Parametric Mechanism Design via Quantifier Elimination*

(Extended Abstract)

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ABSTRACT

This paper proposes an alternative automated mechanism design approach called *parametric mechanism design* via *quantifier elimination* (PMD-QE), which utilizes QE, a symbolic formula manipulation technique. In PMD-QE, we start from a skeleton of mechanisms, which is characterized by a set of parameters, e.g., *critical values*. The range of parameters where the given constraints are satisfied is automatically identified by QE. To demonstrate the potential of this idea, we are able to identify a non-trivial dominant-strategy incentive compatible mechanism for a setting where a bidder has a publicly known budget limit.

Categories and Subject Descriptors

I.2.11 [**ARTIFICIAL INTELLIGENCE**]: Distributed Artificial Intelligence – Multiagent systems; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Algorithm, Economics, Theory

Keywords

Mechanism design, VCG, budget limit, quantifier elimination

1. INTRODUCTION

Traditionally, mechanism design has been a manual endeavor. An innovative approach called *automated mechanism design* (AMD) tries to automatically generate a mechanism from scratch for a given setting and an objective at hand [5]. The basic idea of (traditional) AMD is that, a mechanism can be considered a mapping from an input (possible types of agents) to an output (a possible outcome) that must satisfy certain constraints. AMD creates many decision variables that specify this mapping and formalizes the mechanism design problem as a combinatorial optimization problem. AMD is a very general framework that can flexibly meet various requirements of a designer. It can be applied to a variety of settings that have been extensively studied in (manual) mechanism design and the beyond.

However, to specify AMD as an optimization problem, often the possible inputs must be finite. If the type of each agent, e.g., the value of an agent, is continuous, it often needs to be discretized. Thus, the sizes of the optimization problem tend to be exponentially large. As a result, designing a customized mechanism for a problem instance with large or continuous inputs is virtually impossible, except some settings such as redistribution mechanisms, even by the state-of-the-art optimization packages.

This paper considers a substantially different approach from previous AMD techniques, which we call *parametric mechanism design* via *quantifier elimination* (PMD-QE). Our approach also starts from a skeleton of mechanisms, but the main advantage over the previous approaches is that our framework does not require that the skeleton is feasible for all parameter settings, which makes it much easier to construct the initial skeleton.

Table 1: Comparison between AMD and PMD-QE

	AMD	PMD-QE
Aim	creating a mechanism from scratch	making skeleton mechanisms feasible
Tool	LP/MIP	QE
Possible types	often discretized	continuous
DSIC	enforced as constraints	automatically satisfied
Resulting output	huge table (a single mechanism)	range of parameters (a class of mechanisms)

2. HIGH-LEVEL DESCRIPTION OF OUR APPROACH

Table 1 compares the traditional AMD approach with PMD-QE. The following describes the flow of our approach.

1. We first construct a skeleton of mechanisms that is inspired by theoretical results in mechanism design literature (specifically, we construct the skeleton by specifying the allocation critical values). The skeleton is characterized by a set of parameters $\{c_1, \ldots, c_k\}$. By choosing specific values of these parameters, a concrete mechanism is specified. A concrete mechanism is not

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always feasible. Also, the class of mechanisms covered by the skeleton might be not general enough to represent the whole class of mechanisms.

2. Second, we identify a feasible region over these parameters. More precisely, these parameter values are required to satisfy *feasibility constraints* to guarantee that the obtained mechanism works. We use a symbolic formula manipulation technique called *quantifier elimination* (QE) [1] to identify a feasible region over $\{c_1, \ldots, c_k\}$.

3. Finally, we explore the feasible region to obtain a set of parameters that theoretically or empirically achieves a desirable performance, e.g., efficiency or revenue. Moreover, we clarify the theoretical property of the obtained mechanism, e.g., whether it is optimal in the whole class of mechanisms, or it outperforms existing mechanisms.

