Fairness and False-Name Manipulations in Randomized Cake Cutting

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ABSTRACT

Cake cutting has been recognized as a fundamental model in fair division, and several *envy-free* cake cutting algorithms have been proposed. Recent works from the computer science field proposed novel *mechanisms* for cake cutting, whose approaches are based on the theory of mechanism design; these mechanisms are strategy-proof, i.e., no agent has any incentive to misrepresent her utility function, as well as envy-free. We consider a different type of manipulations; each agent might create fake identities to cheat the mechanism. Such manipulations have been called Sybils or *false-name manipulations*, and designing robust mechanisms against them, i.e., false-name-proof, is a challenging problem in mechanism design literature. We first show that no randomized false-name-proof cake cutting mechanism simultaneously satisfies ex-post envy-freeness and Pareto efficiency. We then propose a new randomized mechanism that is optimal in terms of worst-case loss among those that satisfy false-name-proofness, ex-post envy-freeness, and a new weaker efficiency property. However, it reduces the amount of allocations for an agent exponentially with respect to the number of agents. To overcome this negative result, we provide another new cake cutting mechanism that satisfies a weaker notion of false-name-proofness, as well as ex-post envy-freeness and Pareto efficiency.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*; J.4 [**Social and Behavioral Sciences**]: Economics

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Keywords

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1. INTRODUCTION

Cake cutting is a fundamental model of *fair division* [10, 15, 16] in which agents share a whole cake that is usually represented as an interval, say $[0, 1]$, in a fair manner. This abstract model can be applied to many realistic situations for sharing a divisible good, such as land, time slots of a computational resource, usage of a meeting room, etc. Several cake cutting protocol/algorithms have been developed in the literature, such as cut-and-choose and moving-knife.

In cake cutting, *envy-freeness* (in an ex-post sense) is one of the most studied fairness properties. An allocation/share of the cake is said to be envy-free if, under it, no agent envies any other one, i.e., no agent wants to trade her share for any other's share. For instance, when there are only two agents, any allocation produced from the cut-and-choose protocol satisfies envy-freeness.

In recent years, there are several notable works on cake cutting, which consider the problem as mechanism design and propose *strategy-proof cake cutting mechanisms*, rather than cake cutting protocols. A cake cutting mechanism asks each agent to report her *utility (valuation) function* over the cake, instead of indicating where she wants the cut to occur. Under a strategy-proof cake cutting mechanism, it is guaranteed that reporting a true utility function is a dominant strategy, although she could report any utility function. Many strategy-proof cake cutting mechanisms have been proposed in the computer science field [3, 13, 14]. Especially, the mechanism proposed in Chen et al. [9] is known to be strategy-proof and envy-free, as well as satisfies an efficiency property called Pareto efficiency.

In mechanism design literature, there is another line of research on agents' manipulations: *false-name-proof mechanism design.* Assuming that each agent can use fake identities besides of her true one, they consider designing mechanisms in which no agent has such an incentive to use fake identities. In cake cutting mechanisms implemented in highly anonymous environments, such as over the Internet or a network, each agent might pretend to be multiple agents by adding fake identities to receive more pieces of cake. Furthermore, an agent might also ask her friends/colleagues who are willing to work with her to hand over any pieces of cake they might get.

Let us look at an example of a cake cutting situation with such false-name manipulations. Consider a computational resource that is available from 9am to 5pm in your research institute, where any group in it can apply for usage rights, i.e., time slots. When your group urgently needs the resource due to an upcoming deadline, you want more time slots.

However, under a fair (or more specifically, envy-free) cake cutting mechanism, it seems unlikely that your group will get more slots than other groups who also face the same deadline. In this situation, your group could apply for the resource under "fake groups," e.g., by asking colleagues in different groups who have no upcoming deadline.

In this paper we investigate the effect of such false-name manipulations in randomized cake cutting mechanisms that respects some fairness property. As long as the authors know, this is the first work of false-name-proof mechanism design for cake cutting. We first show two impossibility results: (i) no randomized false-name-proof cake cutting mechanism satisfies *ex-post proportionality*, which is another well-studied fairness property, and (ii) no randomized false-name-proof cake cutting mechanism simultaneously satisfies ex-post envy-freeness and Pareto efficiency. Since all the other properties than false-name-proofness are very traditional and well-studied in the literature, these two results have quite negative implication in false-name-proof cake cutting. They could, however, be natural starting points of further discussion that dig deeper the effect of false-name manipulations in cake cutting and general fair division problems.

We then propose a new randomized cake cutting mechanism that satisfies false-name-proofness, ex-post envy-freeness, and a weak efficiency property called simple allocation. Furthermore, the proposed mechanism is *almost deterministic*, meaning that randomization only occurs in the mechanism for "tie-breaking", and each agent related to the random tie-breaking receives the same ex-post utility under any realization of allocation as long as she truthfully report her utility function. We further show that the proposed mechanism is optimal in terms of the worst-case non-wastefulness ratio among those that satisfy all the three properties. Having this kind of worst case guarantee is theoretically quite appealing when we see a mechanism as an algorithm, although the ratio converges to zero when the number of agents grows.

In the wake of the negative insight that the proposed mechanism only guarantees a very weak efficiency property, we finally show an approach for obtaining positive results, namely, weakening the definition of false-name-proofness. We define a weaker notion of false-name-proofness by assuming that each agent could use only one (possible fake) identity to receive a piece of cake, although she could add any number of fake identities. We then propose a new randomized mechanism that satisfies false-name-proofness in this weaker sense, as well as ex-post envy-freeness, ex-post proportionality, and Pareto efficiency. This positive result under the natural restriction on false-name manipulations sheds light on the possibility of false-name-proof mechanism design, compared to several existing negative results on false-name-proof mechanisms in various domains such as combinatorial auctions.

2. RELATED WORKS

Traditional Envy-Free Cake Cutting.

Gamow and Stern [10] proposed a cake cutting protocol that returns an envy-free allocation when only three agents exist. Brams and Taylor [7] extended the results and proposed an algorithm that returns an envy-free allocation by discrete procedures for any number of agents. In another way, Austin [2] proposed an algorithm that returns an envy-free allocation for two agents, based on the well-known moving-knife protocol. Barbanel and Brams [4] extended this algorithm for three and four agents.

