A Logic of Inferable in Multi-Agent Systems with Budget and Costs

Stefania Costantini[1] **Andrea Formisano**[2] **Valentina Pitoni**[1]
DISIM, Università di L'Aquila, Italy[1]

DMIF, Università di Udine, Italy[2]

Introduction

In Artificial Intelligence, Multi-Agent Systems are able to model many kinds of collective behavior, and Logic is often used to model aspects of the reasoning process of groups of agents. We propose a particular logical framework (Logic of "Inferable") which reasons about whether a group of agents can perform an action, highlighting the concept of *cost* of actions, and of the *budget* which an agent must have available in order to perform this action. However, an agent which has not enough budget can possibly be supported by the group it belongs to.

The focus is on modeling the group dynamics of cooperative agents

Characterization

L-DINF is a logic which consists of a static component and a dynamic one:

- the *static component*, called L-INF, is a logic of explicit beliefs and background knowledge;
- the *dynamic component*, called L-DINF, extends the static one with dynamic operators capturing the consequences of the agents' inferential operations on their explicit beliefs.

Syntax

The language of *L-DINF*, denoted by $\mathcal{L}_{L\text{-}DINF}$, is defined by the following grammar in Backus-Naur form:

$$\begin{aligned}\phi, \psi &::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathsf{B}_i\,\varphi \mid \mathsf{K}_i\,\varphi \mid \\ &\quad do(\phi_A) \mid do^P(\phi_A) \mid can\_do(\phi_A) \mid exec_G(\alpha) \mid [G:\alpha]\,\varphi \\ \alpha &::= \vdash(\varphi,\psi) \mid \cap(\varphi,\psi) \mid \downarrow(\varphi,\psi)\end{aligned}$$

where:

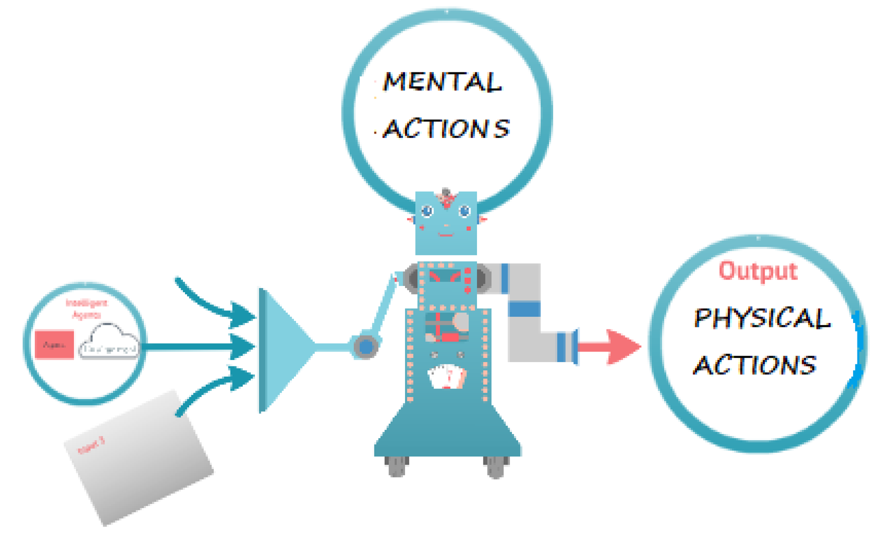
- $\top$, $\bot$, $\rightarrow$, $\leftrightarrow$ are defined from $\neg$ and $\wedge$ in the standard way;
- B_i, K_1, denote modal operators;
- $[G:\alpha]\varphi$ should be read "φ holds after the inferential action α has been performed by at least one of the agents in G, and all agents in G have common knowledge about this fact";
- $exec(\alpha)$ is to be read "α is an inferential action that the agent can perform";
- $do(\phi_A)$ indicates *actual execution* of physical action ϕ_A and $do^P(\phi_A)$ its past execution; $can\_do(\phi_A)$ is a reserved syntax that allows an agent to reason about its own capabilities; $\phi_A \in \mathrm{Atm}_A$ is an atomic propositions representing the physical actions that an agent can perform, including "active sensing" action).

Theorem

We have shown that *L-DINF* is:

- **sound**, using a particular axiomatization;
- **strongly complete**, using a canonical model argument.

Inferential Operations



We consider:

- the notion of executability of agents' inferential actions that require resource consumption, involving a *cost*, which is formalized by a cost function $C : Agt \times \mathcal{L}_{ACT} \times W \longrightarrow \mathbb{N}$;
- in order to execute an action the agent must possess the necessary *budget*, which is formalized by a budget function $B : Agt \times W \longrightarrow \mathbb{N}$;
- when an agent belongs to a group, if that agent does not have enough budget to perform an intended action, it may be supported by its group.

The following operations characterize the basic ways for an agent of forming explicit beliefs via inference:

- $\downarrow(\varphi,\psi)$ is the inferential operation which consists in inferring ψ from φ in case φ is believed and, according to an agent's background knowledge, ψ is a logical consequence of φ;
- $\cap(\varphi,\psi)$ is the inferential operation which consists in closing the explicit belief that φ and the explicit belief that ψ under conjunction;
- $\vdash(\varphi,\psi)$ is the inferential operation which consists in inferring ψ from φ in case φ is believed and, according to the agent's working memory, ψ is a logical consequence of φ. With this last operation, we operate directly on the working memory without retrieving anything from the background knowledge.

We have chosen only these three mental actions because the occurrence of physical actions (performed through some specific agent language which we intend to be independent of) is perceived through the formation of new beliefs; in fact, an agent is aware of whether she performed some physical actions and whether they were successful.

Example

- $\mathbf{K}_i(temperature{-}high \rightarrow do(switch\text{-}on\text{-}conditioner_A))$
- $\alpha : \downarrow(temperature{-}high,\ do(switch\text{-}on\text{-}conditioner_A))$
- $C(i,\alpha,w) = 18$, $B(1,w_1) = 10$, $B(2,w_1) = 7$, $B(3,w_1) = 8$

Agent 1 wants to perform α. She alone does not have enough budget. But, using $[G:\alpha]$, the other agents can lend her part of their budgets to share the cost.The action can thus be performed by the group G by dividing the cost in equal parts, as $\frac{C(1,\alpha_1,w_1)}{|G|} \le \min_{h\in G} B(h,w_1)$.
After the execution of α, the physical action $switch\text{-}on\text{-}conditioner_A$ will have been executed, and the new budgets will be: $B(1,w_2) = 4$, $B(2,w_2) = 1$, $B(3,w_2) = 2$.
All agents can thus infer $\mathbf{B}_i(do^P(switch\text{-}on\text{-}conditioner_A))$, where postfix P stands for 'past execution'.

Conclusions

We discussed some cognitive aspects concerning cooperative executability of actions in a group of agents according to the available budget. So, a group of agents can achieve goals unattainable by a single agent. To model these aspects we proposed the new modal logic L-DINF, for which obtained soundness and strong completeness results.