Uniform Information Exchange in Multi-channel Wireless Ad Hoc Networks

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ABSTRACT

Information exchange is a basic primitive for maintaining the smooth running of a network or a system with multiple communicating agents. Given k packets initially stored at k nodes respectively, the problem is to disseminate the k packets to the whole network with the objective of minimizing the time used. We study this problem in single-hop multi-channel networks of n nodes, and target on devising uniform distributed protocols that do not rely on any prior knowledge of network parameters, such as the network size n or the number of packet holders k. Uniform protocols have better scalability and are more suitable for implementation in reality. Specifically, we propose a uniform distributed protocol that with high probability accomplishes the dissemination in $O(k/\mathcal{F} + \mathcal{F} \cdot \log n)$ rounds, assuming $\mathcal{F}$ available channels. This protocol is asymptotically optimal when k is large ($k \geq \mathcal{F}^2 \cdot \log n$), and provides the best possible linear speedup with multiple channels comparing with the results using a single channel. To the best of our knowledge, this is the first uniform protocol for information exchange in multi-channel networks.

Keywords

Information exchange, multi-channel networks, distributed algorithm, uniform protocol

1. INTRODUCTION

In this paper, we study the information exchange problem in a single-hop, multi-channel radio network. There are k nodes, called the *source nodes*, in the network. At the beginning, each of them holds a packet, and the target is to disseminate these packets to the whole network as quickly as possible. Information exchange is one of the most fundamental operations that are frequently called for in the smooth running of networks or scenarios involving many communicating entities, which include many multi-agent systems.

Using multiple channels obviously can greatly increase the throughput of the network. A lot of works have been devoted to studying the utilization of multiple channels in the derivation of faster communication protocols (e.g. [5, 8, 6, 9, 10, 12, 11, 14, 15, 19, 20]). All existing works however require that the network size n be known in a prior. In ad hoc networks, knowing n is usually a tough task, as it would consume a large amount of time and energy for nodes to compute this global parameter, and hence greatly increase the load of the network. Additionally, in ad hoc networks, the network size could change frequently due to nodes leaving and joining. This consideration necessitates the design of uniform protocols which do not require any prior information about network parameters including the network size n and the number of source nodes k. Uniform protocols have better scalability and are therefore more suitable for implementation in reality. But on the other hand, unknowing network parameters makes nodes unable to estimate the contention level on channels, which may cause a large amount of collisions. Hence, it has been a challenging task to design uniform distributed protocols. In this paper, we propose the first known uniform protocol for information exchange.

1.1 Network Model and Problem Definition

A multi-channel single-hop network is defined as follows. There are n nodes in the network, any pair of which can communicate with each other directly. But n is not known to the nodes. Time is divided into synchronous rounds. There are $\mathcal{F}$ channels available in the network. We use $1, \ldots, \mathcal{F}$ to denote these channels. Even though these $\mathcal{F}$ channels are available to all the nodes, at any time a node can select at most one channel to listen to or transmit on. In other words, each node is equiped with a commonly seen half-duplex transceiver. A node operating on a channel in a given round learns nothing about events on the other channels. When a node v listens to a channel, it can receive a message if and only if there is only one node transmitting on the channel. If two or more nodes transmit on the same channel, a collision occurs and none of these transmissions would be successful. We assume that nodes can detect collisions, i.e., nodes can distinguish collision from silence. Furthermore, we consider the case of non-constant $\mathcal{F}$ (larger than any constant), since otherwise, using a constant number of channels will not break the $\Omega(k)$ lower bound for information

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exchange that always holds in single-channel networks. The algorithm proposed in this paper is randomized, and hence the analysis involves many random events. We say that an event happens with high probability (with respect to n), if it happens with probability $1 - 1/n^c$ for some constant $c > 0$.

The goal of of information exchange is to disseminate some source nodes' packets to the whole network, which is more precisely defined as follows.

Definition 1. (Information Exchange.) *In the information exchange problem, initially k source nodes are holding packets $\{P_1, P_2, \ldots, P_k\}$ respectively. It is required to disseminate all these k packets to the whole network as quickly as possible.*

Denote by K the set of source nodes. Then $|K| = k$. We study the harsh case of the information exchange problem where nodes have no idea about the number of packets k and the set of source nodes K. We asume that multiple packets can be packed in a single message. It is easy to see that if k is small relative to the number of channels $\mathcal{F}$, the benefit of multiple channels will be weakened, since in this case there could be a single node selecting a channel such that its transmission cannot be received by anyone. Thus, throughout this work, we assume that $k \geq \mathcal{F} \log n$, which ensures that when nodes uniformly select the channels, there are multiple nodes operating on each channel with high probability. However, we must point out that our algorithm can also solve the case where k is small.

1.2 Our Results

Uniform Protocol. We give the first known uniform protocol for information exchange in multi-channel networks. Our algorithm can disseminate all k packets to the whole network in $O(k/\mathcal{F} + \mathcal{F} \cdot \log n)$ rounds with high probability when given $\mathcal{F}$ available channels. When k is large ($k \geq \mathcal{F}^2 \log n$), our algorithm shows a linear speedup (consider the $\Omega(k)$ lower bound for single-channel networks). Note that $\Omega(k/\mathcal{F})$ is a trivial lower bound for information exchange with $\mathcal{F}$ available channels. Hence, our protocol is asymptotically optimal when k is large.

Fast Adaptation to Network Change. Our protocol can handle dynamic joining and leaving of nodes efficiently. We have proved that after some nodes joining or leaving, the existing nodes will adapt quickly to a state of "safe range", in which the $\mathcal{F}$ channels will be almost fully made use of. A formal description of this property is given in Theorem 3.

1.3 Related Work

As more and more wireless networks and devices now operate on multiple channels, there has been much attention given to studying the effect of multiple channels on facilitating communication recently [5, 8, 6, 9, 10, 12, 11, 14, 15, 19, 20]. With respect to information exchange in multi-channel single-hop networks, most studies are done under the assumption that each message can carry only one packet. In particular, Holzer et al. [15, 14] proposed deterministic and randomized algorithms with optimal $O(k)$ time to solve the information exchange problem. With the assumption that nodes can listen to and receive messages from multiple channels at the same time, Shi et al. [19] proposed an $O(\log k \log\log k)$ time randomized information exchange protocol using $\Theta(n)$ channels. But with the assumption of unit-size messages, the benefit of utilizing multiple channels is very limited, since in each round, a node can receive at most one packet. Hence, it needs $\Omega(k)$ rounds to complete the information exchange. On the other hand, the packet stored at nodes could be small (e.g., in sensor networks, the data at each node is only a value). It is realistic to consider the case that multiple packets can be packed in a single message. Under this assumption, in [6], Daum et al. proposed a randomized algorithm that accomplishes information exchange in $O(k + \log^2 n/\mathcal{F} + \log n \log\log n)$ rounds with high probability. Their algorithm does not rely on collision detection. Then with collision detection, Wang et al. [20] proposed a protocol that disseminates all the packets in $O(k/\mathcal{F} + \mathcal{F} \cdot \log^2 n)$ rounds with high probability. When k is large ($k \geq \mathcal{F}^2 \log^2 n$), this result is asymptotically optimal considering the trivial lower bound $\Omega(k/\mathcal{F})$. In [22], Yan et al. studied the impact of message size on information exchange in multi-channel networks. Additionally, Gilbert et al. [12] considered the scenario when an adversary can disrupt a number of channels and proposed a randomized algorithm to achieve the almost-complete information exchange. However, all the above results need the prior knowledge of n. To our knowledge, there is not yet any uniform protocol proposed for solving the information exchange problem in single-hop multiple-channel networks.

