On an Argument-centric Persuasion Framework

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ABSTRACT

In this paper, we propose an argument-centric persuasion framework. We first introduce a decision problem, called persuasion satisfiability, which is defined as the problem of determining whether there exists a sequence of arguments that starts from a given initial state, such as beliefs or wishes of the persuadee, and allows for achieving a given purpose of the persuader. This sequence should satisfy different constraints, including particularly upper bound constraints on the weight as well as on the length. We show that this decision problem is NP-complete and propose an encoding in partial weighted MaxSAT framework for solving it. Then, we show that the proposed encoding offers flexibility for dealing with different variants of the persuasion satisfiability problem. Finally, to avoid the explicit use of upper bound constraints on the weight and the length, we consider the notion of Pareto optimality by proposing an approach based on the use of partial weighted MaxSAT, which allows for finding non dominated (optimal) solutions.

KEYWORDS

Computational Persuasion; Argumentation; Knowledge Representation

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1 INTRODUCTION

Persuasion technologies aim at influencing users to make psychological and/or physical changes (thoughts, feelings, behaviors, motivation, etc) in several domains, such as healthcare, education, politics, marketing, etc (for interesting more complete definitions, see e.g. [12, 18]). One of the key approaches in the persuasion activity is the explicit use of convincing arguments [17, 18]. In this context, it is worth mentioning that in [15] the author has proposed interesting and reasonable key requirements for argument-centric persuasion in the particular case of behavior change. These requirements include those essential in this work, which are minimizing the effort involved on the part of the user and maintaining engagement by, for instance, avoiding long sequences of arguments.

In the literature, there are number of works which focus on the use of arguments in the persuasion activity. One can first mention the dialogical approach, where persuasion is defined as a dialogue between two agents trying to convince each other about an issue by exchanging arguments, that is, each agent plays both the role of persuader as well as that of persuadee (e.g. [2–4, 8, 21, 22]). In addition, in [16], the authors have proposed an asymmetric approach, where the word "asymmetric" refers to the fact that the persuadee agent is unable to posit arguments, but can accept and reject the arguments of the persuader agent. The main advantage of an asymmetric persuasion system is that it allows for avoiding natural language processing. As shown in this paper, this approach can be used in our argument-centric persuasion framework. Furthermore, there are works where the persuader system uses models and strategies for selecting the right sequence of arguments to convince the persuadee. To illustrate this point, in [13], the authors have proposed a decision-tree based framework for representing persuasion dialogues. In this general framework, the notion of decision tree is adapted to persuasion for selecting the argument to posit in the current state of dialogue. In the same vein, we propose in this work approaches that are based on the use of encodings in partial weighted MaxSAT framework for selecting the sequences of arguments that the persuader agent has to use. Let us note that other recent interesting studies on argument-centric persuasion have been proposed in [9, 14, 19, 20].

In this paper, we introduce an argument-centric persuasion framework. To do this, we consider the case where the persuasion activity consists in using a sequence of arguments following different approaches, in particular, that where the persuader agent shows that the target objective is a consequence of knowledge, beliefs and wishes of the persuadee agent. An argument in our framework is defined as an ordered pair of sets of literals, which can be seen as a simple intermediate approach between the logic-based one where the arguments are defined using logical formulas (e.g. see [1, 7]) and Dung's abstract one [10] where the internal structure is not considered at all. In this context, our main motivation is enhancing the expressivity of the abstract framework and avoiding at the same time important computational complexity issues, such as the fact that entailment in classical logic is coNP-complete. Moreover, to take into account particular requirements for argument-centric persuasion introduced in [15], we associate a weight to every argument to represent the effort needed on the part of the persuadee to integrate this argument, and we reason on the lengths of the argument sequences in order to prioritize shorter ones.

To formally define our framework, we introduce a decision problem, called persuasion satisfiability, which is defined as the problem of determining whether there exists a sequence of arguments that starts by a given set of literals, called the initial state, and allows for obtaining another given set of literals, called the objective state. This sequence should satisfy different constraints, in particular, upper bound constraints on the weight as well as on the length. The

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initial state can be seen as the set of knowledge, beliefs and wishes of the persuadee agent, while the objective state as the purpose of the persuader agent. From the computational complexity point of view, we show that this decision problem is NP-complete. Note that we propose a simple approach for using the proposed framework in the context of a bidirectional persuasion by allowing the persuadee agent to accept and reject the arguments.