Mechanism Design for Cake Cutting.

Brams [6] investigated the effects of strategic manipulations in cake cutting. Chen et al. [9] proposed a polynomial-time cake cutting mechanism that satisfies strategy-proofness, proportionality, envy-freeness, and Pareto efficiency under piecewise uniform utility (valuation) functions. Maya and Nisan [13] proposed a cake cutting mechanism for two agents, which is strategy-proof and Pareto efficient, and provided several characterization results. Mossel and Tamuz [14] proposed a randomized cake cutting mechanism that satisfies strategy-proofness, proportionality, and Pareto efficiency under general utility functions. Aziz and Ye [3] proposed a randomized cake cutting mechanism that satisfies strategy-proofness and proportionality under piecewise uniform and piecewise constant utility functions.

False-name-proofness.

Yokoo et al. [17, 22] initiated the research on false-name-proof mechanism design, which revealed that the VCG mechanism is not false-name-proof and provided an impossibility theorem. Todo et al. [19] identified a condition called sub-additivity which characterizes false-name-proof allocation rules for combinatorial auctions. Iwasaki et al. [11] analyzed the lower and the upper bounds of the worst-case efficiency that can be obtained by false-name-proof combinatorial auction mechanisms. Even though these results seem quite negative, there are also some works producing positive insights. For example, Alkalay-Houlian and Vetta [1] analyzed the efficiency of VCG in the Nash equilibria when false-name manipulations are possible and showed that it is constant under natural assumptions on agents' valuations. Todo and Conitzer [18] showed that well-studied matching mechanisms are false-name-proof.

3. MODEL

In this section we introduce the cake cutting model considered by this paper. We basically follow the precise model proposed in Chen et al. [9].

Let $\mathcal{N}$ be the set of all potential agents/identities. Let $N = \{1, 2, \dots, n\} \subseteq \mathcal{N}$ be a set of attending agents/identities, while $n := |N|$ indicates the number of elements within the set N. The cake is represented as an interval $[0, 1]$. A piece X of cake is a finite union of intervals in the cake, i.e., $[0, 1]$. Let $\mathrm{len}(I) = y - x$ be the length of a closed interval $I = [x, y] \subseteq [0, 1]$ with $x \leq y$, and let $\mathrm{len}(X) = \sum_{I \in X} \mathrm{len}(I)$ be the length of a piece X of cake, where $I \in X$ indicates each interval that consists of the piece X.

Each agent $i \in N$ has a value density function $v_i : [0, 1] \rightarrow \mathcal{R}_{\geq 0}$, which represents her preference/valuation over the cake. The utility $U_i(X)$ of an agent i with a value density function v_i when she receives a piece of cake X is given as

$$U_i(X) := \sum_{I \in X} \int_I v_i(x) dx.$$

Here the utility function U_i is uniquely determined by the value density function v_i. Therefore, in what follows we assume without loss of generality that each agent has the utility function U_i as its private information. Furthermore,

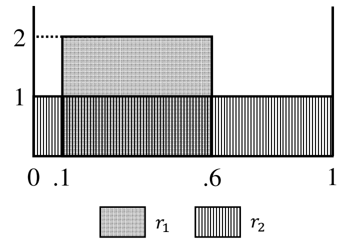

Figure 1: Example of reference pieces: $r_1 = [.1, .6]$ **and** $r_2 = [0, 1]$

we assume that for a given distribution P over some pieces of the cake, an agent i's utility is defined as an expected utility over the distribution, i.e., $U_i(P) = \mathbb{E}_{X \sim P} U_i(X)$. By definition, any utility function U_i is automatically additive and non-atomic. We also assume that it is normalized.

additive: $U_i(X \cup Y) = U_i(X) + U_i(Y)$ for any $X, Y \subseteq [0, 1]$ such that $X \cap Y = \emptyset$,

non-atomic: $U_i([x, x]) = 0$ for any $x \in [0, 1]$, and

normalized: $U_i([0, 1]) = 1$.

Note that we can treat open/half-open intervals as closed intervals with respect to agents' utilities by the non-atomicity property.

In this paper, we further restrict our attention to *piecewise uniform* utility functions [9]. Each agent i with a utility function U_i, which is derived from a value density function v_i, has a *reference piece* $r_i \subseteq [0, 1]$, which represents finite union of intervals, and a non-negative and certain utility for it. More formally, agents' value density functions are restricted to the following form:

$$v_i(x) = \begin{cases} \frac{1}{\text{len}(r_i)} & \text{if } x \in r_i, \\ 0 & \text{otherwise.} \end{cases}$$

Let us emphasize that a reference piece can be a union of intervals. The utility $U_i(X)$ of an agent i with a value density function v_i (when she receives a piece X of cake) is represented, by the reference piece r_i, as:

$$U_i(X) = \frac{\text{len}(X \cap r_i)}{\text{len}(r_i)}.$$

We show an example of piecewise uniform utility functions in Fig. 1, which also indicates their reference pieces. The horizontal axis represents the whole cake, while the vertical axis indicates the valuation for the cake.

Let $\mathcal{U}$ denote the set of all possible piecewise uniform utility functions, which is common among all potential agents. Let $U = (U_i)_{i \in N} \in \mathcal{U}^n$ denote a profile of the utility functions of attending agents N, $U_{-i} = (U_j)_{j \in N \setminus \{i\}} \in \mathcal{U}^{n-1}$ denote a profile of the utility functions of attending agents N except for i, and (U_i, U_{-i}) denote a profile of utility functions when an agent i reports U_i and the other agents report U_{-i}.

A feasible *allocation* A of the cake to a set of attending agents/identities N is represented as a tuple $(A_i)_{i \in N}$, where A_i indicates an allocation to a specific agent $i \in N$, $A_i \cap A_j = \emptyset$ for any pair $i, j (\neq i) \in N$, and $\bigcup_{i \in N} A_i \subseteq [0, 1]$. Let $\mathcal{A}_N$ denote the set of all feasible allocations to N. Furthermore, let $\Delta(\mathcal{A}_N)$ denote the set of all possible probability distributions over the set $\mathcal{A}_N$.

