Heterogeneous Facility Location without Money

(Doctoral Consortium)

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ABSTRACT

The facility location problem is arguably the prototypical problem in the mechanism design without money's research agenda. Motivated by the intrinsic limitations of the classical model on both (*i*) adequately modelling several real life scenarios and (*ii*) admitting truthful mechanisms having good approximation ratio, we introduce and study a novel, more realistic model of facility location, wherein facilities are heterogeneous and the agent's cost model is dependent on the kind of facilities she is interested in. In this context, we study truthful mechanisms that optimize both utilitarian and non-utilitarian objective functions.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent systems; J.4 [**Social and Behavioral Science**]: Economics

Keywords

Mechanism Design, Facility Location, Approximation Algorithms

1. INTRODUCTION

Approximate Mechanism Design without money is a recent research agenda in the field of algorithmic game theory that aims at modelling strategic scenarios where the impossibility of performing monetary exchanges between the mechanism and the agents prevents the use Vickery-Clarke-Groves (VCG) mechanisms. The facility location problem is, arguably, the benchmark problem in this new research agenda. In the setting studied in the literature so far, that we name *homogeneous facility location* hereinafter, a set of agents located on a network require access to public facilities, and bid their location on the network (which is undisclosed and private information) to an allocation mechanism. The mechanism, on input the agents' bids, must then determine the location of the public facilities so to optimize a certain function of the individual costs (typically the distance to the nearest facility) incurred by the agents. Agents can misreport their location if they can gain by doing so, hence allocation mechanisms must be strategyproof, meaning that it is a dominant strategy for each agent to report her location truthfully to the mechanism.

Following this research agenda, and in the effort of both providing a more realistic model for several real-life scenarios and to overcome the inapproximability results for strategyproof mechanisms, in [7] we proposed and analyzed a novel model for the facility location problem named *heterogeneous facility location without money*. In particular, the main novelties we introduced in our model are: (*i*) multiple heterogeneous (i.e. serving different purposes) facilities, (*ii*) agents' locations being disclosed to the mechanism and (*iii*) agents bidding for the set of facilities they are interested in (as opposed to bidding for their position on the network, as in the homogeneous facility location model). We studied our proposed model under two different objective functions, namely: social cost and Min-Max.

1.1 Related Work

The homogeneous facility location problem has been extensively studied under both utilitarian and non-utilitarian objective functions. In [5] Procaccia and Tennenholtz initiated the field of approximate mechanism design without money by studying the problems of truthfully locating one and two homogeneous facilities, wherein agents can lie about their location on a continuous line. They focus on both social cost and min-max objective functions. For 2-facility location and utilitarian objective, they propose the Two-Extremes algorithm, that places the two facilities in the leftmost and rightmost location of the instance, and prove that it is group strategyproof (i.e. not susceptible to manipulation by any coalition of agents) and has an $\mathcal{O}(n)$ approximation ratio (n being the number of agents). This lower bound has later been shown to be tight by [3] and by the characterization of truthfulness given in [1], where the authors prove that Two-Extremes is the *only deterministic anonymous SP* mechanisms with *bounded approximation ratio* for the 2-facility location problem on the line. Lu et al. [4] prove a 1.045 lower bound and an $n/2$-approximate upper bound for randomized mechanisms, thus improving the bounds given in [5]. The literature on the Min-Max objective function is quite rich in the case of mechanism design with money (mainly because it shows the tension between approximation and truthfulness – being VCG not applicable) but sparse in the case of moneyless mechanisms. Procaccia and Tennenholtz [5] prove in their model tight bounds for min-max approximation with 1 facility and nearly tight results with 2 facilities. Koutsoupias [2] studies moneyless *SP* mechanisms approximating min-max objective for scheduling selfish unrelated machines whose execution times can be verified.