Information exchange has also been extensively studied since 1970s [4, 13, 18] in single-channel networks. In single-channel networks, information exchange is also known as contention resolution [2] or k-selection [16]. Assuming collision detection as in this work, a randomized adaptive protocol with expected running time of $O(k + \log n)$ was presented by Martel in [17]. Kowalski [16] improved the protocol in [17] to $O(k + \log\log n)$ by making use of the expected $O(\log\log n)$ selection protocol in [21]. When requiring high probability results, the best known randomized algorithm was introduced in [1], which solves the k-selection problem in $O(k + \log^2 n)$ rounds without assuming collision detection. Note that in the single-channel networks, the trivial lower bound for k-selection is $\Omega(k)$. Hence the result in [1] is asymptotically optimal for $k \in \Omega(\log^2 n)$. By assuming that the channel can provide feedback on whether a message is successfully transmitted, an uniform randomized protocol with running time $O(k)$ is introduced in [2] for single-channel networks. However, the error probability of the protocol in [2] is $1/k^c$, rather than $1/n^c$. For deterministic solutions, adaptive protocols for k-selection were presented with running time $O(k \log(n/k))$ in [4, 13, 18], assuming collision detection.

1.4 Outline

Section 2 introduces some preliminary results that help the analysis. Section 3 introduces our protocol. Section 4 analyzes the performance of our protocol; particularly, we give an upper bound on the time needed to accomplish (with high probability) information exchange. Furthermore, we show the "self-stabilization" property of our protocol in Section 5. Section 6 summarizes our work, followed by a discussion.

2. PRELIMINARIES

In this section, we review some useful results concerning randomness.

Lemma 1 (Chernoff Bound.). *Consider a set of random variables $0 \le X_1, X_2, \ldots, X_n \le c$ for some parameter $c > 0$. Let $X := \sum_{i=1}^{n} X_i$ and $\mu := \mathbb{E}[X]$. If X_i's are independent or negatively associated, then for any $\delta > 0$ it holds that*

$$Pr[X \ge (1+\delta)\mu] \le \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{\frac{\mu}{c}}.$$

In details, for $\delta \le 1$, the bound can be upper bounded by

$$Pr[X \ge (1+\delta)\mu] \le \exp\{-\frac{\delta^2\mu}{3c}\};$$

for $\delta > 1$, it holds that

$$Pr[X \ge (1+\delta)\mu] \le \exp\{-\frac{\delta \ln(1+\delta)\mu}{2c}\}.$$

On the other hand, for any $0 < \delta < 1$ it holds that

$$Pr[X \le (1-\delta)\mu] \le \left(\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right)^{\frac{\mu}{c}} \le \exp\{-\frac{\delta^2\mu}{c}\}.$$

Next, we present some useful conclusions about a classic procedure, "throw balls into bins". These conclusions have essentially been proved in existing works such as [7].

Lemma 2. *Consider H bins and l balls with weights $0 \le w_1, w_2, \ldots, w_l \le \zeta$. Assume that $\sum_{i=1}^{l} w_i = \alpha \cdot H$ where $\alpha \ge 0.01$ is a constant. Balls are thrown into bins uniformly at random. Then, if ζ is small enough, with probability $1 - \exp\{-\Omega(H)\}$ there are at least $H \cdot 31/32$ bins in which the total weight of balls is between $\alpha \cdot 15/16$ and $\alpha \cdot 2$.*

Lemma 3. *Consider H bins and $l > H \cdot \Delta$ balls, where $\Delta > 2$. Balls are thrown to bins uniformly at random. If Δ is big enough, then with probability $1 - \exp\{-\Omega(H)\}$ there are at least $H \cdot 31/32$ bins that contain at least 2 balls.*

Corollary 1. *Consider H bins and $l > H \cdot \Delta$ balls with weights $0 \le w_1, w_2, \ldots, w_l \le \zeta$. Assume that $\sum_{i=1}^{l} w_i = \alpha \cdot H$ where $\alpha \ge 0.01$ is a constant. Balls are thrown to bins uniformly at random. Then, if Δ is big enough and ζ is small enough, then with probability $1 - \exp\{-\Omega(H)\}$ there are at least $H \cdot 15/16$ bins in which there are at least 2 balls, and the total weight is between $\alpha \cdot 15/16$ and $\alpha \cdot 2$.*

At the end of this section, we introduce a result given in [3].

Lemma 4. *Consider a set of l nodes, $v_1, v_2, \ldots, v_l$, transmitting on a channel. For node v_i, it transmits with probability $0 < p(v_i) < 1/2$. Let w_0 denote the probability that the channel is idle; and w_1 the probability that there is exactly one transmission on the channel. Then, $w_0 \cdot \sum_{i=1}^{l} p(v_i) \le w_1 \le 2 \cdot w_0 \cdot \sum_{i=1}^{l} p(v_i)$.*

3. UNIFORM INFORMATION EXCHANGE

In this section, we introduce our Uniform Information Exchange (UIE) protocol. The pseudo-code of the protocol is given in Algorithm 1 and Algorithm 2.

UIE Protocol. There are two states for the nodes: *active* and *inactive*. Intuitively, the active nodes are trying to transmit messages over the network, while the inactive nodes just listen for incoming messages. Initially, all the source nodes are *active*, and the others are *inactive*.

In the protocol, an active node will become inactive when it successfully transmits its message to other active nodes. In this way, on one hand, the number of active nodes is constantly decreasing, and on the other hand, it ensures that at any time the active nodes possess all k packets. Hence, when there is only one active node left, it can send all the k packets to all the other nodes. The utilization of multiple channels can speed up the reduction of active nodes. By the transmissions on multiple channels, the active nodes can be reduced on all channels in parallel. However, when the number of active nodes becomes small, it cannot guarantee that for a particular channel, there are multiple active nodes operating on it. As a result, even if an active node successfully transmits on a channel, its message may not be received by other active nodes. In other words, the multiple channels are not efficient any more. Additionally, the protocol needs to ensure that when the surviving active node transmits, all other nodes listen on the same channel. Hence, we set a primary channel, which serves two purposes: first, it is used for reducing active nodes when the number of active nodes is small; second, it is used by the surviving active node to disseminate the packets.