Now we are ready to define *randomized cake cutting mechanisms* (shortly, randomized mechanisms). A randomized mechanism f is a union of functions $f^N : \mathcal{U}^n \to \Delta(\mathcal{A}_N)$ for each $N \subseteq \mathcal{N}$. That is, f^N maps a profile of utility functions reported by N to a probability distribution over A_N. For simplicity, we usually abbreviate f^N to f if it is clear from the context. For a given profile U of utility functions, let $f(U)$ denote the distribution that a mechanism f returns, and $f_i(U)$ denote the distribution over the pieces of the cake allocated to agent i according to $f(U)$. Furthermore, we sometimes say an allocation A is *realizable* by a randomized mechanism f under a profile U if $f(U)$ assigns non-zero probability to A, and represent this by $A \sim f(U)$.

3.1 Properties

Cake cutting mechanisms are expected to satisfy 'good' properties, three of which we introduce here: ex-post envy-freeness, strategy-proofness, and false-name-proofness.

We first define *ex-post envy-freeness*, which is one representative property of fairness concepts. Intuitively it requires that no agent envies any other agent. More precisely, no agent gets better utility by trading her own piece of the cake for a piece allocated to any other agent. Note that this property is defined in the *ex-post* sense, meaning that the above must hold for *any* realization of allocation.

Definition 1 (Ex-Post Envy-Freeness). *A randomized cake cutting mechanism f is said to satisfy* ex-post envy-freeness *if for any $N \subseteq \mathcal{N}$, any $U \in \mathcal{U}^n$, any $A \sim f(U)$, and any $i, j (\neq i) \in N$, $U_i(A_i) \geq U_i(A_j)$.*

We next define an incentive property called *strategy-proofness*, which requires that reporting a true utility function to a mechanism is the best strategy, i.e., truth-telling is a dominant strategy, for every agent.

Definition 2 (Strategy-Proofness). *A randomized cake cutting mechanism f is said to satisfy* strategy-proofness *if for any $N \subseteq \mathcal{N}$, any $i \in N$, any $U_{-i} \in \mathcal{U}^{n-1}$, any $U_i \in \mathcal{U}$, and any $U'_i \in \mathcal{U}$,*

$$U_i(f_i((U_i, U_{-i}))) \geq U_i(f_i((U'_i, U_{-i}))).$$

In this paper we discuss *false-name-proofness*, which is a stronger incentive property than strategy-proofness. False-name-proofness requires that reporting a true utility function to a mechanism *only using one identity* is a dominant strategy for every agent, even though she could add fake identities and pretend to be multiple agents. In other words, under a false-name-proof mechanism, an agent's expected utility when she reports her true utility function using only one identity is (weakly) greater than the expected utility for the union of the pieces of cakes she gets under multiple identities.

Definition 3 (False-Name-Proofness). *A randomized cake cutting mechanism f is said to satisfy* false-name-proofness *if for any $N \subseteq \mathcal{N}$, any $i \in N$, any $U_{-i} \in \mathcal{U}^{n-1}$, any $U_i \in \mathcal{U}$, any $U'_i \in \mathcal{U}$, any $S \subseteq \mathcal{N} \setminus N$, and any $U_S \in \mathcal{U}^k$ s.t., $k = |S|$,*

$$U_i(f_i((U_i, U_{-i}))) \geq \sum_{j \in \{i\} \cup S} U_i(f_j((U'_i, U_{-i}, U_S))),$$

where (U'_i, U_{-i}, U_S) indicates the profile of utility functions when an agent i reports U'_i under her true identity and U_S under fake identities S and the other agents report U_{-i}.

By setting $S = \emptyset$, we can easily observe that if a mechanism is false-name-proof, then it is also strategy-proof. In the rest of this paper (except for Section 4.1), we will study ex-post envy-free and false-name-proof cake cutting mechanisms.

4. PRELIMINARY IMPOSSIBILITIES

Fairness is one of the most studied concepts of cake cutting algorithms/mechanisms. Therefore, in this paper, we start our discussion by investigating the relationships between false-name-proofness and fairness properties. We focus on two fairness properties; *ex-post proportionality* and ex-post envy-freeness. In the following two subsections, we show two impossibility results: (i) there exists no mechanism that satisfies false-name-proofness and ex-post proportionality, and (ii) there exists no mechanism that satisfies false-name-proofness, ex-post envy-freeness, and Pareto efficiency.

4.1 Impossibility with Proportionality

Proportionality is another well-known fairness property in the literature of fair division. As an exercise, we first derive an impossibility result by taking ex-post proportionality into account, instead of ex-post envy-freeness, as a fairness property in conjunction with false-name-proofness.

Let us first define the proportionality property. It intuitively requires that every agent receives a utility of at least $1/n$ for her own perspective, where n is the number of attending agents/identities. The following is its formal description:

DEFINITION 4 (EX-POST PROPORTIONALITY). *A randomized cake cutting mechanism f is said to satisfy* ex-post proportionality *if for any $N \subseteq \mathcal{N}$, any $U \in \mathcal{U}^n$, any $A \sim f(U)$, and any $i \in N$, $U_i(A_i) \geq \frac{1}{n}$ holds.*

Ex-post proportionality is implied by ex-post envy-freeness in the case that the entire cake is distributed. It seems obvious that this property is incompatible with false-name-proofness, since adding a large enough number of fake identities increases the manipulator's (expected) share of the whole cake arbitrarily close to one in any proportional cake cutting mechanism. We now formally show this intuition.

PROPOSITION 1. *There exists no randomized cake cutting mechanism that satisfies false-name-proofness and ex-post proportionality.*

PROOF. For the sake of contradiction, we assume that there exists a cake cutting mechanism f that satisfies false-name-proofness and ex-post proportionality.

