Welfare Losses in Commodity Storage Games

(Extended Abstract)

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ABSTRACT

We develop a game theoretic model of a shared commodity storage facility whose injection/ejection and space resources have been allocated to multiple selfish firms. We assume a setting where each firms injectability and deliverability depend on overall inventory within the store. This is most common in settings that involve the storage of liquids or gases such as natural gas or oil. We analyze the strategic interaction that occurs in a pre-commitment game where firms commit to inject until a certain time and subsequently withdraw the commodity until it is exhausted. We contrast equilibrium and welfare maximizing outcomes whilst casting our analysis in terms of a natural gas storage facility. We also study the effects of incorporating a "use-it-or-lose-it" policy on overall welfare. Our results indicate that significant inefficiencies can occur in both settings.

1. INTRODUCTION

We study the problem of managing a capacitated warehouse for storing non-perishable commodity products. In particular, we look at the storage of liquid and gaseous commodities in slow-cycling facilities that cannot be filled or emptied in single time-periods. The warehouse is *capacitated* in terms of the rate at which goods can be injected or withdrawn from the store. Consider the case of natural gas storage. Most facilities consist of partially depleted gas fields that have undergone a lengthy and expensive conversion process. Furthermore, a cushion gas needs to be maintained in the store in order to maintain stability. This requires a capital investment in the order of billions of dollars and therefore induces a great incentive to operate the facility in a manner that is economically efficient. The worldwide shortage of gas storage facilities and the increased volatility of prices have contributed to a rapid increase in their value [4].

When multiple agents purchase rights they are being assigned three resources 1) injection capacity (a fraction of pump resources), 2) withdrawal capacity (a fraction of the extraction resources) and 3) space. We look at the typical setting where a storage manager sells these resources on a pro rata basis so that each agents holds the same ratio of injection:ejection:space resources. During the term of a storage contract, the agents can then make nominations on which actions the store operator should take on their behalf.

2. PRE-COMMITMENT STORAGE GAME

The operating characteristics of actual storage facilities present challenges because decisions exhibit dependencies on past behavior. When the commodity is withdrawn from storage it cannot be released again [1]. Also, when the commodity is released the deliverability of the remaining inventory in storage generally decreases. Likewise, the rate of injection decreases as the inventory in storage increases. In the case of the gaseous commodities, this occurs because of the change in pressure. For liquid storage, the pressure also drops as the hydrostatic pressure drops in line with the height of the stored liquid [2]. The strategic interaction between multiple leaseholders of storage capacity (firms) arises because injectability and deliverability depends on the stored inventory, which in turn depends upon the strategic decisions of rival suppliers. In practice, given the strong seasonality in prices and operational difficulty in switching from/to injection and withdrawal, energy traders often decide on an injection/withdrawal policy for the forthcoming year.Many firms decide to inject continuously until a given date in late Autumn and then withdraw from the store. We capture this form of pre-commitment in a normal form game and examine equilibrium outcomes. We also assume that firms are risk-neutral.

We present a model in which two firms each own leases for $\frac{1}{2}$ of the storage capacity in the same facility. We cast this model in terms of a natural gas setting. It can be easily translated to another commodity setting in which periodicity of the price is regular, *e.g.* pumped-hydro storage for electricity generation on a daily basis. We assume that the gas price has a strong seasonal element, we propose a normal form model in which firm 1 injects gas until a date d_1 at which point the firm withdraws until they have no remaining gas in the store. The cost to firm i of injecting for d_i days (to store q_i units of gas), $C_i(d_i)$, depends on the strategy of the other firm(s). We assume, without loss of generality, that $d_i \leq d_{i+1}$,$\forall i \in \{1, \ldots, n-1\}$. Given the strong seasonality, we assume that the daily average price for day i in a contract is given by $p_i = \alpha + \beta * Cos(\frac{2\pi i}{365})$, where α is a constant that ensures non-negative prices, and β is a coefficient calibrated with historical prices. There is a clearly discernable seasonality effect in prices and this is widely accepted in the relevant literature [2, 3] Gas utilities also have access to liquid forward and future contracts prices that provide an indication of the markets expectation for price movements. The maximum rate of injection is always applied. The maximum injection rate is $c_{min}(I) = -K_1\sqrt{\frac{1}{I+I_b}} + K_2$, for some constants K_1 and K_2 that relate to the physical attributes of the storage facility.

The total cost, C, of gas injected from day d_0 until day d_1 for each agent is given by the sum of cashflows until day d_1. $C_{1,2}(d_0, d_1)$ $\frac{1}{2}\int_{d_0}^{d_1} -K_1\sqrt{\frac{1}{I+I_b}} + K_2 p_i \delta i$. This equation indicates that the pump-

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ing resources are divided among the two competing agents equally, so we are assuming they have an equal share in the storage capacity. In period 2, however, only firm 2 is pumping gas and the pressure within the store is decreasing because the total inventory I, a function of both firms strategy, is being reduced by firm 1's withdrawal.

