Multi-Agent Search using Sensors with Heterogeneous Capabilities

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ABSTRACT

In this thesis we address the problem of multi-agent search. We formulate two deploy and search strategies based on optimal deployment of agents in search space so as to maximize the search effectiveness in a single step. We show that a variation of centroidal Voronoi configuration is the optimal deployment. When the agents have sensors with different capabilities, the problem will be heterogeneous in nature. We introduce a new concept namely, *generalized Voronoi partition* in order to formulate and solve the heterogeneous multi-agent search problem. We address a few theoretical issues such as optimality of deployment, convergence and spatial distributedness of the control law and the search strategies. Simulation experiments are carried out to compare performances of the proposed strategies with a few simple search strategies.

1. INTRODUCTION

Autonomous agents equipped with necessary sensors carry out the search operation in an unknown environment called the search space $Q \subset \mathbb{R}^d$, a convex polytope, where the uncertainty or the lack of information is known *a priori* as a probability density function $\phi(q)$, $q \in Q$. The optimal deployment of agents so as to maximize uncertainty reduction in a given step forms the basis for two search strategies we propose, namely, *sequential deploy and search* and *simultaneous deploy and search* strategies.

2. HOMOGENEOUS MULTI-AGENT SEARCH

Here we assume that all the agents are equipped with same kind of sensors to perform search task. We propose two deploy and search strategies.

2.1 Sequential Deploy and Search (SDS)

At each iteration, after deploying themselves *optimally*, the sensors gather information about Q, reducing the density function as [1],

$$\phi_{n+1}(q) = \phi_n(q) \min_i \{\beta(\|p_i - q\|)\} \tag{1}$$

Cite as: Extended Thesis Abstract for Doctoral Mentoring Program Guruprasad K. R., *Proc. of 7th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2008)*, Padgham, Parkes, Müller and Parsons (eds.), May, 12-16., 2008, Estoril, Portugal, pp. 1734-1735

where, $\phi_n(q)$ is the density function at the n-th iteration, $\beta : \mathbb{R} \mapsto [0,1]$, a function of Euclidian distance of a given point in space from the agent, acts as the factor of reduction in uncertainty by the sensors, and p_i is the position of the i-th sensor. At a given $q \in Q$, only the agent with the smallest $\beta(\|p_i - q\|)$, that is, agent which can reduce the uncertainty by the largest amount is active. It is clear that $\beta \in [0,1]$.

We are looking for deployment of agents in Q, maximizing per iteration reduction in the uncertainty ϕ. Consider the following objective function to be maximized.

$$\mathcal{H}_n = \sum_i \int_{V_i} \phi_n(q)(1 - \beta(\|p_i - q\|))dQ \tag{2}$$

where V_i is the Voronoi partition corresponding to the i-th agent, and $p_i \in Q$ is the position of the i-th agent.

We consider $\beta(r) = 1 - ke^{-\alpha r^2}, \quad k \in (0,1) \quad \text{and } \alpha > 0$, and show that the critical points of (2) are

$$p_i = \tilde{C}_{V_i} \tag{3}$$

where $\tilde{C}_{V_i}$ are the centroids of corresponding Voronoi partition with respect to the density $\tilde{\phi}_n(q) = \phi_n(q)ke^{-\alpha r^2}$, which is a variation of centroidal Voronoi configuration [2, 3].

Let us consider the system dynamics as $\dot{p}_i = u_i$ and consider the control law to make the agents move towards the corresponding centroids

$$u_i = -k_{prop}(p_i - \tilde{C}_{V_i}) \tag{4}$$

Using LaSalle's invariance principle, we show, that the trajectories of the agents governed by the control law (4), starting from any initial condition $\mathcal{P}(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}_n$.

We also show that the *sequential deploy and search* strategy is spatially distributed in the Delaunay graph, and the average uncertainty can be reduced to any arbitrarily small value in finite number of iterations.

2.2 Combined Deploy and Search (CDS)

In this strategy, as the agents are being deployed optimally, the search task, that is, the uncertainty update is done simultaneously (in discrete time intervals in case of discrete implementation). The objective function does not have the subscript n here. As ϕ is varying with time (implicitly), $\mathcal{H}$ too varies with time. We show, that at any time instant t, the optimal deployment is given by (3). We use same control law (4).

We show that the *combined deploy and search* strategy is spatially distributed in the Delaunay graph, and the average uncertainty can be reduced to any arbitrarily small value in finite time.