We first consider the following case: there are only two agents $\{i, j\}$, who have the same utility function, i.e., $U_i = U_j \in \mathcal{U}$. Let $A = (A_i, A_j)$ be an arbitrary allocation realizable by the mechanism under the profile $U = (U_i, U_j)$. From ex-post proportionality, $U_i(A_i) \geq \frac{1}{2}$ and $U_j(A_j) \geq \frac{1}{2}$. Furthermore, since $A_i \cap A_j = \emptyset$ from feasibility, $U_i = U_j$ implies $U_i(A_j) \geq \frac{1}{2}$ and $U_j(A_i) \geq \frac{1}{2}$. Therefore, under the normalization assumption, it must hold that $U_i(A_i) = U_j(A_j) = \frac{1}{2}$ for any realizable allocation A, and thus $U_i(f_i(U)) = U_j(f_j(U)) = \frac{1}{2}$.

We then consider the following case: there is also another agent k with the same utility function, i.e., $U_k = U_i = U_j$. Let $B = (B_i, B_j, B_k)$ be an arbitrary allocation realizable by the mechanism under the profile $\tilde{U} = (U_i, U_j, U_k)$. By almost the same argument as above, we have $U_i(B_i) = U_j(B_j) = U_k(B_k) = \frac{1}{3}$ for any B, and thus $U_i(f_i(\tilde{U})) = U_j(f_j(\tilde{U})) = U_k(f_k(\tilde{U})) = \frac{1}{3}$.

Here, consider that the agent i in the first case adds a fake identity k to make the situation identical to the second case. By this false-name manipulation, agent i's utility increases from $U_i(f_i(U)) = \frac{1}{2}$ to $U_i(f_i(\tilde{U})) + U_i(f_k(\tilde{U})) = \frac{2}{3}$, which contradicts the assumption that the mechanism is false-name-proof. □

Note that the statement is also true even if we just require *interim* envy-freeness, meaning that no agent envies any other agent *in expectation*. The 'tightness' of the impossibility, i.e., the necessity of both of the properties for deriving the contradiction in the proof, can be easily verified. For example, a mechanism that does not allocate any piece at all, which has been sometimes referred to as the 'empty allocation' mechanism, is obviously false-name-proof, but not proportional. On the other hand, we can design a proportional mechanism based on the well-known moving-knife algorithm, but it cannot be false-name-proof.

4.2 Impossibility with Envy-Freeness

We now return to our main subject: the compatibility of false-name-proofness and ex-post envy-freeness. Some readers may already be aware that the 'empty allocation' mechanism (the false-name-proof one mentioned in Section 4.1) also satisfies ex-post envy-freeness. However, since it is completely inadequate in terms of the quality of allocations, it possesses no interest at all in practical applications. Therefore, in this subsection we introduce an efficiency property called *Pareto efficiency*, which guarantees the quality of the allocations obtained by the mechanisms.

Pareto efficiency, which is one of well-studied efficiency properties in the literature of social choice and mechanism design, requires that the allocation obtained by a mechanism must always be 'socially optimal.' More precisely, for a given allocation obtained by a Pareto efficient mechanism, there exists no other allocation that weakly raises the utilities of all the attending agents/identities and strictly raises the utility of at least one.

DEFINITION 5 (PARETO EFFICIENCY). *For a given $N \subseteq \mathcal{N}$ and $U \in \mathcal{U}^n$, an allocation $A' \in \mathcal{A}_N$ is said to* Pareto dominates *another allocation $A \in \mathcal{A}_N$ if $U_i(A'_i) \geq U_i(A_i)$ holds for any $i \in N$, with inequality strict for some $j \in N$. A randomized cake cutting mechanism f is said to satisfy* Pareto efficiency *if for any $N \subseteq \mathcal{N}$, and any $U \subseteq \mathcal{U}^n$, and for any $A \sim f(U)$, there exists no allocation $A' \in \mathcal{A}_N$ that Pareto dominates A.*

The following theorem shows the incompatibility between false-name-proofness and ex-post envy-freeness under Pareto efficiency, which is one of our main contribution in this paper. Since its proof closely resembles the proof of Proposition 1, we only sketch it here.

THEOREM 1. *There exists no randomized cake cutting mechanism that satisfies false-name-proofness, ex-post envy-freeness, and Pareto efficiency.*

PROOF SKETCH. For the sake of contradiction, we assume that there exists a mechanism f that satisfies false-name-proofness, ex-post envy-freeness, and Pareto efficiency.

We first consider that there are only two agents $\{i, j\}$, who have the same utility function $U_i = U_j$ with a reference piece $[0, 1]$ and show that $U_i(f_i(U)) = U_j(f_j(U)) = \frac{1}{2}$ for $U = (U_i, U_j)$.

Let $A = (A_i, A_j)$ be an arbitrary allocation realizable by the mechanism under the profile U. Since both reference pieces are $[0, 1]$, $U_i(A_i) = \text{len}(A_i \cap [0, 1])/\text{len}([0, 1]) = \text{len}(A_i)$ and $U_j(A_j) = \text{len}(A_j)$. From ex-post envy-freeness, $\text{len}(A_i) = \text{len}(A_j)$ holds. Furthermore, from Pareto efficiency, $A_i \cup A_j = [0, 1]$ must hold; otherwise we can find an allocation that Pareto dominates A by just allocating the remaining pieces $[0, 1] \setminus \{A_i \cup A_j\}$ of the cakes to either of the agents with keeping the original allocations A_i and A_j still assigned to each agent. Therefore, it must be the case that $U_i(A_i) = U_j(A_j) = \frac{1}{2}$ for any such realizable allocation A, and thus $U_i(f_i(U)) = U_j(f_j(U)) = \frac{1}{2}$.

We next consider the following case: there is also another agent k with the same utility function, i.e., $U_k = U_i = U_j$, with the reference piece $[0, 1]$. By almost the same argument as above, $U_i(f_i(\tilde{U})) = U_j(f_j(\tilde{U})) = U_k(f_k(\tilde{U})) = \frac{1}{3}$ holds for the profile $\tilde{U} = (U_i, U_j, U_k)$.

The rest of the proof is identical with the proof of Proposition 1. □

We then verify the tightness of the impossibility. As we have already seen, the 'empty allocation' mechanism is false-name-proof and ex-post envy-free but not Pareto efficient. A serial-dictatorship mechanism, based on such unmanipulable signals as log-in timestamp, is false-name-proof and Pareto efficient but not ex-post envy-free. Finally, a randomized mechanism that will be presented in Section 6 is Pareto efficient and ex-post envy-free (and even strategy-proof) but not false-name-proof.