Specifically, there are two processes in the protocol: the multiple-channel transmission process and the primary-channel transmission process. In the multiple-channel transmission process, active nodes operate on multiple channels to reduce the number of active nodes, while in the primary-channel transmission process, nodes operate on the primary channel. Note that because nodes have no idea about any network parameters, it is hard for nodes to determine when the multiple-channel transmission process should finish. Hence, in the protocol, these two processes are in parallel, rather than consecutive. Specifically, there are four slots in each round: in the first two slots, active nodes operate on multiple channels, and in the other slots, nodes operate on the primary channel. We set the first channel as the special primary channel. We next introduce the protocol in more detail.

Each active node v maintains two parameters $p(v)$ and $q(v)$. Denote the values of $p(v)$ and $q(v)$ in a round t by $p_t(v)$ and $q_t(v)$, respectively. In particular, $p_t(v)$ and $q_t(v)$ are the transmission probabilities of node v for the multi-channel transmission process and the primary-channel transmission process in round t, respectively. Initially, $p_0(v) := q_0(v) := \zeta$, where $0 < \zeta < 1$ is a constant (determined in Lemma 2). Let $m_t(v)$ denote the set of packets received by node v by round t. Initially, for a source node v initiated with packet P, $m_0(v) := \{P\}$. And for other nodes, $m_0(v) := \emptyset$.

The operations in the four slots of each round t are as follows:

- **Slot 1.** In this slot, the inactive nodes do nothing. Each active node v selects a channel from the $\mathcal{F}$ candidates uniformly at random, and then transmits with probability $p_t(v)$ on the selected channel. If it does not transmit, it listens on the selected channel. If v receives a message containing a set of packets m', it updates $m_{t+1}(v) := m' \cup m_t(v)$.

  At the end of Slot 1, v updates the transmission probability p according to the following rule: if v listens and detects no transmission on the selected channel, $p_{t+1}(v) := \min\{\zeta, 2 \cdot p_t(v)\}$; otherwise, $p_{t+1}(v) := p_t(v)/2$.

- **Slot 2.** In this slot, the inactive nodes still do nothing. For an active node v, if it has received a message in Slot 1, it transmits an acknowledgement on the selected channel. Otherwise, v listens on the selected channel.

  If an active node v transmitted in slot 1 and detects transmissions on the selected channel in Slot 2, the state of v switches to *inactive*.

- **Slot 3.** In this slot, all nodes operate on the primary channel (Channel 1). Specifically, all inactive nodes listen, and an active node v transmits with probability $q_t(v)$. At the end of Slot 3, active nodes update the transmission probability $q_t(\cdot)$ using the same rule as in Slot 1.

- **Slot 4.** For each (active or inactive) node v, if v received a message in Slot 3, it transmits an acknowledgement.

  For an active node v, if v transmitted in Slot 3 and detects transmissions in this slot, it changes its state to *inactive*

Algorithm 1: UIE

Initialization: for node v at time 0

$p(v) := q(v) := \zeta$;
if *initially have a packet* P **then**
  $m(v) := \{P\}$;
  $state(v) := Active$;
else
  $m(v) := \{\}$;
  $state(v) := Inactive$;

Active State: for node v at time $t \geq 0$

Slot 1-2: pick a Channel r uniformly at random and call $channel-use(r, m(v), p(v))$;
Slot 3-4: call $channel-use(1, m(v), q(v))$;

Inactive State: for node v at time $t \geq 0$

Slot 1-2: do nothing;
Slot 3:
listen on Channel 1;
if *receive a message containing a set of packets* m' **then** $m(v) := m(v) \cup m'$;;
Slot 4: if *received a message in Slot* 3 **then** transmit on Channel 1;;

We state the correctness of the UIE protocol in the following Theorem 1.

Theorem 1. *Consider an execution of the UIE Protocol. When there is exactly one active node left, say node v in round T, then $m_T(v) = \bigcup_{v \in K} m_0(v)$. Recall that K is the set of all source nodes.*

Proof. Denote the set of active nodes in round t by A_t. Then $A_t \subseteq A_{t-1} \subseteq \ldots \subseteq A_1 \subseteq A_0 = K$ holds for any $t > 0$, according to the protocol. Then the conclusion follows from the fact that $\bigcup_{v \in A_t} m_t(v) = \bigcup_{v \in A_{t-1}} m_{t-1}(v)$ holds for any $t > 0$, which is true because when an active node v becomes inactive in some round t, it means $m_t(v)$ is known to some other active node u which is still active in round $t+1$. In detail, if an active node received acknowledgement or detected collisions in Slot 2, then it means its message has been received by some other active nodes; if an active node received an acknowledgement or detected collisions in Slot 4, then it means its message has been received by all the other nodes in the network, including the active ones if any exists. $\square$

Algorithm 2: $channel-use(i, s, w)$

Slot 1:
on Channel i, transmit a message containing packets in s with probability w and listen with probability $1 - w$;
if *listened* **then**
  if *Channel i is idle* **then**
    $w := \min\{2w, \zeta\}$;
  else if *received a message containing a set of packets* m' **then**
    $s := s \cup m'$;
    $w := w/2$;
  else
    // Channel i is busy
    $w := w/2$;
else
  // transmitted
  $w := w/2$;

Slot 2:
if *received a messge in Slot* 1 **then** transmit on Channel i;;
if *transmited in Slot* 1 **then**
  listen on Channel i;
  if *receive a message OR Channel i is busy* **then** $state(v) := Inactive$;;

Remark 1 (Termination of Listening). *One may note that in the proposed protocol, the nodes will eventually all reach an* inactive *state and perform "listen" afterwards. In case a total termination of the protocol in execution is required, there is a simple way to do it: We can add a special slot in each round, in which all nodes that are still active (having messages to disseminate) will transmit; when there is only one active node left, i.e. all messages have been successfully disseminated, there will be a successful transmission in this special slot. Then the last active node can inform the whole network to terminate once its message has been successfully transmitted. With the consideration of dynamic network changes (see details in Section 5), we did not include such a termination condition in the proposed protocol.*

4. ANALYSIS OF THE PROTOCOL

In this section, we prove that with k source nodes, our protocol can disseminate all k packets to the whole network in $O(k/\mathcal{F} + \mathcal{F} \cdot \log n)$ rounds with high probability. Recall that $\mathcal{F}$ is the number of available channels and n is the number of nodes in the network. Formally, this conclusion is summarized in Theorem 2.

