Strategyproof Deterministic Lotteries under Broadcast Communication

(Short Paper)

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ABSTRACT

The design of deterministic and fair mechanisms for selection among a set of self-motivated agents based solely on these agents' input is a major challenge in multiagent systems. This challenge is especially difficult when the agents can only communicate via a broadcast channel. We propose the notion of selection games: a special case of zero-sum games where the only possible outcomes are selections of a single agent among the set of agents. We assume the lack of an external coordinator, and therefore we focus on mechanisms which have a solution where the agents play weakly dominant strategies. Our first major result shows that dominated strategies could be added to any selection mechanism, so that the resulting mechanism becomes quasi-symmetric. For fairness, we require the mechanism to be non-imposing; that is, the mechanism should allow any agent to be selected in such a solution. We first show that such mechanisms do not exist when there are two or three agents in the system. However, surprisingly, we show that such mechanisms exist when there are four or more agents. Moreover, in our second major result, we show that there exist selection mechanisms that implement any distribution over the agents, when the agents play mixed dominant strategies. These results also have significance for electronic commerce, ranking systems, and social choice.

Categories and Subject Descriptors

G.3 [**Probability and Statistics**]: Distribution functions; J.4 [**Social and Behavioral Sciences**]: Economics

Keywords

selection games, ranking games, quasi-symmetry

1. INTRODUCTION

The design of deterministic and fair mechanisms for selection among a set of self-motivated agents based solely on these agents' input is a major challenge in multiagent systems. This challenge is especially difficult when the agents can only communicate via a broadcast channel.

This problem can manifest itself when several machines, controlled by different entities, share a local network in order to run some distributed computation or service. Such system may require a designated entity which, as a byproduct or by design, is provided with better service. In this setting, all communications between machines are, by the nature of the network, broadcast, and due to the upgraded service, the agents are self-interested.

Another example of this problem is a group of people trying to coordinate random allocation of some sort of resource online, when they do not trust each other, and have a common broadcast forum they may use (such as a mailing list or bulletin board), but no trusted center.

A similar problem can be found in the domain of electronic commerce: A common marketing tactic is to conduct lotteries among customers. However, legal constraints limit the use of such lotteries, leading businesses to apply alternative means for fair distribution of prizes. Therefore, we again see a need for deterministic and fair selection mechanisms that mimic the outcome generated by such lottery while relying only on the agents' inputs.

One simple solution for this problem in the context of leader election is collective coin flipping [4]. In this context, the idea is to try and minimize the influence of agents and coalitions on the result of the lottery in the face of Byzantine failures. Some powerful mechanisms showing a relatively small amount of influence have been introduced (see [9] for a discussion of such results). However, these approaches do not utilize the incentive structure of the agents, and thus agents must be given some level of influence, which may be abused to ensure self-selection.

This problem of influence has been addressed by applying cryptography in the form of one-way functions in order to circumvent the problem of open communication. This approach however is not secure in the information-theoretic sense, as agents with an unlimited computational power are able to reverse the one-way function and respond. On the positive side, such protocols are secure even in the face of coalitions.

In order to solve the aforementioned problems, we suggest applying a game-theoretic approach. We propose mechanisms where although agents can manipulate the result, they do not have any incentive to do so. As we assume a broadcast channel, we require a solution in dominant strategies. That is, even after all but one agent have exposed their inputs, the remaining agent will not have an incentive to deviate from its dominant strategy.

In order to capture the agents' incentives, we define the notion of selection games. These games are a special case of zero-sum games where the only possible outcomes are selections of a single agent among the set of agents. We assume that the agents will play (a mixture of) weakly dominant strategies if such strategies exist. We therefore focus on selection games where a desired outcome is

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attained when all agents play such strategies. Our aim in the design of such mechanisms is that the mechanisms will be fair and will implement a desired probability distribution on the outcomes.

Fairness in this context is captured by the notion of quasi-symmetry. Quasi-symmetry means that all agents have the same strategy set, and that the outcome depends only on the number of times each strategy is played by all players, and not on the identity of the players. In fact, in a quasi-symmetric game all agents have exactly the same influence on the outcome. Note however, that the agents' preferences may have a major effect on the strategies actually selected, and thus on the practical influence of each agent in the game.