5. ALTERNATIVE EFFICIENCY

Due to Theorem 1 which we provided in the previous section, we need to abandon Pareto efficiency as an efficiency property, as long as both false-name-proofness and ex-post envy-freeness are hard requirements. In this section, we introduce a weaker efficiency property called *simple allocation*, which can be satisfied by false-name-proof and ex-post envy-free randomized mechanisms.

A key idea for introducing a new efficiency property is that we should reduce the amount of allocations when there is conflict between agents/identities' reference pieces, since during the proof of Theorem 1 we only considered preference profiles such that all the agents have the same preference. This idea sounds reasonable from the perspective of mechanism design; because an agent who is going to cheat could manipulate the existence of a conflict between reference pieces by false-name manipulations, e.g., adding a fake identity having a utility function with the same reference piece, false-name-proof mechanisms should not allocate such pieces which more than one agent wants.

Therefore, as an extreme starting point, we define the new efficiency property in the following manner. For any interval of the cake, when it is solely required by exactly one agent, it must be allocated to that agent. In other words, if more than one agent wants to receive an interval of the cake (i.e., there is a conflict between reference pieces over it), then it does not have to be allocated to any agent.

To define the property, we introduce a few additional notations. For a given profile U of the utility functions associated with reference pieces $r_1, \ldots, r_n$, and a given agent i, $\hat{r}_i \subseteq r_i$ is the piece that the agent i solely requires, i.e.,

$$\hat{r}_i = r_i \setminus \bigcup_{j \in N \setminus \{i\}} r_j.$$

We sometimes say below that a piece $X \subseteq [0, 1]$ is *with conflicts* if there is a set of at least two agents, say $\{i, j\}$, in N with utility functions associated with reference pieces, say r_i and r_j, such that $X = r_i \cap r_j$.

DEFINITION 6 (SIMPLE ALLOCATION). *A randomized cake cutting mechanism f is said to satisfy* simple allocation *if for any $N \subseteq \mathcal{N}$, any $U \in \mathcal{U}^n$, any $A \sim f(U)$, and any $i \in N$, $\hat{r}_i \subseteq A_i$ holds.*

To make the paper self-contained, we explicitly show the intuition that the new property is weaker than Pareto efficiency in the following lemma.

LEMMA 1. *If a randomized cake cutting mechanism satisfies Pareto efficiency, then it also satisfies simple allocation.*

PROOF SKETCH. Consider a mechanism f that does not satisfy simple allocation but does satisfy Pareto efficiency. Then under some utility profile and realized allocation, at least one agent i misses an interval of the length non-zero within $\hat{r}_i$. Since such a missed interval is not required by any other agent by definition, assigning it to i with keeping all the remaining allocations the same Pareto dominates the original allocation. □

5.1 Mechanism Satisfying Simple Allocation

We then propose a randomized cake cutting mechanism that satisfies false-name-proofness, ex-post envy-freeness, and simple allocation and provide an example that shows how it works. For defining the mechanism, let us first introduce a concept called *minimal valid partition*.

DEFINITION 7 (MINIMAL VALID PARTITION). *Given a set N of attending agents and a profile of utility functions associated with reference pieces $(r_i)_{i \in N}$, we say a set of intervals $\mathcal{I}$ is a* valid partition *of the interval $[0, 1]$ if*

- *each $I \in \mathcal{I}$ is a continuous interval,*
- *$\forall I, I' \in \mathcal{I}, I \cap I' = \emptyset$,*
- *$\bigcup_{I \in \mathcal{I}} I = [0, 1]$, and*
- *$\forall I \in \mathcal{I}, \forall i \in N$, either $I \subseteq r_i$ or $I \cap r_i = \emptyset$ holds.*

We say $\mathcal{I}$ is a minimal valid partition *if $|\mathcal{I}|$ is minimized within all valid partitions for a given N and $(r_i)_{i \in N}$.*

Note that finding a minimal valid partition is easy; we first collect the start and end points of intervals within the reference pieces of all agents. Let $L = (p_0, \ldots, p_l)$ be the list of these points sorted in an ascending order, where duplicated points are eliminated, $p_0 = 0$, and $p_l = 1$ hold. Then, the minimal valid partition must be $\mathcal{I} = \{[p_k, p_{k+1}] \mid 0 \leq k \leq l - 1\}$. By using the concept of minimal valid partition, we define a randomized cake cutting mechanism.

MECHANISM 1. *Given a set N of attending agents, and a profile U of utility functions, let $\mathcal{I}$ be a minimal valid partition. For each $I \in \mathcal{I}$, let $W(I, U)$ be $\{i \in N \mid I \subseteq r_i\}$. Divide each I into $2^{|T|-1}$ intervals of the same length, and assign exactly one arbitrary interval (among $2^{|T|-1}$) uniformly at random to each agent $i \in T$, where $T = W(I, U)$.*

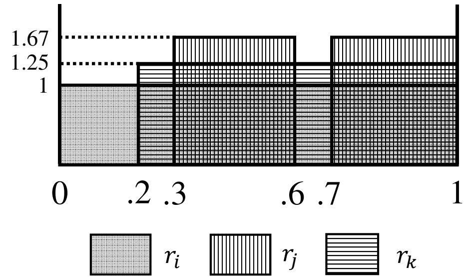

Figure 2: Three agents i, j, k with reference pieces $r_i = [0, 1]$, $r_j = [.3, .6] \cup [.7, 1]$, and $r_k = [.2, 1]$.

Example 1. *Consider there are three agents $N = \{i, j, k\}$, whose utility functions $U = (U_i, U_j, U_k)$ are associated with reference pieces $r_i = [0, 1]$, $r_j = [.3, .6] \cup [.7, 1]$, and $r_k = [.2, 1]$, respectively (please see Fig. 2).*

Then, the minimal valid partition is $\{[0, .2), [.2, .3), [.3, .6), [.6, .7), [.7, 1]\}$. These intervals are allocated as follows.