Theorem 2. *Consider information exchange on a network of size n with $\mathcal{F}$ available channels. For the case where there are initially $k \leq n$ source nodes, the following conclusions hold:*

1. *There exists a constant $\nu > 0$ such that with high probability, there is only one active node left at time $T^* := \nu(k/\mathcal{F} + \mathcal{F} \cdot \log n)$.*
2. *For time T^* when there is only one active node v left, at time $T^{**} := 2 \cdot T^* + \log n = O(T^*)$ it holds with high probability that every node in the network knows the k packets initially maintained by the source nodes and node v becomes inactive.*

PROOF. The first conclusion follows directly from the Lemmas 5 and 12 which will be given later in Section 4.1 and 4.2, respectively. Here we first prove the second conclusion.

Since at time T^* there is only one active node v left, we know that $q_{2T^*}(v)$ will get back to ζ if node v is still active at time $2T^*$. Note that if node v transmits with probability ζ on the primary channel (Slot 3) for the subsequent $\log n$ rounds, then with high probability there exists one round in which node v transmits and consequently all the nodes in the network will receive the message. As shown in Theorem 1, the message transmitted by v contains all k packets. Hence, all nodes will get these packets in the received message. Finally, since the inactive nodes that received a message in Slot 3 transmit on the primary channel, then node v detects transmissions in Slot 4 and becomes inactive. □

We next briefly introduce the analysis process for the first conclusion in Theorem 2. Recall that there are two parallel processes in our algorithm: the multi-channel transmission process (the first two slots in each round) and the primary-channel transmission process (the last two slots in each round). As discussed before, when there are many active nodes (more than $\mathcal{F} \cdot \log n$), multiple channels should be efficient in reducing the number of active nodes. When the number of active nodes is reduced to something small (less than $\mathcal{F} \cdot \log n$), the utilization of multiple channels might not be efficient any more, since for a particular channel, there might not be multiple nodes selecting it. In this case, we have to rely on the primary-channel transmission process to reduce the number of active nodes. Therefore, we divide the analysis into two parts. The first part analyzes how long it takes to decrease the number of active nodes to $\mathcal{F} \cdot \log n$ and the second part deals with how long it takes to further reduce the number of active nodes to one. More precisely, let A_t denote the set of active nodes in a round t. Let T be the first round in which the number of active nodes drops below $\mathcal{F} \cdot \log n$. That is, for any time $t < T$, it holds that $|A_t| \geq \mathcal{F} \cdot \log n$, and for any time $t \geq T$, it holds that $|A_t| < \mathcal{F} \cdot \log n$. Then the whole analysis is divided into two parts by T: The first part concerns the time period from 0 to $T - 1$, and the second part considers the algorithm execution since T. In the first part of the analysis, we mainly analyze the efficiency of the multi-channel transmission process in reducing the number of active nodes, and in the second part, we are mainly concerned about the efficiency of the primary-channel transmission process.

In the rest of this section, we assume that $k \geq \mathcal{F} \cdot \log n$. Otherwise, we can jump directly to the second part of the analysis.

4.1 Efficiency of Multiple Channels

In this section, we analyze the first part, i.e., the period from time 0 to the first round when the number of active nodes drops below $\mathcal{F} \cdot \log n$. The conclusion is summarized in the following Lemma 5.

LEMMA 5. *There exists $T = O(k/\mathcal{F})$ such that in round T it holds with high probability that $|A_T| < \mathcal{F} \cdot \log n$.*

The main idea in proving Lemma 5 is to find a proper $\gamma' > 0$ such that after the protocol has been running for $T' = O(\log n)$ rounds, within any period of γ' rounds subsequently with $|A_t| \geq \mathcal{F} \cdot \log n$, there are (with constant probability) $\Omega(\mathcal{F})$ active nodes that switch from the *active* state to the *inactive* state. Then, with high probability, there are $k - \mathcal{F} \cdot \log n < k$ active nodes switching to *inactive*, in a period of $O(\log n + k/\mathcal{F})$ (which is $O(k/\mathcal{F})$ for $k > \mathcal{F} \cdot \log n$) rounds. To prove Lemma 5, we need to introduce and prove a series of "small" lemmas at first, and leave the proof of Lemma 5 to the end of this section. We next do some preparation for proving Lemma 5.

By using more channels, it is natural to expect that the number of successful transmissions is increased accordingly. Specifically, it is expected that in a round, there should be $\Omega(\mathcal{F})$ successful transmissions with $\mathcal{F}$ channels. In the following Lemma 6, we show that if a "safe range" on the total transmission probability of all active nodes is satisfied, the above expectation is true.

LEMMA 6 (SAFE RANGE). *Consider the Uniform Information Exchange Protocol. For a round $t > 0$ with $|A_t| \geq \mathcal{F} \cdot \log n$, if there exist constants $\alpha_1, \alpha_2 \geq 1$ such that $\alpha_1 \cdot \mathcal{F} \leq \sum_{v \in A_t} p_t(v) \leq \alpha_2 \cdot \mathcal{F}$, then with constant probability there are $\Omega(\mathcal{F})$ active nodes switching to the inactive state in the second slot.*

PROOF. For the convenience of the argument, we introduce a series of random variables $X^i(v)$ with $i = 1, \cdots, \mathcal{F}$ and $v \in A_t$. The variable $X^i(v)$ takes value $p_t(v)$ if node v selects Channel i in the 1st slot of round t; otherwise, $X^i(v) := 0$. Furthermore, denote $X^i := \sum_v X^i(v)$. By Corollary 1, with probability $1 - \exp\{-\Omega(\mathcal{F})\}$, there are at least $\mathcal{F} \cdot 15/16$ channels, such that for each of them there are at least two active nodes selecting it and the total transmission probability of these active nodes is between $\alpha_1 \cdot 15/16$ and $2 \cdot \alpha_2$. Next, we show that in such cases there are $\Omega(\mathcal{F})$ active nodes switching to the *inactive* state with constant probability.

With $b_i \in [0, 1/2]$ for $i = 0, 1, \ldots$, it holds [6] that

$$4^{-\sum_i b_i} \leq \prod_i (1 - b_i) \leq e^{-\sum_i b_i}. \tag{1}$$

Hence, for Channel i with X^i between $\alpha_1 \cdot 15/16$ and $2 \cdot \alpha_2$, it is idle with probability at least $4^{-4\alpha_2}$, and there is exactly one transmission on the channel with probability at least $\alpha_1 \cdot 4^{-4\alpha_2} \cdot 15/16$ (by Lemma 4). If there are at least two active nodes selecting a channel and there is only one node transmitting on the channel, then the transmission will succeed and the one transmitting in the first slot will sense transmissions in the second slot. According to the algorithm, the node that transmitted will switch to the *inactive* state. Therefore there are at least $\mathcal{F} \cdot 15/16$ channels such that for each of them there is an active node switching to the *inactive* state with probability at least $\alpha_1 \cdot 4^{-4\alpha_2} \cdot 15/16$. In expectation, there are $C \cdot \mathcal{F}$ new *inactive* nodes where $C := (1 - \exp\{-\Omega(\mathcal{F})\})\alpha_1 \cdot 4^{-2\alpha_2} \cdot 15/16$ which is at least $\alpha_1 \cdot 4^{-2\alpha_2} \cdot 15/32$ when $\mathcal{F}$ is large enough. Using the Chernoff bound over the $\mathcal{F}$ channels, it holds that with constant probability (given α_1, α_2, and large enough $\mathcal{F}$), there are $\Omega(\mathcal{F})$ active nodes switching to the *inactive* state in the second slot of round t. This completes the proof. □

