delta)$ and $\Delta = d(\Delta)$; (v) Δ is a *preferred* extension iff Δ is a maximal admissible extension; (vi) Δ is a *stable* extension iff $\Delta = n(\Delta)$; (vii) Δ is the *grounded* extension iff Δ is the least fixed point of the defense function d.

Aggregation Model. A *property* σ of extensions can be regarded as a subset of $2^{\textit{Arg}}$, namely, $\sigma \subseteq 2^{\textit{Arg}}$. Then the set of the extensions under a semantics is a property, e.g., completeness is the set of the complete extensions of *AF*. For any formula φ in $\mathcal{L}_{AF}$, we let $\mathrm{Mod}(\varphi) = \{\Delta \subseteq \textit{Arg} \mid \Delta \vDash \varphi\}$, namely, $\mathrm{Mod}(\varphi)$ denotes the set of all models of φ. Obviously, $\sigma = \mathrm{Mod}(\varphi)$ is a property. When using a formula φ to characterize such a property, φ is referred to as an *integrity constraint*.

Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. Let $N = \{1, \cdots, n\}$ be a finite set of *agents*. Suppose that each agent $i \in N$ reports an extension $\Delta_i \subseteq \textit{Arg}$. Then $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_n)$ is referred to as a *profile* of extensions. An *aggregation rule* is a function $F\colon (2^{\textit{Arg}})^n \to 2^{\textit{Arg}}$, mapping any given profile of extensions to a subset of *Arg*.

Quota rules. Let N be a finite set of n agents, and let $q \in \{1, \cdots, n\}$. The *quota rule* with quota q is defined as the aggregation rule mapping any given profile $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_1) \in (2^{\textit{Arg}})^n$ of extensions to the set including exactly those arguments accepted by at least q agents:

$$F_q(\boldsymbol{\Delta}) = \{A \in \textit{Arg} \mid \#\{i \in N \mid A \in \Delta_i\} \geqslant q\}. \tag{3}$$

Preservation. Let $\sigma \subseteq 2^{\textit{Arg}}$ be a property of extensions of *AF*. Then an aggregation rule $F\colon (2^{\textit{Arg}})^n \to 2^{\textit{Arg}}$ for n agents is said to *preserve* σ if $F(\boldsymbol{\Delta}) \in \sigma$ for every profile $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_n) \in \sigma^n$.

Solid Semantics

To obtain the most acceptable arguments, we formally introduce solid admissibility in this section. We argue that the most acceptable arguments should satisfy two criteria: (*i*) they should have defenders as many as possible, and (*ii*) they should avoid the undesirable interference of some arguments. We will show that arguments in admissible extensions satisfy the criteria. Firstly, we strengthen Dung's defense. A set of arguments *solidly defends* an argument iff this set defends (in Dung's sense) this argument and contains all the defenders of each element of this set.

Solid defense. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. $\Delta \subseteq \textit{Arg}$ solidly defends (or *s-defends*) $C \in \textit{Arg}$ iff for any $B \in \textit{Arg}$, if $B \rightharpoonup C$, then $\Delta \rightharpoonup B$ and $\overline{B} \subseteq \Delta$.

Solid defense function. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. The *solid defense function* $d_s\colon 2^{\textit{Arg}} \longrightarrow 2^{\textit{Arg}}$ of *AF* is defined as follows. For any $\Delta \subseteq \textit{Arg}$:

$$d_s(\Delta) = \{C \in \textit{Arg} \mid \Delta \text{ s-defends } C\} \tag{4}$$

Dung's Fundamental Lemma has a counterpart in solid semantics. The following lemma states that when we have a s-admissible extension, if we put into this extension an argument that is s-defended by this extension, then the new set is still a s-admissible extension.

S-Fundamental Lemma. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$, a s-admissible extension $\Delta \subseteq \textit{Arg}$, and two arguments $C,\ C' \in \textit{Arg}$ which are s-defended by Δ. Then (i) $\Delta' = \Delta \cup \{C\}$ is s-admissible and (ii) Δ' s-defends C'.

Solid admissibility. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. $\Delta \subseteq \textit{Arg}$ is a *s-admissible* extension iff $\Delta \subseteq n(\Delta)$ and $\Delta \subseteq d_s(\Delta)$.

The definition above states that a set of arguments is a s-admissible extension iff the set is conflict-free and s-defends each of its elements.

We develop some solid semantics based on solid admissibility. These semantics strengthen Dung's semantics in the sense that for a *solid extension* Δ, there exists a Dung's extension Γ such that Δ is a subset of Γ.

Solid semantics. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. For any $\Delta \subseteq \textit{Arg}$:

- (i) Δ is a *s-complete* extension iff $\Delta \subseteq n(\Delta)$ and $\Delta = d_s(\Delta)$;
- (ii) Δ is a *s-preferred* extension iff Δ is a maximal s-admissible extension;
- (iii) Δ is a *s-stable* extension iff $\Delta = n(\Delta)$ and for any argument $A \notin \Delta$, $\overline{A} \subseteq \Delta$;
- (iv) Δ is the *s-grounded* extension iff Δ is the least fixed point of d_s.