- *$[0, .2)$ is given to agent i.*
- *$[.2, .3)$ is divided into two intervals, e.g., $[.2, .25)$ and $[.25, .3)$. Each of i and k receives one of them with equal provability $1/2$.*
- *$[.3, .6)$ is divided into four intervals. e.g., $[.3, .375)$, $[.375, .45)$, $[.45, .525)$, and $[.525, .6)$. Each of i, j, and k receives one of them with equal probability $1/4$. One interval remains unallocated.*
- *$[.6, .7)$ is divided into two intervals, e.g., $[.6, .65)$ and $[.65, .7)$. Each of i and k receives one of them with equal provability $1/2$.*
- *$[.7, 1]$ is divided into four intervals, e.g., $[.7, .775)$, $[.775, .85)$, $[.85, .925)$, and $[.925, 1]$. Each of i, j, and k receives one of them with equal probability $1/4$. One interval remains unallocated.*

The proposed mechanism guarantees false-name-proofness by reducing the amount of allocations for each agent exponentially with respect to the number of attending agents, motivated by the false-name-proof mechanism proposed in Tsuruta et al. [20] for redistribution auctions. The following theorem shows this intuition as well as other desirable properties.

Theorem 2. *Mechanism 1 satisfies false-name-proofness, ex-post envy-freeness, and simple allocation.*

Proof Sketch. The proposed mechanism obviously satisfies simple allocation, since for each i, $\hat{r}_i$ is solely allocated to the agent i for any realization by its definition.

We next show ex-post envy-freeness. For a given profile U and a given agent i, its reference piece is divided into intervals in $\mathcal{I}$. For each $I \in \mathcal{I}$, where $i \in T = W(I, U)$, by definition, i receives the same length of the interval in I with each of the other agents $j \in T \setminus \{i\}$, for every such I and every realization from the mechanism. Therefore, no other agent in $N \setminus \{i\}$ could have a strictly better utility, with respect to i's utility function, than her own utility under any realization, which implies ex-post envy-freeness.

We finally prove that the mechanism satisfies false-name-proofness. Let us fix U_{-i} and focus on an agent i. Let $\mathcal{I}$ be the minimal valid partition computed by the mechanism when i truthfully report her utility function using only one identity. For each $I \in \mathcal{I}$, where $i \in T = W(I, U)$, by the definition of the mechanism, she receives an expected utility of $1/2^{|T|-1}$ of $U_i(I)$ under truth-telling. We can also easily see that she could not receive more than this by any misreport (while might receive a smaller expected utility by shrinking her reference piece r_i), as long as she does not use fake identities. Furthermore, for any given false-name manipulation that reports U'_i under her true identity and U_S under a set S of fake identities, where associated reference pieces are r'_i and r_j for $j \in S$ respectively, using one reference piece $r^*_i = r'_i \cup \bigcup_{j \in S} r_j$ under her true identity without using any fake one gives her at least as same utility as it. Therefore, there exists no beneficial false-name manipulation, which completes the proof. □

Another good property of the proposed mechanism is that it is "almost deterministic," meaning that randomization only occurs for determining allocation of pieces with conflicts, as a kind of tie-breaking, and each agent related to the random tie-breaking receives the same ex-post utility under any realization of allocations as long as she truthfully reports her utility function.

Let us note here that the proposed mechanism is even not strategy-proof *in ex-post sense*. That is, between two specific realizations, one of which for a truth-telling case and the other for a misreporting case, an agent could better off. For example, consider the following case: there are only two agents i, j, whose utility functions U_i and U_j are associated with reference pieces $r_i = [0, .5]$ and $r_j = [0, 1]$, respectively. Then, from the distribution $f((U_i, U_j))$, agent i receives one between, e.g., $[0, .25)$ and $[.25, .5)$, both of which give her an ex-post utility of $1/2$. On the other hand, when i misreports her utility functions as U'_i with a reference piece $r'_i = [0, 1]$, agent i receives, e.g., $[0, .5)$ under a realization of allocation (and $[.5, 1]$ under the other realization), which gives her an ex-post utility of 1, although the expected utility is $1/2$ as we have already verified in the proof above.

Since exponentially decreasing amount of allocation is unhappy for the agents, some readers may think the proposed mechanism is not a "good" mechanism. In the next subsection, we show optimality of this mechanism in terms of worst-case non-wastefulness ratio: how much portion of the desired intervals are allocated to the agents.

5.2 Worst-Case Optimality

As we have already seen at the beginning of this section, the simple allocation property is developed in a quite pessimistic way; because allocating pieces with conflicts all such pieces with conflicts are thrown out. Therefore, although the proposed mechanism satisfies simple allocation, one might deem it still very poor in terms of efficiency due to its reduction of allocation with respect to the number of agents. Actually, it is true that when all attending n agents have the same reference piece $[0, 1]$, only $n/2^{n-1}$ of the whole cake is allocated for any realization, which converges to zero. We could show in Theorem 3 and Proposition 2, however, that in terms of the worst-case non-wastefulness, the proposed mechanism is optimal among those that satisfy false-name-proofness and ex-post envy-freeness.

Definition 8 (Worst-Case Non-Wastefulness Ratio). *For a given n, a randomized cake cutting mechanism f is* *said to have a* worst-case non-wastefulness ratio *(WCNWR) of* $\alpha_n \in \mathcal{R}_{\geq 0}$ *if*

$$\alpha_n \leq \underset{N \ s.t.,\ |N|=n, U \in \mathcal{U}^n, A \sim f(U)}{\arg\min} \frac{\text{len}\left(\bigcup_{i\in N} A_i\right)}{\text{len}\left(\bigcup_{i\in N} r_i\right)}.$$

Intuitively, the worst-case non-wastefulness ratio of a mechanism shows how much portion of the desired intervals are allocated to the agents. Having a bigger worst-case non-wastefulness ratio is considered better in terms of efficiency in the worst-case. If a mechanism is Pareto efficient, then it has the worst-case non-wastefulness ratio of one. The following theorem shows the upper bound of the worst-case non-wastefulness ratio of mechanisms that satisfy false-name-proofness and ex-post envy-freeness.