With the above Lemma 6, now the proof idea becomes clear, we only need to show that once initiated, the network will fall into the safe range very soon, and then stays in this range as long as there are enough active nodes, i.e $|A_t| \geq \mathcal{F} \cdot \log n$. At the very beginning of the protocol, nodes are initiated with constant transmission probabilities, i.e. $p_0(\cdot) = \zeta$. Therefore, the summation of the initial transmission probabilities might be as large as $n \cdot \zeta$. We need to consider how long it takes for the summation $\sum_{v \in A_t} p_t(v)$ to drop below $\mathcal{F} \cdot \alpha_2$, where $\alpha_2 > 0$ is the constant defined by the safe range.

LEMMA 7. *For a round t with $\sum_{v \in A_t} p_t(v) = \alpha \cdot \mathcal{F}$, it holds that $Pr[\sum_{v \in A_t} p_{t+1}(v) \leq \alpha \cdot \mathcal{F} \cdot 3/4] \geq 7/8$ for large enough α.*

PROOF. To show the conclusion, we need to look at some execution details of the UIE Protocol. Note that there are two parts concerning randomness. One part is in the channel selection, and the other part is in the transmission selection. Consider the channel selection part first, in which a random instance σ is a mapping from the $|A_t|$ active nodes to the $\mathcal{F}$ channels. Recall that the probability of successful transmission on a channel is closely related to the total transmission probability of nodes selecting this channel. We call an instance *fair* if under it there are at least least $\mathcal{F} \cdot 15/16$ channels such that on each of them the total transmission probability (of nodes selecting this channel) is at least $\alpha \cdot 15/16$. By Lemma 2, a fraction of $1 - \exp\{-\Omega(\mathcal{F})\}$ of instances are fair. We next consider such a fair instance σ.

Let X_σ be the random variable that indicates the value of $\sum_{v \in A_{t+1}} p_{t+1}(v)$, conditioned on channel selection instance σ. Clearly, $X_\sigma \leq 2 \sum_{v \in A_t} p_t(v)$, and for different instances σ, X_σs are mutually independent. For a channel c, if without confusion, we also use c to denote the set of active nodes selecting channel c in the instance σ. Denote by X_σ^c the random variable that indicates the value of $\sum_{v \in c \cap A_{t+1}} p_{t+1}(v)$. Hence, $X_\sigma = \sum_c X_\sigma^c$.

Focus on a channel c with $\sum_{v \in c \cap A_t} p_t(v) \geq 15\alpha/16$. The probability that there is at least one transmission on channel c is at least $1 - \exp\{-15\alpha/16\}$, by Equation (1). According to the UIE Protocol, the nodes that selected channel c all halve their transmission probabilities if channel c is not idle in round t. Hence,

$$\begin{aligned} & Pr[X_\sigma^c = \sum_{v \in c \cap A_t} \frac{p_t(v)}{2} | \sum_{v \in c \cap A_t} p_t(v) \geq \alpha \cdot \frac{15}{16}] \\ \geq & \quad 1 - \exp\{-\alpha \cdot \frac{15}{16}\}, \end{aligned}$$

which is at least 31/32 when α is large enough. Hence in expectation, there are at least $(31/32) \cdot (\mathcal{F} \cdot 15/16)$ channels c with $X_\sigma^c = \sum_{v \in c \cap A_t} p_t(v)/2$.

Note that once the instance σ is given, the total transmission probability $\sum_{v \in c \cap A_t} p_t(v)$ for each channel c is specified. Then for different channels, the random variables X_σ^cs are mutually independent. Hence, by the Chernoff bound in Lemma 1, there are at least $(15/16) \cdot (\mathcal{F} \cdot 15/16)$ channels with $X_\sigma^c = \sum_{v \in c \cap A_t} p_t(v)/2$ with probability $1 - \exp\{-\Omega(\mathcal{F})\}$. Hence, with probability $1 - \exp\{-\Omega(\mathcal{F})\}$,

$$\begin{aligned} X_\sigma & \leq \mathcal{F} \cdot \left(\frac{15}{16}\right)^2 \cdot \frac{15\alpha}{16} \cdot \frac{1}{2} + (\alpha \cdot \mathcal{F} - \alpha \cdot \mathcal{F} \cdot \left(\frac{15}{16}\right)^3) \cdot 2 \\ & < \alpha \cdot \mathcal{F} \cdot 3/4. \end{aligned}$$

Finally it holds that $Pr[\sum_{v \in A_{t+1}} p_{t+1}(v) \leq \alpha \cdot \mathcal{F} \cdot 3/4] \geq (1 - \exp\{-\Omega(\mathcal{F})\}) \cdot (1 - \exp\{-\Omega(\mathcal{F})\})$ which is at least 7/8 for large $\mathcal{F}$. The last thing to note is in the above analysis we did not consider the effect when an active node becomes *inactive*, which only makes the summation decrease and hence is not harmful. $\square$

LEMMA 8 (GOING DOWN). *There exists a constant $\alpha_2' > 1$, such that among $\gamma \log n$ rounds (not necessarily consecutive) with $\sum_{v \in A_t} p_t(v) \geq \alpha_2' \cdot \mathcal{F}$ and sufficiently large $\gamma > 0$, there are at least $\frac{3}{4}\gamma \log n$ rounds with $\sum_{v \in A_{t+1}} p_{t+1}(v) < \frac{3}{4} \sum_{v \in A_t} p_t(v)$, with probability $1 - O(n^{-1})$.*

PROOF. Let $T := \gamma \log n$, and X_t be the random variable that indicates the value of $\sum_{v \in A_{t+1}} p_{t+1}(v) / \sum_{v \in A_t} p_t(v)$. Then by Lemma 7, it holds that $Pr[X_t \leq 3/4] \geq 7/8$. Let Y_t be the binary random variable that takes value 1 if $X_t \leq 3/4$. Note that given $\sum_{v \in A_t} p_t(v) > \alpha_2' \cdot \mathcal{F}$, $\mathbb{E}[Y_t] \geq 7/8$ always hold. Hence, $\mathbb{E}[\sum_{t=1}^T Y_t] \geq T \cdot 7/8$, and it holds that $Pr[\sum_{t=1}^T Y_t \leq T \cdot 3/4] = O(n^{-1})$ by the Chernoff bound. That is, with probability $1 - O(n^{-1})$, there are at least $T \cdot 3/4$ rounds t with $\sum_{v \in A_{t+1}} p_{t+1}(v) / \sum_{v \in A_t} p_t(v) \leq 3/4$, which completes the proof. $\square$