Let us illustrate an example of single-item auctions with agent 1 and 2, whose values are v_1 and v_2. A dominant-strategy incentive compatible (DSIC) mechanism in this setting is characterized by the *critical value* of each agent. Critical value q_i for agent i means that when i's value exceeds q_i, she wins the item and pays q_i. Assume q_i is given as a linear function of the value of the other agent v_j, i.e., $q_i = a_i + b_i v_j$. Such critical values describe a skeleton of mechanisms characterized by the parameters $\{a_1, a_2, b_1, b_2\}$. For example, by setting $a_1 = a_2 = 0, b_1 = b_2 = 1$, we obtain the VCG mechanism. Note that this skeleton can represent only a restricted subclass of mechanisms, since we consider only the case where a critical value is given by a linear function. These parameters must be chosen to satisfy the *allocation feasibility* constraint, i.e., since only one item exists,

$$\forall v_1 \forall v_2 \exists q_1 \exists q_2 (((q_1 \geq v1) \vee (q2 \geq v2)) \wedge ((q_1 = a_1 + b_1 v_2) \wedge (q_2 = a_2 + b_2 v_1))) \tag{1}$$

must hold. As long as these parameters satisfy this constraint, the obtained mechanism works and is automatically guaranteed to be DSIC.

One distinguished feature of our approach is that we utilize QE to unravel the feasibility constraints. That is, we apply QE to identify the feasible region of the parameters. QE reduces first-order formulas to their equivalent quantifier-free forms. For example, given a first-order formula $\exists x\ (x^2 + ax + b \leq c)$ (here, $\exists$ is the quantifier, and a, b, c are the parameters), we can apply QE to reduce it to an equivalent quantifier-free formula: $a^2 - 4b + 4c \geq 0$, which defines the feasible region of a, b, c.

Another one is that we can utilize the theoretical results in mechanism design literature to develop new concrete mechanisms that satisfy desirable properties. PMD-QE can directly apply a certain theoretical result to construct a new concrete mechanism. Note that the skeleton of mechanisms does not need to be feasible for all the parameter settings. A QE solver will automatically find the range of parameters so that the obtained mechanisms become feasible. Thus, it can serve as a tool that fills the gap between mechanism design theory and concrete mechanisms.

3. CASE STUDY

We consider a case where one agent has a *public* budget limit, i.e., an agent has a limit on the payment she can make, and the limit is known to the mechanism designer [4, 2]. In this case, DSIC and Pareto efficiency are incompatible. Thus, we search for a mechanism that satisfies the mandatory sales constraint, which requires a mechanism to allocate an item to some agent in all cases. We consider the following scenario. Two agents 1 and 2, participate in a single-item auction. Agent 1 has a public budget limit w_1 which is a constant value. She cannot pay more than w_1, even if her value v_1 exceeds w_1. Agent 1 is more likely to have a higher value. More precisely, let $F_1(v_1)$ denote the cumulative distribution function of v_1 over the continuous interval of $[0, 1]$. Also, let $F_2(v_2)$ denote the cumulative distribution function of v_2 over the continuous interval of $[0, \bar{v}_2]$, where $\bar{v}_2 < 1$.

We conduct PMD-QE for prespecified piece-wise linear critical values and call the obtained class of mechanisms VCG-b$^+(\lambda)$:

1. If $v_1 < w_1$ or $v_2 < w_1$, apply VCG.

2. If $v_1 \geq w_1$ and $w_1 \leq v_2 < \lambda$, allocate the item to agent 1 at payment w_1.

3. If $v_1 \geq w_1$ and $\lambda \leq v_2$, allocate it to agent 2 at payment λ.

If we set $\lambda = w_1$, it is equivalent to a variant of VCG where if the declared value exceeds the budget limit, the value is replaced to the budget limit and is applied to the standard VCG. If we set $\lambda = \bar{v}_2$, it is equivalent to another variant where if the budget limit matters, the item is always allocated to agent 1.

In addition, we prove that VCG-b$^+(\lambda)$ is the most efficient within deterministic, no positive transfer, and DSIC mechanisms, by choosing appropriate threshold λ.

THEOREM 1. *Assume that λ is set to*

$$E(v_1 | v_1 \geq w_1) = \frac{\int_{w_1}^{1} v f_1(v) dv}{1 - F_1(w_1)}.$$

VCG-b$^+(\lambda)$ yields the highest expected social surplus among all mechanisms that are deterministic, no positive transfer, and DSIC.

$f_1(v)$ is the first-order differentiation of $F_1(v)$, i.e., the probability density function of v and $E(v_1 | v_1 \geq w_1)$ indicates an expected value of v_1 conditional on $v_1 \geq w_1$.

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