Our first main result shows that any selection game can be extended to a quasi-symmetric game preserving all agents' dominant strategies. Hence, if we are able to implement some desired outcome in dominant strategies, we can also do so fairly, in the sense of quasi-symmetry. We then show, in our second main result, that *any* distribution over at least four agents may be implemented under mixed dominant strategies. These two results together offer a practical means of conducting arbitrary deterministic lotteries in a fair manner. Alternatively, this result can be viewed as providing a means for leader election for an arbitrary distribution.

These results are applicable to several types of popular online lotteries. For example, businesses conduct sweepstakes where the chances of winning are proportional to some participation indicator (such as points, products purchased, etc.). Our general distribution result directly applies in this context. Another example is in the selection of ads to be shown based on agents' bids[13]. If we wish that the selection will be random based on the relative value of bids, then our machinery becomes highly relevant.

Several additional results are also obtained in this paper. We show that, surprisingly, in the special case where there are only two or three agents, *no* selection game exists where all agents have nonzero probability of being elected. Formally, this is captured by the fact no non-imposing selection games exist for two or three agents.

A similar type of mechanism where agents' strategies only affect the welfare of other agents has been discussed with regard to impartial division of a dollar [6]. In that setting, allocation of a divisible good is determined by agents' opinions on the other agent's shares. In our setting, the problem is the indivisibility of the good (or service), while we assume that the share proportions are agreed upon. These two methods can be used in sequence for impartial allocation of an indivisible good: First decide on the size of shares, and then apply our method to randomly allocate the good based on these proportions.

As mentioned above, our selection games setting is related to the leader election problem in distributed computing [10]. In particular, the study of the so-called *cheater's edge*[3] can be applied to our domain. That study focuses on limiting the probability of a failed agent being elected under a Byzantine failure model, which allows for arbitrary failures. Specifically, that paper presents a protocol $\mathcal{P}_0$ that guarantees that under at most one fault, the faulty agent will be elected with probability of at most $\frac{1}{n}$, which is the minimal attainable probability. This can be seen as a special case of our second main result, as this protocol implements the uniform distribution.

In section 2 we define the selection games setting and solution concepts in this setting. In section 3 we prove our first major result with regard to the existence of quasi-symmetric selection games. In sections 4 and 5, we discuss non-imposing selection games and selection games for general distributions respectively. In section 6 we demonstrate an application of our results for impartial allocation of an indivisible good. Finally, in section 7 we discuss the impact of this work on related fields.

2. SELECTION GAMES

In order to begin our discussion of selection games, we first formally define the notion of a selection game as a special case of a zero-sum normal form game:

DEFINITION 1. *A* selection game *is a tuple $G = (N, S, v)$, where $N = \{1, \ldots, n\}$ is a set of players ($n > 1$), $S = (S_1, S_2, \ldots, S_n)$ is a vector of strategy sets $S_i = \{s_i^1, \ldots s_i^{m_i}\}$ for each player, and v is a function $v : \mathbb{S} \mapsto N$ that maps every strategy profile $s \in \mathbb{S}$ to a winner $v(s) \in N$, where $\mathbb{S}$ the set of strategy profiles $\mathbb{S} = S_1 \times S_2 \times \cdots \times S_n$ in the game G.*

A selection game can be mapped to a zero-sum normal form game with the utility function

$$u_i(s) = \begin{cases} 1 & v(s) = i \\ 0 & \text{otherwise}. \end{cases}$$

This definition means that all the results that apply to general zero-sum games apply to selection games as well. Specifically, classical games such as *Matching Pennies* or even *Chess* (assuming no ties) are in fact two-player selection games.

In Game Theory, whenever a type of game is discussed we try to define solution concepts for that type of game. Such a solution concept may be pureor mixed:

DEFINITION 2. *A* (pure) solution concept for selection games *is a function $\mathcal{C} : \mathbb{G} \mapsto \wp(\mathbb{S})$ that maps every selection game $G \in \mathbb{G}$ to a set of strategy profiles in that game.*

A mixed solution concept for selection games *is a function $\mathcal{C} : \mathbb{G} \mapsto \wp(\Delta(S_1) \times \Delta(S_2) \times \cdots \times \Delta(S_n))$ that maps every selection game to a set of mixed strategy profiles in that game.*

Note that any pure solution concept can be mapped to an equivalent mixed solution concept.