THEOREM 3. *For any* $n \geq 2$*, any randomized mechanism that satisfies false-name-proofness and ex-post envy-freeness has a worst-case non-wastefulness ratio of at most* $n/2^{n-1}$.

PROOF. We prove the theorem by induction on n, for arbitrary randomized mechanism f that satisfies false-name-proofness and ex-post envy-freeness. We can obviously see that the statement is true for $n = 2$, i.e., the base case, since the ratio cannot exceed one by definition.

We then go to the induction step. Assuming that the statement is true for $n = m(\geq 3)$, we show that the statement is also true for $n = m + 1$. For the sake of contradiction, we assume $\alpha_{m+1} > (m + 1)/2^m$. Consider a profile $\tilde{U} \in \mathcal{U}^{m+1}$ with the same reference piece $[0, 1]$ (as the proof of Proposition 1). By definition of α_{m+1}, it must hold, for any $i \in N$ with $|N| = m + 1$ and for any realization $B \sim f(\tilde{U})$, that $\alpha_{m+1} \leq \text{len}\left(\bigcup_{i\in N} B_i\right)/\text{len}\left(\bigcup_{i\in N} r_i\right)$. Therefore, $(m+1)/2^m < \text{len}\left(\bigcup_{i\in N} B_i\right)/\text{len}\left(\bigcup_{i\in N} r_i\right)$ holds for any $i \in N$ and any realization B. Furthermore, from ex-post envy-freeness and the fact that all the reference pieces are $[0, 1]$, each of the $m + 1$ agents must have the same amount. Thus, it must hold that $\text{len}(B_i) > 1/2^m$ for any agent/identity $i \in N$.

On the other hand, from the assumption that the statement is true for $n = m$, $\alpha_m \geq m/2^{m-1}$ holds. This implies in conjunction with ex-post envy-freeness that $\text{len}(A_i) \leq 1/2^{m-1}$ holds for any $i \in N'$ and for any realization $A \sim f(U)$, where $|N'| = m$ and $U \in \mathcal{U}^m$ is the profile of identical utility functions associated with the reference piece $[0, 1]$. Therefore, an agent in N' can make the situation identical to the above one by creating a fake identity with the same utility function, which increases her utility from $\leq 1/2^{m-1}$ to $> 2/2^m = 1/2^{m-1}$. □

We now complete the optimality result by intuitively showing that the proposed mechanism matches the upper bound, i.e., has the worst-case non-wastefulness ratio of $n/2^{n-1}$ for arbitrary $n \geq 2$.

PROPOSITION 2 (OPTIMALITY W.R.T. WCNWR). *For any* $n \geq 2$*, Mechanism 1 has the worst-case non-wastefulness ratio of* $n/2^{n-1}$.

PROOF SKETCH. Let f denote the proposed mechanism. By definition, we can see, for any $n \geq 2$, any $U \in \mathcal{U}^n$, and any realization $A \sim f(U)$, that the ratio is described as

$$\frac{\sum_{I\in\mathcal{I}} \text{len}(I) \cdot \frac{|T|}{2^{|T|-1}}}{\text{len}\left(\bigcup_{i\in N} r_i\right)}.$$

The denominator is bounded from above by 1, while $\frac{|T|}{2^{|T|-1}}$ is bounded from bottom by $|N|/2^{|N|-1}$. Furthermore, by the definition of the valid partition, $\sum_{I\in\mathcal{I}} \text{len}(I) = 1$ holds. Therefore, the ratio is bounded from bottom by $\frac{n}{2^{n-1}}$. □

Let us note that the optimality with respect to the worst-case non-wastefulness ratio seems strongly depending on ex-post envy-freeness, as we used it in the proof of the upper bound. Actually we remain unsure whether Mechanism 1 is optimal even among all mechanisms that satisfy false-name-proofness, including those that do not satisfy ex-post envy-freeness.

OPEN QUESTION 1. *Does there exist any cake cutting mechanism that is false-name-proof and has a better worst-case non-wastefulness ratio than Mechanism 1?*

Furthermore, even though we focus on randomized mechanisms that satisfy false-name-proof and ex-post envy-freeness, it is also still open to clarify whether there exists a mechanism that *Pareto dominates* Mechanism 1, meaning that for every realization under any profile of utility functions it is not Pareto dominated by Mechanism 1, and for some realization under some profile it Pareto dominates Mechanism 1. In other words, is there any mechanism that strictly better than Mechanism 1 with respect to efficiency and also matches the best possible WCNWR?

6. BEYOND THE IMPOSSIBILITY

As we have already seen in the previous section, we could not come up with any mechanism that respects good efficiency properties in the worst case, as long as we keep all the other requirements unchanged. In this section we define a weaker notion of false-name-proofness, which conceptually assumes that each agent can use exactly one (possibly fake) identity to receive a piece of cake, although she could use multiple fake identities. Under this assumption, we modify Mechanism 1 to improve the worst-case non-wastefulness ratio and satisfy the weaker notion of false-name-proofness, as well as remain ex-post envy-free.

In practice, it seems natural to assume that receiving a piece of cake, e.g., a usage right/time slot of a shared computational server, needs direct contact with a mechanism designer or a third party for confirmation. In this sense, it is reasonable to consider the following weaker notion of false-name-proofness as robustness against false-name manipulations.

DEFINITION 9 (WEAK FALSE-NAME-PROOFNESS). *A randomized cake cutting mechanism* f *is said to satisfy* weak false-name-proofness *if for any* $N \subseteq \mathcal{N}$*, any* $i \in N$*, any* $U_{-i} = \mathcal{U}^{n-1}$*, any* $U_i \in \mathcal{U}$*, any* $U'_i \in \mathcal{U}$*, any* $S \subseteq \mathcal{N} \setminus N$*, and any* $U_S \in \mathcal{U}^k$ *s.t.,* $k = |S|$*,*

$$U_i(f_i((U_i, U_{-i}))) \geq \max_{j\in\{i\}\cup S} U_i(f_j((U'_i, U_{-i}, U_S))).$$

That is, under a weakly false-name-proof mechanism, the utility when an agent reports her true utility function is never smaller than the maximal utility she can receive *under a single identity* by any false-name manipulation. The weaker definition of false-name-proofness was also used in Todo and Conitzer [18] in the context of two-sided matching. It is obvious that this weaker notion of false-name-proofness still implies strategy-proofness.