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2. LOCATING HETEROGENEOUS FACILITIES WITHOUT MONEY

The *heterogeneous 2-facility location problem on the line* ([7]), consists of locating facilities on a *linear unweighted graph* on input the bids of the agents for the facilities they are interested in. More specifically, we are given a set of agents $N = \{1, \ldots, n\}$; an undirected unweighted linear graph $G = (V, E)$, where $V \supseteq N$; a set of facilities $\mathfrak{F} = \{\mathsf{F}_1, \mathsf{F}_2\}$. Agents' *types* are subsets of $\mathfrak{F}$, called their *facility set*. We denote the true type of agent i as $T_i \subseteq \mathfrak{F}$. A mechanism M for the facility location problem takes as input a vector of types $\mathcal{T} = (T_1, \ldots, T_n)$ and returns as output a *feasible* allocation $M(\mathcal{T}) = (F_1, F_2)$, such that $F_i \in V$, for $i \in \{1, 2\}$, and $F_1 \neq F_2$. Given a feasible allocation $\mathcal{F} = (F_1, F_2)$, the cost of agent i is defined as $cost_i(\mathcal{F}) = \sum_{j \in T_i} d(i, F_j)$, where $d(i, F_j)$ denotes the length of the shortest path from i to F_j in G. Naturally, agents seek to minimize their cost, and they could misreport their facility sets to the mechanism if this reduces their cost. We let $T'_i \subseteq \mathfrak{F}$ denote a declaration of agent i to the mechanism. We are interested in the following class of mechanisms. A mechanism M is *truthful* (or *strategyproof*, *SP*, for short) if for any agent i, any declaration T'_i and any declarations of the other agents $\mathcal{T}_{-i}$, the following holds: $cost_i(\mathcal{F}) \leq cost_i(\mathcal{F}')$, where $\mathcal{F} = M(\mathcal{T})$ and $\mathcal{F}' = M(T'_i, \mathcal{T}_{-i})$. A *randomized* mechanism M is a *truthful in expectation* if the *expected cost* of every agent is minimized by truthtelling. In our work, we are interested in truthful mechanisms M that return allocations $\mathcal{F} = M(\mathcal{T})$ minimizing a certain *objective function* $obj(\mathcal{F})$, dependent on the costs of individual agents. In particular, we considered two objective function: (i) the *social cost* function, namely: $cost(\mathcal{F}) = \sum_{i \in N} cost_i(\mathcal{F})$ and (ii) the *Min-Max* function, namely: $mc(\mathcal{F}) = \max_{i \in N} cost_i(\mathcal{F})$. A mechanism M is optimal if:

$$M(\mathcal{T}) \in \underset{\mathcal{F} \text{ feasible}}{\operatorname{argmin}}\ obj(\mathcal{F})$$

where obj is either $cost$ or mc. An allocation on declaration vector $\mathcal{T}$ is optimal, denoted as $OPT(\mathcal{T})$, if $obj(OPT(\mathcal{T})) = \min_{\mathcal{F} \text{ feasible}} obj(\mathcal{F})$. When truthfulness and optimality are incompatible, as is often the case, we have to content ourselves with *approximate mechanisms*. A mechanism M is α-approximate if $obj(M(\mathcal{T})) \leq \alpha \cdot obj(OPT(\mathcal{T}))$. Table 1 summarizes our results.

	Social Cost		**Min-Max**	
	LB	UB	LB	UB
Deterministic	9/8	$n-1$	3/2	3
Randomized	1		4/3	3/2

Table 1: Summary of our results

2.1 Social cost objective function

We studied the heterogeneous facility location problem under the social cost function in [6]. We proved a $9/8$ approximation lower bound for deterministic strategyproof algorithms and proposed a deterministic $(n-1)$–approximate algorithm, named TwoExtremes (an adaptation of the algorithm proposed in [5]), that allocates facility F_1 (F_2, respectively) to the leftmost (rightmost, respectively) location of the subgraph induced by agents requesting it. This simple algorithm is the only anonymous bounded-approximation strategyproof mechanism we know for heterogeneous facility location, and, on the basis of a parallel with the characterization of truthful mechanisms for homogeneous facility location given in [1], we conjecture that it is impossible to improve this upper bound. The difficulty in obtaining better approximation guarantees arguably lies in the fact that agents can easily exploit to their advantage the cases when a clash in the allocation of two facilities occurs, i.e. both facilities should be accommodated on the same node. Motivated by this, we then turned our attention to randomized mechanisms. We devised an *optimal randomized algorithm* named RandOPT which successfully treats the cases where collisions in the allocation occur via randomizing over optimal outcomes.

2.2 Min-Max objective function

In [8] we investigated the heterogeneous facility location problem under the Min-Max objective function. In this setting, we proved a 3/2 lower bound for deterministic strategyproof algorithms. Furthermore, we observed that algorithm TwoExtremes proposed in [6] is SP (truthfulness depends on the agents' cost model, which is the same under both the social cost and Min-Max objective function). We also proved that algorithm TwoExtremes is 3–approximate in the Min-Max setting.

Randomization provably helps also under the Min-Max objective function, alas to a lesser extent than the social cost setting. In fact, we proved that Min-Max cannot be approximated within a factor lower than $4/3$ of the optimal value while preserving strategyproofness. We contrast this with a randomized algorithm named RandAVG that attains a $3/2$ approximation ratio. RandAVG works by randomizing over solutions that (in expectation) locate each facility F_k on the middle node of the linear subgraph of G induced by agents requesting facility F_k.

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