We can now present several solution concepts in weakly dominant strategies for normal-form games:

DEFINITION 3. *Let $G = (N, S, v)$ be a selection game and let $i \in N$ be some agent. The weakly dominant strategy set for i in G, denoted by $D_G(i)$, is the set of all strategies $s_i \in S_i$ such that for all strategy profiles $s' \in \mathbb{S}$: $v(s'_i, s'_{-i}) = i \Rightarrow v(s_i, s'_{-i}) = i$.*

Note that our definition of weakly dominant strategies allows for several weakly dominant strategies that an agent is indifferent between, as we do not require a strict preference over every other strategy. This is required in order to allow multiple outcomes to possibly be selected while all agents still play "dominant" strategies. For this to happen, at least one agent must have more than one "dominant" strategy and be indifferent between these strategies.

DEFINITION 4. *Let G be a selection game. The* Weakly Dominant Strategies *pure solution concept $\mathcal{C}_{WD}$ is the set of all strategy profiles s where every player plays a weakly dominant strategy. That is, for all $i \in N : s_i \in D_G(i)$.*

The Mixed Dominant Strategies *solution concept $\mathcal{C}_{MD}$ is the set of all mixed strategy profiles s where every player plays a weakly dominant mixed strategy. That is, for all $i \in N$: $s_i \in \Delta(D_G(i))$.*

The Uniform Dominant Strategies *mixed solution concept $\mathcal{C}_{UD}$ consists of the mixed strategy profile s^U where for all $i \in N$: s_i^U is a uniform mixture over $D_G(i)$.*

EXAMPLE 1. *Consider the selection game $G = (N, S, v)$, where $N = \{1, 2, 3, 4\}$, $S = (\{a_1, b_1\},\ \{a_2, b_2\},\ \{a_3, b_3\},\ \{\varepsilon\})$, and v is described below:*

	a_3		b_3	
	a_2	b_2	a_2	b_2
a_1	1	4	2	2
b_1	1	3	4	3

In this game all strategies for all players are weakly dominant. Therefore, all 8 strategy profiles are solutions in weakly dominant strategies. Furthermore, any mixture $((\alpha, 1-\alpha), (\beta, 1-\beta), (\gamma, 1-\gamma), (1))$ *is a solution in mixed dominant strategies. However, there is only one solution in uniform dominant strategies, which is when the three active agents play their "a" strategy with a probability of exactly* $\frac{1}{2}$.

3. QUASI-SYMMETRY

A basic requirement on mechanisms for deterministic lotteries is fairness in the sense that all agents have the same influence, as captured by the notion of quasi-symmetry. In a quasi-symmetric game all agents have the same options (i.e. strategy set), and the outcome is determined only by the number of times each strategy is played, with no regard to which agent played what strategy:

DEFINITION 5. *A selection game* $G = (N, S, v)$ *is called quasi-symmetric if all strategy sets are equal (*$S_i = S_j \forall i, j \in N$*) and for every permutation* $\pi : N \mapsto N$ *and for every strategy profile* $s \in \mathbb{S}$*:* $v(\pi(s)) = v(s)$.

Note that this definition is different from *symmetry* as defined for normal-form games, which requires the payoffs be also permuted. Such symmetry, however, is impossible with selection games because it implies that when all agents play the same strategy there will be a tie.

A classical example of a quasi-symmetric selection game is *matching pennies*, where one player is selected if two different strategies are played and the other is selected if the same strategy is played by both players.

Our first major result shows that any selection game can be extended to a quasi-symmetric one while preserving the dominant strategy sets for the agents. That is, in any selection game we can add dominated strategies for the agents to produce a quasi-symmetric selection game.

THEOREM 1. *Let* $G = (N, S, v)$ *be a selection game where all agents' strategy sets are disjoint. Then, there exists a quasi-symmetric selection game* $G' = (N, S'^n, v')$ *such that for all* $s \in \mathbb{S}$*:* $v'(s) = v(s)$*, and for every agent* $i \in N$*:* $D_G(i) = D_{G'}(i)$.

The idea of the proof is to allow every agent play any other agent's strategy, but make sure that all agents will prefer to play their own strategies (by making other agents' strategies dominated), and thus the original game is always played.