LEMMA 9 (FAST ADAPTATION). *There exists a constant $\alpha_2' > 1$, such that during any period of $\gamma \log n$ rounds with sufficiently large $\gamma > 0$, the probability that within the considered period there is a round t with $\sum_{v \in A_t} p_t(v) \leq \alpha_2' \cdot \mathcal{F}$ is $1 - O(n^{-1})$.*

PROOF. Denote $T := \gamma \log n$. Without loss of generality, assume that the period of T rounds starts from $t = 1$ and ends at $t = T$, with $\sum_{v \in A_t} p_t(v) > \alpha_2' \cdot \mathcal{F}$ always holds. Note that

$$\sum_{v \in A_T} p_T(v) = \sum_{v \in A_0} p_0(v) \cdot \Pi_{t=0}^{T-1} \frac{\sum_{v \in A_{t+1}} p_{t+1}(v)}{\sum_{v \in A_t} p_t(v)}.$$

Then by Lemma 8, with probability at least $1 - O(n^{-1})$, it holds that

$$\begin{aligned} \sum_{v \in A_T} p_T(v) & \geq \sum_{v \in A_0} p_0(v) \cdot \left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot 2\right)^{\frac{T}{4}} \\ & = \sum_{v \in A_0} p_0(v) \cdot \left(\frac{27}{32}\right)^{\frac{T}{4}}, \end{aligned} \tag{2}$$

where the first inequality holds by by coupling the "evolution" factors $\sum_{v \in A_{t+1}} p_{t+1}(v) / \sum_{v \in A_t} p_t(v)$. Since it holds that $\sum_{v \in A_0} p_0(v) < n$ and $T = \gamma \log n$, we know that $\sum_{v \in A_T} p_T(v)$ is at most $\alpha_2' \cdot \mathcal{F}$ for large enough γ. $\square$

In the above, we have shown that the adaptation process of the total transmission probability (from the initial state to $\Theta(\mathcal{F})$) takes $O(\log n)$ rounds with high probability. Meanwhile, we also showed that when the total transmission probability increases beyond the upper bound of the safe range, the total transmission probability of active nodes shows a trend of going down. To finally show that in most of the rounds, the safe range is satisfied, we still need to show that if the total transmission probability of active nodes becomes very small, the trend is that it will go up. This is illustrated in Lemma 10, which is proved using a similar argument as that for Lemma 8. Due to the space limit, the detailed proof is omitted.

Lemma 10 (Going up). *There exists a constant $\alpha_1' > 0$, such that among $\gamma \log n$ rounds (not necessarily consecutive) with $\sum_{v\in A_t} p_t(v) \le \alpha_1' \cdot \mathcal{F}$ and sufficiently large $\gamma > 0$, there are at least $\frac{3}{4}\gamma \log n$ rounds with $\sum_{v\in A_{t+1}} p_{t+1}(v) \ge \frac{4}{3}\sum_{v\in A_t} p_t(v)$, with probability $1 - O(n^{-1})$.*

Now we are ready to show that in most of the rounds after the adaptation process, the total transmission probability of active nodes is in the safe range.

Lemma 11 (Stable). *Let t_0 be the first round in which $\sum_{v\in A_t} p_{t_0}(v)$ drops below $\alpha_2' \cdot \mathcal{F}$. In the subsequent $T := \tau \cdot \log n$ rounds where $\tau > 0$ and n are large enough, the following hold:*

*(i) **hardly going high:** there are at least $T \cdot 3/4$ rounds t with $\sum_{v\in A_t} p_t(v) \le \alpha_2 \cdot \mathcal{F}$, where $\alpha_2 > \alpha_2'$ is a constant.*

*(ii) **hardly going low:** there are at least $T \cdot 3/4$ rounds t with $\sum_{v\in A_t} p_t(v) \ge \alpha_1 \cdot k$, where $\alpha_1 < \alpha_1'$ is a constant.*

Proof. We prove the two conclusions one by one.

Proof for "hardly going high". Consider the period from $t = t_0$ to $t = t_0 + T$. Define a *wave* to be an interval $[t_1, t_2]$ with $t_2 > t_1 + 19$, such that for rounds $t \in [t_1, t_2]$ it holds that $\sum_v p_t(v) > \alpha_2' \cdot \mathcal{F}$, and for rounds $t = t_1 - 1, t_2 + 1$ it holds that $\sum_{v\in A_t} p_t(v) \le \alpha_2' \cdot \mathcal{F}$. Then for any round t not in a wave, $\sum_{v\in A_t} p_t(v)$ is at most $\alpha_2 \cdot \mathcal{F}$ where $\alpha_2 := \alpha_2' \cdot 2^{10}$.

Assume there are at least $T \cdot 1/4$ rounds t with $\sum_{v\in A_t} p_t(v) > \alpha_2 \cdot \mathcal{F}$. Otherwise, the lemma holds. Let $\mathcal{A}$ denote the event that the assumption is true. Next, we show that $\mathcal{A}$ will never happen when n is large enough. Let T' be the number of rounds t with $\sum_{v\in A_t} p_t(v) > \alpha_2 \cdot \mathcal{F}$. Clearly, these rounds are all on waves, and by the assumption, $T' \ge \frac{1}{4}T$. Let $\mathcal{B}$ denote the event that among all these rounds, there are $T' \cdot 3/4$ rounds t with $X_t \le 3/4$. Recall that X_t is the random variable that takes value $\sum_{v\in A_{t+1}} p_{t+1}(v) / \sum_{v\in A_t} p_t(v)$. Assume that $\tau > 4\gamma$, where γ is from Lemma 8. Then $T' > \gamma \log n$, and hence by Lemma 8, it holds that $Pr[\mathcal{B}|\mathcal{A}] = 1 - O(n^{-1})$, which is positive when n is large enough. However, as shown in the following argument, events $\mathcal{B}$ and $\mathcal{A}$ do not happen together, which leads to the conclusion that $\mathcal{A}$ will never happen when n is large enough.