The revenue from gas withdrawn for firm 1, R_1, after day d_1 is given by the sum of cashflows from day d_1 until the store is emptied of all gas that firm 1 injected on day d_{e1}. The rate of change of total inventory, I, is a piecewise function that signifies the cashflows whilst other firm(s) are still injecting. The pressure decrease accelerates as more firms start withdrawing gas. In the case of revenue for Firm 1 in our two player game: $R_1(d_1, d_{e1}) = \frac{1}{2}\int_{d_i}^{d_{i+1}} K_3\sqrt{I} - K_1\sqrt{\frac{1}{I+I_b}} + K_2 p_i \delta i$. The revenue for Firm 2 is expressed similarly.

In the storage game for natural gas, each firm may choose $d_i \in \{1, \ldots, 365\}$. We construct an example using arbitrarily chosen values to describe realistic physical characteristics for a realistic gas storage facility. To model the injection rate, we let $K_1 = 300000$ and $K_2 = -0.0004$. These constants reflect pump capacity and maximum pressure levels, respectively. We can determine the injection rate as a function of the total inventory in the store. We let $K_3 = 500$ and this reflects the diameter of the exit pipe. We calculate *best response functions* for each firm.

Non-transferability Policy Results.

In order to determine whether or not there is a pure strategy Nash equilibrium in this game, we examine the best response functions of both firms in Figure 1. We see that there is no unique PSNE $\{d_1, d_2\}$ at which d_1 is a best response to d_2.

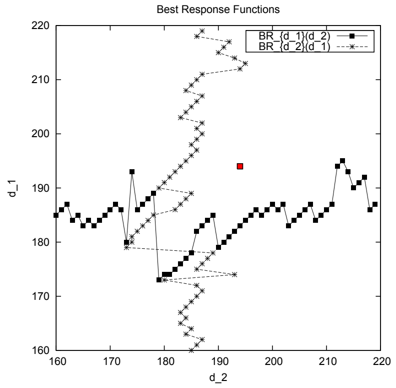

Figure 1: Best response functions.

The social welfare maximizing outcome is for both firms to inject until day d_{194}, $(194, 194)$. This would result in a profit of $Eur15.751m$ for each firm. However, due to the game-theoretic incentive to prematurely withdraw gas, each company faces the prospect of a pareto sub-optimal equilibrium. A single mixed strategy Nash equilibria exists in this game and it is clear that the expected welfare losses are significant. The firms randomize over the following strategies ({178,179},{178,179}); with an approximate welfare loss of $Eur0.708m$.

Use-It-Or-Lose-It Policy Results.

Many storage facilities operate a "use-it-or-lose-it" policy whereby if you choose to inject a commodity, your ejection resources can be freely redistributed to other firms and vice versa. Injecting firms future deliverability is reduced at an accelerated level by the faster withdrawal of others. The laggard firm faces the prospect being unable to withdraw as much gas as he would like when gas prices are at their peak. On the other hand, the injectability increases and additional gas can be pumped into the store before prices peak.

The British gas regulator insists on this "use-it-or-lose-it" policy for the operation of gas storage facilities in the UK. In this section we examine the impact of this policy in a competitive setting and we aim to determine whether this will lead to an expected increase or decrease in welfare for storage leaseholders.

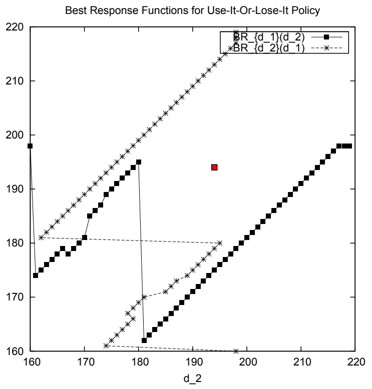

Figure 2: Best responses (Use-It-Or-Lose-It policy).

Figure 2 clearly illustrates the impact that this policy has on the best response functions of the two firms. A single mixed strategy Nash equilibria exists in this game and it is clear that the expected welfare losses are significant. The same mixed strategy Nash Equilibrium exists in this game as previously. However, the best response functions are further from the pareto-optimal ({194},{194}). This indicates that there is a stronger incentive for firms to unilaterally deviate by engaging in premature withdrawal.

3. CONCLUSION

We provide equilibrium results for a realistic gas storage field and find that strategic behavior significantly reduces the societal value of the facility. Given the enormous volatility in some of the relevant commodity spot markets (*e.g.* natural gas or electricity), it is not only important for firms to cushion the effects of price spikes via storage but it is also critical for the economic wellbeing of entire nations that their energy storage assets in are utilized in an efficient manner. Our results have significant implications for the economic efficiency of storage facilities, price stability in the relevant commodity markets and the operational policies for storage companies. This work provides an increased incentive for further research into mechanism design techniques for improving economic efficiency in commodity storage.

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