4. NON-IMPOSITION

One application of selection games is in the context of elections among members of some group, also known as Ranking Games [5] or Ranking Systems [1]. In this context, a very simple requirement is non-imposition, which means that for every agent there is a possible outcome in which it is elected[2]. However, in the context of selection games, we want to ensure this situation will actually occur when the game is played. Therefore, we require that all outcomes must be possible in a *solution* of the game:

DEFINITION 6. *Let* $\mathcal{C}$ *be a (pure) solution concept for selection games. A selection game* $G = (N, S, v)$ *is called* non imposing under solution concept $\mathcal{C}$ *if for all* $i \in N$ *there exists some solution* $s \in \mathcal{C}(G)$ *such that* $v(s) = i$.

As the following proposition shows, if the number of agents is at most three, it is impossible to satisfy even this modest fairness requirement under weakly dominant strategies. Needless to say, this prevents the implementation of general distributions.

PROPOSITION 1. *There exists no selection game that is non-imposing under weakly dominant strategies with* $|N| \leq 3$.

PROOF. Assume $|N| = 2$. Let (s_1, s_2) be a solution under weakly dominant strategies where $v(s_1, s_2) = 1$ and let (s'_1, s'_2) be a solution under weakly dominant strategies where $v(s'_1, s'_2) = 2$. By weak dominance of s'_1: $v(s'_1, s_2) = 1$, but by weak dominance of $s_2 : v(s'_1, s_2) = 2$, which is a contradiction.

Assume $|N| = 3$. Let (s_1^1, s_2^1, s_3^1), (s_1^2, s_2^2, s_3^2), (s_1^3, s_2^3, s_3^3) be solutions under weakly dominant strategies where $v(s_1^i, s_2^i, s_3^i) = i$. By dominance of strategies $\{s_1^2, s_2^3, s_3^1\}$, we have: $v(s_1^2, s_2^1, s_3^1) = 1$; $v(s_1^2, s_2^3, s_3^2) = 2$; $v(s_1^3, s_2^3, s_3^1) = 3$. Now consider the strategy profile (s_1^2, s_2^3, s_3^1). If $v(s_1^2, s_2^3, s_3^1) = 1$, then by dominance of s_1^3: $v(s_1^3, s_2^3, s_3^1) = 1 \neq 3$, in contradiction to the above. Similarly, if $v(s_1^2, s_2^3, s_3^1) = 2$ then $v(s_1^2, s_2^1, s_3^1) = 2 \neq 1$ and if $v(s_1^2, s_2^3, s_3^1) = 3$ then $v(s_1^2, s_2^3, s_3^2) = 3 \neq 2$, which leads to a contradiction. $\square$

We will discuss the significance of this result to ranking systems in our discussion in Section 7.

5. IMPLEMENTATION OF GENERAL DISTRIBUTIONS

We shall now present our second main result: the existence of selection games that implement any distribution under mixed dominant strategies. First, we must define the concept of implementation of a distribution:

DEFINITION 7. *A selection game* $G = (N, S, v)$ implements a distribution $\mathcal{D} \in \Delta(N)$ under mixed solution concept $\mathcal{C}$ *if there exists some solution* $s \in \mathcal{C}(G)$ *such that for all* $i \in N$*:* $Pr\left[v(s) = i\right]$ *is distributed according to* $\mathcal{D}$.

A selection game $G = (N, S, v)$ *is called* uniform under mixed solution concept $\mathcal{C}$ *it implements the uniform distribution under* $\mathcal{C}$.

The existence of selection games that are uniform under uniform dominant strategies has been previously shown by [3] in the context of leader election. We shall now extend this result to any rational distribution:

DEFINITION 8. *A probability distribution* $\mathcal{D}$ *over a finite set* N *is called* rational *if for all* $i \in N$*:* $\mathcal{D}(i) \in \mathbb{Q}$.

THEOREM 2. *Let* N *be a finite player set where* $|N| \geq 4$*, and let* $\mathcal{D}$ *be a rational distribution over* N*. There exists a selection game that implements* $\mathcal{D}$ *under uniform dominant strategies.*

The implementation of any rational distribution is based on an extension of the idea presented in Example 1. Only three agents determine the outcome in a way that their strategies do not affect whether or not they are selected. The game is then specifically designed to ensure that the exact ratios specified in the distribution are implemented.