Now we show that $\mathcal{B}$ and $\mathcal{A}$ do not happen together. Actually, it is sufficient to show that $\mathcal{B}$ will not happen. Recall that event $\mathcal{B}$ happens meaning that a fraction of 3/4 rounds in waves satisfy $X_t \le 3/4$. To show this is impossible, we focus on a single wave $[t_1, t_2]$, and prove that among these $t_2 - t_1 + 1$ rounds, there are less than $(t_2 - t_1 + 1) \cdot 3/4$ rounds t with $X_t \le 3/4$. Assume the opposite, and then the value of $\sum_{v\in A_{t_2}} p_{t_2}(v)$ is at most $\sum_{v\in A_{t_1}} p_{t_1}(v) \cdot (27/32)^{(t_2-t_1+1)/4}$ (using the coupling technique). Recalling that in a wave $t_2 - t_1 + 1 > 20$, we have $\sum_{v\in A_{t_2}} p_{t_2}(v) < \sum_{v\in A_{t_1}} p_{t_1}(v) \cdot (27/32)^5 < \sum_{v\in A_{t_1}} p_{t_1}(v)/2$. Since in round $t = t_1 - 1$, $\sum_{v\in A_t} p_t(v) < \alpha_2' \cdot \mathcal{F}$, which implies that $\sum_{v\in A_{t+1}} p_{t_1}(v) \le 2\alpha_2' \cdot \mathcal{F}$. Hence, $\sum_{v\in A_{t_2}} p_{t_2}(v) < \alpha_2' \cdot \mathcal{F}$, which contradicts the definition of the wave. Hence the assumption does not hold, which completes the proof.

Proof for "hardly going low". Consider the period from $t = t_0$ to $t = t_0 + T$. Define a *hole* to be an interval $[t_1, t_2]$ with $t_2 > t_1 + 19$, such that for rounds $t = t_1, \ldots, t_2$ it holds that $\sum_v p_t(v) < \alpha_1' \cdot \mathcal{F}$, and for rounds $t = t_1 - 1, t_2 + 1$ it holds that $\sum_v p_t(v) \ge \alpha_1' \cdot \mathcal{F}$. Then for any round t not in a hole, $\sum_v p_t(v)$ is at least $\alpha_1 \cdot k$ where $\alpha_1 := \alpha_1'/2^{10}$.

Assume there are at least $T \cdot 1/4$ rounds t with $\sum_{v\in A_t} p_t(v) < \alpha_1 \cdot \mathcal{F}$. Otherwise, the lemma holds. Let $\mathcal{A}$ denote the event that the assumption is true. Next, we show that $\mathcal{A}$ will never happen when n is large enough. Let T' be the number of rounds t with $\sum_v p_t(v) < \alpha_1 \cdot \mathcal{F}$. Clearly, these rounds are all in holes, and by the assumption, $T' \ge \frac{1}{4}T$. Let $\mathcal{B}$ denote the event that among all these rounds, there are $T' \cdot 3/4$ rounds t with $X_t \ge 4/3$. Recall that X_t is the random variable that takes value $\sum_{v\in A_{t+1}} p_{t+1}(v) / \sum_{v\in A_t} p_t(v)$. Assume that $\tau > 4\gamma$, where γ is from Lemma 10. Then $T' > \gamma \log n$, and hence by Lemma 10, it holds $Pr[\mathcal{B}|\mathcal{A}] = 1 - O(n^{-1})$, which is positive when n is large enough. However, as shown in the following argument, $\mathcal{B}$ and $\mathcal{A}$ do not happen together, which leads to the conclusion that $\mathcal{A}$ will never happen when n is large enough.

Now we show that $\mathcal{B}$ and $\mathcal{A}$ never happen together. Actually, it is sufficient if we show $\mathcal{B}$ never happen. Recall that if event $\mathcal{B}$ happens, it means a fraction of 3/4 of the considered rounds satisfy $X_t \ge 4/3$. To show this is impossible, we focus on a single hole $[t_1, t_2]$, and prove that among these $t_2 - t_1 + 1$ rounds, there are less than $(t_2 - t_1 + 1) \cdot 3/4$ rounds t with $X_t \ge 4/3$. Assume the opposite, and then the value of $\sum_{v\in A_{t_2}} p_{t_2}(v)$ is at least $\sum_{v\in A_{t_1}} p_{t_1}(v) \cdot (32/27)^{(t_2-t_1+1)/4}$ (using the coupling technique). Recalling that in a hole $t_2 - t_1 + 1 > 20$, we have $\sum_{v\in A_{t_2}} p_{t_2}(v) > \sum_{v\in A_{t_1}} p_{t_1}(v) \cdot (32/27)^5 > \sum_{v\in A_{t_1}} 2 \cdot p_{t_1}(v)$. Since at $t = t_1 - 1$, $\sum_{v\in A_t} p_t(v) > \alpha_1' \cdot \mathcal{F}$, which implies that $\sum_{v\in A_{t_1}} p_{t_1}(v) \ge \alpha_1' \cdot \mathcal{F}/2$. Hence, $\sum_{v\in A_{t_2}} p_{t_2}(v) > \alpha_1' \cdot \mathcal{F}$, which contradicts the definition of the hole. Hence, the hypothesis does not hold, which completes the proof. □

Now, we are ready to prove Lemma 5.

***Proof of* Lemma 5**. At first, recall that by Lemma 6, in any round t with $|A_t| \ge \mathcal{F} \cdot \log n$ and $\alpha_1 \cdot \mathcal{F} \le \sum_{v\in A_t} p_t(v) \le \alpha_2 \cdot \mathcal{F}$, there exists constants $0 < c_1, c_2 < 1$ such that with probability at least c_1 there are $c_2 \cdot \mathcal{F}$ active nodes switching to the *inactive* state.

Define T_1 as the first round t such that the summation $\sum_{v\in A_t} p_t(v)$ drops below $\alpha_2 \cdot k$. By Lemma 9, we know that $T_1 = O(\log n)$. After T_1, by applying Lemma 11 it follows that for any period of length at least $T' := \max\{2 \cdot k/(\mathcal{F} \cdot c_1 \cdot c_2), \tau \cdot \log n\}$, with high probability, there are $T'/2$ rounds t in which $\sum_{v\in A_t} p_t(v)$ is between $\alpha_1 \cdot \mathcal{F}$ and $\alpha_2 \cdot \mathcal{F}$. Then we know that for large enough $\tau > 0$, with high probability there is a round $t < T_1 + T'$ that satisfies $|A_t| < \mathcal{F} \cdot \log n$. Otherwise, based on the above argument and using the Chernoff bound, it is easy to show that up to round $T_1 + T'$, there are more than k active nodes switching to the *inactive* state with high probability, which is impossible.

Hence, there exists constant $\gamma' > 0$ with $T := \gamma'(\log n + k/\mathcal{F}) \ge T_1 + T'$, such that with high probability there is a round $t \le T$ that satisfies $|A_t| < \mathcal{F} \cdot \log n$. Recall that we assume $k \ge \mathcal{F} \cdot \log n$ (otherwise, we can ignore this section and only consider the analysis in Section 4.2), which implies $T = O(k/\mathcal{F})$. □

4.2 Efficiency of the Primary Channel

In this section, we analyze the "second part" of the algorithm execution: the execution after the round when the number of active nodes drops below $\mathcal{F} \cdot \log n$. The conclusion is summarized in Lemma 12. Note that here in this part of the analysis, we do not consider the decrease of active nodes due to successful transmissions in the multi-channel transmission process. Since the multi-channel transmission process makes the decrease of active nodes much faster, the assumption will not affect the correctness of the analysis.