We provide a few formal analysis results such as stability and convergence of the proposed control laws, and spatial distributedness of the strategies under a few constraints such as limit on maximum speed of agents, agents moving with constant speed and limitation on sensor range for both the strategies.

The simulation experiments show that the proposed strategies perform fairly well. The proposed strategies have been compared with a few simple search strategies such as greedy and random search, and the results are encouraging.

3. GENERALIZED VORONOI PARTITION

Here we present a new scheme of partitioning a given space. The concept is based on Voronoi decomposition. In case of Voronoi decomposition, a distance measure such as the Euclidean distance forms the basis on which the space is partitioned. A few generalizations such as weighted (multiplicatively and additively weighted) Voronoi partitions have been used in some applications [4]. We propose a partitioning scheme based on a collection of functions called node functions along with nodes or generators.

Consider a space $Q \subset \mathbb{R}^d$, a set of points called *nodes* or *generators* $\mathcal{P} = \{p_1, p_2, \ldots, p_N\} \in Q$, with $p_i \neq p_j$, whenever $i \neq j$, and a strictly decreasing function $f_i : \mathbb{R}^+ \mapsto R$, called *node function* for the i-th node. The collection $\{f_i\}$ satisfies the condition that $f_i - f_j$ is analytic $\forall i, j \in [1, N]$. Define a collection $\{V_i\}$, $i \in [1, N]$, with mutually disjoint interiors, such that $Q = \cup_{i \in [1,N]} V_i$, where V_i is defined as

$$V_i = \{q \in Q | f_i(\|p_i - q\|) \geq f_j(\|p_j - q\|), j \neq i, \quad j \in [1, N]\} \tag{5}$$

We call $\{V_i\}$, $i \in [1, N]$, as a *generalized Voronoi partition* of Q with nodes $\mathcal{P}$ and node functions f_i.

4. HETEROGENEOUS LOCATIONAL OPTIMIZATION

Here we use the concept of generalized Voronoi partitions to solve heterogeneous locational optimization problem. Let $Q \subset \mathbb{R}^d$ be a convex polytope; $\phi : Q \mapsto [0, 1]$, be a density distribution function; $\mathcal{P} = (p_1, p_2, \ldots, p_N) \in Q^N$ be the configuration of N agents, with $p_i \neq p_j$ whenever $i \neq j$; $f_i : \mathbb{R}^+ \mapsto \mathbb{R}$, $i \in [1, N]$, be continuous, strictly decreasing function corresponding to i-th node and $V_i \subset Q$ be the generalized Voronoi cell corresponding to the i-th node.

Consider the objective function to be maximized,

$$\mathcal{H}(\mathcal{P}) = \sum_i \int_{V_i} f_i(\|q - p_i\|)\phi(q) dQ \tag{6}$$

We show that the optimal configuration is a variation of centroidal Voronoi configuration with density $\tilde{\phi}(q) = -\phi(q)$ $\partial f_i(r_i)/\partial (r_i)^2$. We also show, that the trajectories of the agents governed by the control law (4), starting from any initial condition $\mathcal{P}(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}$.

5. HETEROGENEOUS MULTI-AGENT SEARCH

We devise two deploy and search strategies namely, SDS and CDS as counterparts of homogeneous multi-agent search strategies, based on heterogeneous locational optimization problem. Here we consider $f_i(.) = 1 - \beta_i(.)$, and as a special case use $\beta_i(r) = 1 - k_i e^{-\alpha_i r^2}$ with $k_i \in (0, 1)$ and $\alpha_i > 0$, $i \in [1, N]$.

We show, that The trajectories of the agents governed by the control law (4), starting from any initial condition $\mathcal{P}(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}_n$ for SDS strategy.

We also show spatial distributedness and convergence for both CDS and SDS.

A few simulation experiments were carried out to analyse the strategies numerically and further experiments are going on.

6. CONCLUSIONS

We addressed the problem of multi-agent search with agents having sensors with homogeneous and heterogeneous capabilities. Two search strategies namely *sequential deploy and search* and *combined deploy and search* have been proposed and shown to successfully reduce the uncertainty to any desired value in finite time in a spatially distributed way. In order to formulate and solve the heterogeneous multi-agent search problem we introduced the concept of a *generalized Voronoi partition.* The standard Voronoi partitions and its variations were shown as special cases of the general partitioning scheme. We formulated and solved a heterogenous locational optimization problem and intend to analyze the problem with sensor range limits. A few simulation experiments were carried out to evaluate the performance of the proposed strategies and compare them with a few simple strategies such as greedy and random search and the results are encouraging.

7. REFERENCES

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