The fact that only three agents determine the result of the game should be seen as an advantage of this mechanism, as it requires a constant number of log-length broadcast messages in order to guarantee random selection according to any pre-determined distribution. In contrast to classical mechanisms in social choice, having all agents participate is not, in itself, an important property to be satisfied.

Applying Theorem 1, this result can be extended to quasi-symmetric games.

COROLLARY 1. *Let N be a finite player set where $|N| \geq 4$. Let $\mathcal{D}$ be a rational distribution over N. There exists a quasi-symmetric selection game G that implements $\mathcal{D}$ under uniform dominant strategies.*

The results above extend to irrational distributions as well, but only if we replace the requirement of uniform dominant strategies with mixed dominant strategies.

6. IMPARTIAL ALLOCATION OF AN INDIVISIBLE GOOD

de Clippel et al. [6] suggest a method for impartial allocation of a divisible good (such as a dollar) under dominant strategies. In their system, every participant announces their opinion on the relative shares of the other participants, and the amount allocated to each of the participants is determined solely by the claims of the others.

The impartial division method is based upon the assumption that the good could be arbitrarily divided among the agents, and leaves open the problem of allocating an indivisible good or selecting a winner, as in our setting. Such allocation can be accomplished if the participants had a common coin they could all agree on. This is exactly what our selection game implementation provides.

It is worthy to note that both systems have impossibility results for 3 agents, and the impossibility result of impartial division can be seen as a special case of Proposition 1.

Combining the two methods, we suggest the following protocol for the impartial allocation of an indivisible good on a broadcast channel:

- Agents announce their opinions on the fair shares for the other agents.
- All agents compute the "dollar division" among the agents using the method in [6].
- Agents compute a selection game for the resulting division.
- The three agents assigned the smallest shares (ties broken arbitrarily) uniformly choose and broadcast their random strategy for the selection game.
- All agents compute the result and agree on the winner.

This protocol solves the allocation problem for an indivisible good, and involves only two stages of communication: one for selecting the division of the good as if it were divisible, and one for randomly selecting an agent to get the good based on this division.

Note that the second-stage random selection cannot be sent until all first-stage announcements are done, otherwise an agent may manipulate by adjusting the shares such that the players active in the resulting selection game will elect him.

7. DISCUSSION AND IMPLICATIONS

A slight variation of the selection games setting discussed in this paper is when the incentive structure is reversed, and each agent prefers *not* to be selected. This incentive structure arises when agents need to allocate some kind of errand which they would rather not do (but would otherwise like to see allocated randomly). Any selection game where all agents have only weakly dominant strategies, and thus are indifferent between all their strategies, retains this feature in the reverse incentive structure. As our second main result was built using this kind of games, it equally applies in the reverse utility setting. If the utility structure is known, the results regarding quasi-symmetry could also be applied in this setting.

If communications were allowed to be private, standard cryptographic techniques such as Collective Coin Flipping [4] could be used. These techniques enable us to implement any rational distribution in strong Nash equilibrium, and thus tackle the problem of collusion which could not be addressed in the broadcast communications setting, giving a solution in strong Nash equilibrium.

Selection games are as a special case of ranking games [5], where the agents care only about whether or not they are ranked first (or last). Mechanism design in this setting has been studied in our work on ranking systems [1, 2]. For example, when outgoing links are considered as votes, any page ranking system, such as PageRank [11] or the HITS algorithm [8] can be described as a ranking system, implying a selection game in which agents care only about being ranked first. Our results can therefore be interpreted also from the perspective of the study of existence of non-imposing incentive compatible ranking systems, with the important difference that the ranking systems setting allows ties, which are not allowed in selection games. In particular, although we have shown that under the linear utility function there exists an incentive-compatible non-imposing ranking system for three agents [2], we have shown in Proposition 1 that no such selection mechanism exists.

It is also interesting to put this work in perspective of work on incentive-compatible social choice. The celebrated Gibbard-Satterthwaite theorem [7, 12] shows an impossibility result for non-imposing[1] incentive compatible mechanisms if there are at least three candidates in a social choice setting. In contrast, our results show that in a selection game setting, although impossibility is obtained when there are *at most* three agents, a constructive possibility result is obtained for four or more agents.

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[1] In fact, Gibbard-Satterthwaite's results only require that the domain size be at least 3, which also prevents non-imposition.