LEMMA 12. *Consider a round T with $|A_T| \le \mathcal{F} \cdot \log n$. There is a constant $\mu > 0$ such that at time $T^* \le T + \mu \cdot \mathcal{F} \cdot \log n$ there is only one active node left with high probability.*

PROOF. The proof for this lemma depends on a special case of the proof for Lemma 5, where $\mathcal{F} = 1$ and the transmission probability refers to $q(\cdot)$. Hence, we only give a brief sketch.

After time T with $|A_T| \le \mathcal{F} \cdot \log n$, it takes at most $O(\log n)$ rounds for the summation $\sum_{v \in A_t} q_t(v)$ to fall down to a range between β_1 and β_2. Here, β_1 and β_2 are constants such that for any round t with $\beta_1 \le \sum_{v \in A_t} q_t(v) \le \beta_2$, there is one active node switching to the *inactive* state in the 4th slot with constant probability. Afterward, consider a round $T' := T + \mu \cdot \mathcal{F} \cdot \log n$ where $\mu > 0$ is a large enough constant. Then with high probability there is a time round $t < T'$ such that $|A_t| = 1$. Otherwise, during the period from T to T', with high probability there are more than $\mathcal{F} \cdot \log n$ active nodes switching to the *inactive* state in the 4th slot, which is impossible. $\square$

5. STABILIZATION

As mentioned in the introduction, our protocol has the ability to handle dynamic joining and leaving of nodes. Here we explain this feature more precisely.

When nodes join the network, there are two possible situations. In the first situation, the joining nodes are all inactive, i.e. they have no information to spread. In this case, the only thing that changes is the network size, which decides the number of rounds for accomplishing the information exchange with high probability. Recall that in our protocol the nodes operate without having to know the network size. Hence, the active nodes need to do nothing to adapt to these newly joining nodes.

In the second situation, there are active nodes joining the network. This will increase the summation of the transmission probabilities, as well as the network size. To adapt to these newly joining active nodes, the existing active nodes will have to adjust their transmission probabilities. This is automatically achieved because in our protocol, any single node makes decisions only according to the response from the channel it selected and operated on.

When nodes leave, the situation is similar. The behaviors of active nodes are affected only because the summation of the transmission probabilities is changed.

In summary, after nodes' joining and leaving, the network adapts to the change through adjusting the sum of the transmission probabilities to become "safe" again. As shown in Theorem 3, this adaption is done very quickly.

THEOREM 3. *Consider the case when the number of active nodes is always at least $\mathcal{F} \cdot \log n$. For a round t^* with $p^* = \sum_{v \in A_{t^*}} p_{t^*}(v)$ outside the safe range $[\alpha_1 \cdot \mathcal{F}, \alpha_2 \cdot \mathcal{F}]$, with high probability $\sum_{v \in A_t} p_t(v)$ will fall back into the safe range in $\Phi = O(\log(\max\{\frac{p^*}{\mathcal{F}}, \frac{\mathcal{F}}{p^*}\}) + \log n)$ rounds.*

PROOF. Recall that in the proof of Lemma 9, in order to show that the summation $\sum_{v \in A_t} p_t(v)$ goes below $\alpha_2 \cdot \mathcal{F}$, we considered $T := 4 \cdot T'$ rounds such that $T' \ge \gamma \log n$ for a large enough constant γ, during which there are $3 \cdot T'$ rounds with a decrease of $\sum_{v \in A_t} p_t(v)$ by a factor 3/4 (by Lemma 8) and T' rounds with an increase of $\sum_{v \in A_t} p_t(v)$ by a factor at most 2. Then, after these T rounds, the summation $\sum_{v \in A_t} p_t(v)$ will be decreased by a factor of $(27/32)^{T'}$ with high probability. Since the network is initiated with $\sum_{v \in A_0} p_0(v) \le \zeta \cdot n$, we know that it is enough to set $T := O(\log n)$ for the network to become "safe".

In a similar approach, it is easy to show that for any round t^* with $p^* := \sum_{v \in A_{t^*}} p_{t^*}(v) > \alpha_2 \cdot \mathcal{F}$, by the round $t' := t^* + \max\{4 \cdot \log(32 \cdot p^*/(27 \cdot \alpha_2 \cdot \mathcal{F})), 4\gamma \log n\}$, the summation $\sum_{v \in A_{t'}} p_{t'}(v)$ becomes smaller than $\alpha_2 \cdot \mathcal{F}$ with high probability.

For the case that $p^* < \alpha_1 \cdot \mathcal{F}$, the proof idea is similar. Note that during $T := 4 \cdot T'$ rounds with $\sum_{v \in A_t} p_t(v) < \alpha_1 \cdot \mathcal{F}$, where $T' \ge \gamma \log n$ for a large enough constant γ, there are $3 \cdot T'$ rounds with an increase of $\sum_{v \in A_t} p_t(v)$ by a factor 4/3 (Lemma 10) and T' rounds with a decrease of $\sum_{v \in A_t} p_t(v)$ by a factor 1/2. Overall, after these T rounds, the summation $\sum_{v \in A_t} p_t(v)$ will be increased by a factor of $(32/27)^{T'}$ with high probability. Hence, by setting $t' := t^* + \max\{4 \cdot \log(27 \cdot \alpha_1 \cdot \mathcal{F}/(32 \cdot p^*)), 4\gamma \log n\}$, the summation $\sum_{v \in A_{t'}} p_{t'}(v)$ becomes larger than $\alpha_1 \cdot \mathcal{F}$ by round t' with high probability. $\square$

REMARK 2. *Note that when some nodes (active or inactive) turn faulty, the network will automatically adjust itself according to our protocol in just the same way as the joining and leaving of nodes.*

6. CONCLUSION

In this paper, we considered the information exchange problem of k source nodes in single-hop multiple-channel networks of n nodes. With $\mathcal{F}$ available channels, we proposed a protocol that solves the information exchange problem in $O(k/\mathcal{F} + \mathcal{F} \cdot \log n)$ rounds, with high probability. Our algorithm is uniform in n and k, which is the first known uniform algorithm for information exchange in multi-channel networks. And the proposed protocol is asymptotically optimal when k is large.

In our protocol, when detecting transmissions, a node will decrease its transmission probability to avoid collisions. Then if there exist jamming signals on a channel, an analysis similar to that would show that even for the case when jamming only affects a constant fraction of the available channels, the total transmission probability (i.e. $\sum_{v \in A_t} p_t(v)$) may tend to become very small. The affects the primary channel strategy even more significantly, since a fixed channel may be jammed all the time. This problem motivates us to consider jamming resilience of the proposed protocol and other similar protocols in the future.

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