The following mechanism is a slight modification of Mechanism 1, whose behavior is described in Example 2.

Mechanism 2. *Given a set N of attending agents and a profile U of utility functions, let $\mathcal{I}$ be a minimal valid partition. For each $I \in \mathcal{I}$, let $W(I, U)$ be $\{i \in N \mid I \subseteq r_i\}$. Divide each I into $|T|$ intervals of the same length, and assign exactly one arbitrary interval (among $|T|$) uniformly at random to each agent $i \in T$, where $T = W(I, U)$.*

Example 2. *Consider the same situation with Example 1. By the definition of the mechanism,*

- $[0, .2)$ *is given to agent i.*
- $[.2, .3)$ *is divided into two intervals. Each of i and k receives one of them with equal provability $1/2$.*
- $[.3, .6)$ *is divided into three intervals. Each of i, j, and k receives one of them with equal probability $1/3$.*
- $[.6, .7)$ *is divided into two intervals. Each of i and k receives one of them with equal provability $1/2$.*
- $[.7, 1]$ *is divided into three intervals. Each of i, j, and k receives one of them with equal probability $1/3$.*

Now we show that the mechanism satisfies all the requirements introduced in this paper, namely, ex-post proportionality, ex-post envy-freeness, Pareto efficiency, and weak false-name-proofness.

Theorem 4. *Mechanism 2 satisfies ex-post envy-freeness, ex-post proportionality, Pareto efficiency, and weak false-name-proofness.*

Proof Sketch. We can show ex-post envy-freeness by the same argument with the proof of Theorem 2. Furthermore, Pareto efficiency obviously holds since the mechanism allocate every piece within the interval $\bigcup_{i \in N} r_i$ to an agent who have non-zero utility for it.

We next show ex-post proportionality. For each agent i, consider the set of intervals I in the minimal valid partition $\mathcal{I}$ such that $i \in T = W(I, U)$. Note that by definition, the union of such intervals i coincides with i's reference piece r_i. Here, for each of such I, agent i receives $1/|T|$ of it under any realization. Thus, her utility is described as $\sum_I (U_i(I)/|T|)$ regardless of realization, where the summation is taken over those interval I with $I \subseteq r_i$. Therefore,

$$\sum_I \frac{U_i(I)}{|T|} \geq \frac{\sum_I U_i(I)}{|N|} = \frac{U_i(r_i)}{|N|} = \frac{1}{|N|}$$

holds, which implies ex-post proportionality.

We finally prove that the mechanism satisfies weak false-name-proofness. Let us fix U_{-i} and focus on an agent i. Let $\mathcal{I}$ be the minimal valid partition computed by the mechanism when i truthfully report her utility function U_i using only one identity. For each $I \in \mathcal{I}$, where $i \in T = W(I, U)$, by the definition of the mechanism, she receives an expected utility of at most $1/|T|$ of $U_i(I)$ when she does not use fake identities. That is, the mechanism satisfies strategy-proofness.

We then consider an arbitrary false-name manipulation by agent i with adding a set S of fake identities. We first replace all the utility functions by i's true utility function U_i, i.e., the mechanism takes (U_i, U_{-i}, U_S) as the argument, where $U_S = (U_i, \ldots, U_i) \in \mathcal{U}^{|S|}$. In this case, it holds by the definition of the mechanism that $U_i(f_i((U_i, U_{-i}))) \geq U_i(f_i((U_i, U_{-i}, U_S)))$. Furthermore, all the identities in $\{i\} \cup S$, including her true one, must receive the same utility from ex-post envy-freeness. Therefore, no identity is receiving a strictly higher expected utility than $U_i(f_i((U_i, U_S)))$ with respect to its utility function U_i (note that in this case all those identities have the same utility function U_i). By replacing each of the utility functions of $\{i\} \cup S$ one by one, we can emulate any possible false-name manipulation by i that adds the set S of fake identities. Here, from strategy-proofness, no identity can better off compared to the above case with respect to U_i at each step to the target false-name manipulation. This argument holds for any S, which completes the proof. □

On the other hand, as we can obviously see from Theorem 1 and the fact that Mechanism 2 satisfies both ex-post envy-freeness and Pareto efficiency, it is not false-name-proof. For instance, when there are only two agents, say i, j, whose utility functions are identical and associated with the reference piece $[0, 1]$, agent i could better off by adding a fake k also with the same reference piece and totally obtains a utility of $2/3$ under the two identity, instead of just $1/2$ by truth-telling under its true identity only (as we already see in the proof of Theorem 1).

7. CONCLUSION

In this paper, we investigate the effect of false-name manipulations in fair cake cutting mechanisms. We first show two impossibility results: (i) the incompatibility of false-name-proofness and ex-post proportionality, and (ii) the incompatibility between false-name-proofness, ex-post envy-freeness, and Pareto efficiency. We then propose a new randomized cake cutting mechanism that satisfies false-name-proofness, ex-post envy-freeness, and a very weak efficiency property called simple allocation. Our proposed mechanism is optimal among those satisfying false-name-proofness and ex-post envy-freeness with respect to the worst-case non-wastefulness ratio. We finally determine that by naturally weakening the requirement of false-name-proofness, we can design a randomized cake cutting mechanism that satisfies ex-post proportionality, ex-post envy-freeness, and Pareto efficiency as well as weak false-name-proofness.

In addition to some open questions mentioned above, it would also be an interesting future direction to consider different equilibrium concepts, such as Nash equilibria, when false-name manipulations are possible. In the literature of social choice and fair division, analyzing the cake cutting protocol based on the Nash equilibria is a very natural approach [8]. Even in false-name-proof mechanism design, recent papers have considered the Nash equilibria to analyze the quality of solutions by mechanisms that are not fully false-name-proof [1, 12]. Other approaches for preventing false-name manipulations, such as introducing participation costs [21], might work as well.

8. ACKNOWLEDGMENTS

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