Characterization for Solid Semantics

We can capture solid semantics by using propositional formulas with the techniques in [1]. These formula are used for aggregation in the next section. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. For any $\Delta \subseteq \textit{Arg}$,

- Δ is s-self-defending iff $\Delta \vDash \mathrm{IC}_{SS}$ where $\mathrm{IC}_{SS} \equiv \bigwedge_{C \in \textit{Arg}} \Big[C \to \bigwedge_{\substack{B \in \textit{Arg} \\ B \rightharpoonup C}} \Big(\big(\bigvee_{\substack{A \in \textit{Arg} \\ A \rightharpoonup B}} A\big) \wedge \big(\bigwedge_{\substack{A \in \textit{Arg} \\ A \rightharpoonup B}} A\big)\Big)\Big]$;
- Δ is s-reinstating iff $\Delta \vDash \mathrm{IC}_{SR}$ where $\mathrm{IC}_{SR} \equiv \bigwedge_{C \in \textit{Arg}} \Big[\bigwedge_{\substack{B \in \textit{Arg} \\ B \rightharpoonup C}} \Big(\big(\bigvee_{\substack{A \in \textit{Arg} \\ A \rightharpoonup B}} A\big) \wedge \big(\bigwedge_{\substack{A \in \textit{Arg} \\ A \rightharpoonup B}} A\big)\Big) \to C\Big]$;
- Δ is s-stable iff $\Delta \vDash \mathrm{IC}_{SST}$ where $\mathrm{IC}_{SST} \equiv \bigwedge_{B \in \textit{Arg}} \Big[\big(B \leftrightarrow \bigwedge_{\substack{A \in \textit{Arg} \\ A \rightharpoonup B}} \neg A\big) \wedge \big(\neg B \to \bigwedge_{\substack{A \in \textit{Arg} \\ A \rightharpoonup B}} A\big)\Big]$;
- Δ is s-admissible iff $\Delta \vDash \mathrm{IC}_{SA}$ where $\mathrm{IC}_{SA} \equiv \mathrm{IC}_{CF} \wedge \mathrm{IC}_{SS}$;
- Δ is s-complete iff $\Delta \vDash \mathrm{IC}_{SC}$ where $\mathrm{IC}_{SC} \equiv \mathrm{IC}_{SA} \wedge \mathrm{IC}_{SR}$;
- Δ is s-preferred iff Δ is a maximal model of IC_{SA};
- Δ is s-grounded iff Δ is the least model of IC_{SC}.

Main Results

From a skeptical view, it is not cautious to accept an argument that is indirectly attacked and indirectly defended by some argument. But such arguments may be acceptable in Dung's semantics. Theorem 1 states that they never occur in s-admissible extensions. Interestingly, this theorem also guarantees that any argument in any odd-length cycle never occur in s-admissible extensions.

Theorem 1. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$ and a s-admissible extension $\Delta \subseteq \textit{Arg}$. If an argument $A \in \textit{Arg}$ is controversial w.r.t. an argument $B \in \textit{Arg}$, then $B \notin \Delta$.

Although no quota rule preserves Dung's admissibility for any argumentation framework [2], we show that there exist quota rules (e.g. the strict majority rule) preserve solid admissibility for any argumentation framework.

Theorem 2. Given $AF = \langle \textit{Arg}, \rightharpoonup \rangle$. Any quota rule F_q for n agents with $q > \frac{n}{2}$ preserves solid admissibility for *AF*.

Comparison

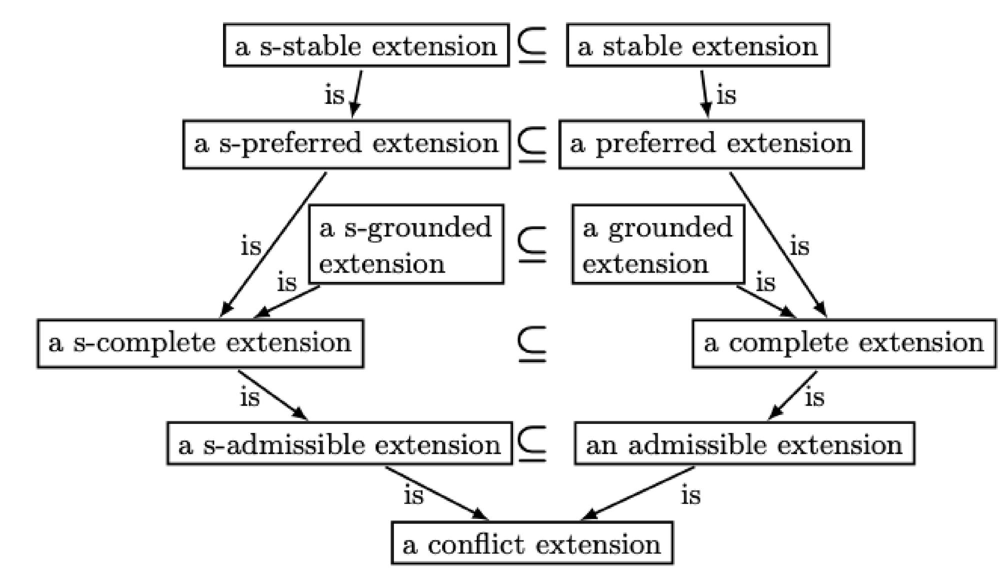

Fig. 1: An overview of solid semantics and Dung's semantics

We can tune the parameters for attackers and defenders to obtain defenses with different levels of strength in *graded semantics* [5]. But It is impossible to characterise solid semantics by tuning the parameters since different attackers may have different numbers of counter-attackers. In *prudent semantics* [3], whenever an argument A is controversial w.r.t. an argument B, both prudent semantics and solid semantics can prevent A and B from occurring in the same extension. But there is a difference between these two types of semantics. Both A and B can occur in a prudent extension separately. However, B is excluded from any s-admissible extension, while A might occur in some s-admissible extension.

References

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