Then, we propose an encoding in partial weighted MaxSAT framework for solving the persuasion satisfiability problem. Using the fact that partial weighted MaxSAT is an optimization problem, our encoding allows for finding a solution of the smallest weight. Next, we show that the proposed partial weighted MaxSAT encoding offers flexibility for dealing with different variants of the persuasion satisfiability problem, such as allowing the persuader to hide parts of conclusions of the used arguments to avoid inconsistency, or considering a conflict graph between arguments.

As said before, we use in the persuasion satisfiability problem upper bound constraints on the length and on the weight. The choice of the bounds may be arbitrary without specific knowledge about the considered case, which can be seen as a drawback of our approach. Thus, to avoid the use of upper bounds constraints, we consider the notion of Pareto optimality. Indeed, we propose a method based on the use of partial weighted MaxSAT encodings that allows for finding solutions that are not dominated (optimal in Pareto sense). More precisely, a sequence of arguments is optimal if there is no sequence that has a smaller length without a greater weight, or a smaller weight without a greater length.

2 A PERSUASION FRAMEWORK

In this section, we introduce a simple argument-centric persuasion framework. We first describe the approches that we consider in the persuasion activity. Then, we formally introduce our framework and, in particular, the persuasion satisfiability problem. After that, we provide some computational complexity results. Finally, we describe a basic bidirectional approach within our framework.

2.1 Persuasion Approaches

In this work, we consider that the persuasion activity consists in using a sequence of arguments that allows the persuader agent to achieve a target objective by using one of the following three approaches:

- **Strong approach:** the persuader agent shows that the target objective is a consequence of knowledge, beliefs and wishes of the persuadee agent.
- **Weak approach:** the persuader agent shows that the target objective has as a consequence wishes of the persuadee agent. In other words, the persuadee may satisfy her/his purpose by accepting/doing the purpose of the persuader, but the latter is not a requirement to satisfy the persuadee purpose.
- **Mixed approach:** the persuader agent uses both the strong and the weak approaches for convincing the persuadee agent. For instance, the persuader can use the strong approach to achieve a part of the purpose and the weak approach to achieve the remaining part.

For instance, using the strong approach, the persuader agent can use the argument stating that the fact that the persuadee knows that a given product is efficient and not expensive has as a consequence that this product should be purchased: the purchase of the considered product by the persuadee agent is the purpose of the persuader agent and the fact that the considered product is efficient and not expensive are beliefs of the persuadee agent. In addition, using the weak approach, the persuader agent can use the argument stating that playing sports has as a consequence that the persuadee agent may develop friendships: playing sports is the purpose of the persuader agent and developing friendships is a wish of the persuadee agent. It is worth noting that the previous argument does not mean that playing sports is the unique way to develop friendships. In summary, in the strong approach the persuader starts with knowledge, beliefs and wishes of the persuadee, while in the weak approach the persuader starts with the target objective. Regarding the mixed approach, the persuader agent can first use the weak approach with the argument that stop smoking allows for avoiding dangerous carcinogens, and then the strong approach with the argument that avoiding dangerous carcinogens allows for avoiding dangerous diseases: the purpose of the persuader is convincing the persuadee to stop smoking, and the persuadee wishes to avoid any dangerous disease. As a side note, the mixed approach can be used by combining in different other manners the strong and weak approaches.

In this paper, we define the notion of argument as an ordered pair of sets of literals. In a sense, this can be seen as an abstraction of the standard logic-based argument structure (e.g. see [1, 7]). Indeed, a logic-based argument is defined as an ordered pair of a set of formulas representing the support and a formula representing the conclusion. Thus, instead of using logical formulas, we use here literals. Our aim in this context is to use a simple intermediate approach between the logic-based one and Dung's abstract one [10] where the internal structure is not considered at all.

It is noteworthy that our framework is inspired, in part, from the requirements for argument-centric persuasion introduced in [15], in particular, the requirements 3 and 6. Indeed, we associate a weight to each argument in order to represent the effort needed on the part of the persuadee to integrate this argument, and we reason on the lengths of the arguments sequences in order to prioritize shorter ones.

2.2 Framework Definition

We here define the notion of persuasion frame and some related notions. Then, we introduce a decision problem, called persuasion satisfiability, which is defined as the problem of determining whether there exists a sequence of arguments that starts by a given set of literals, called the initial state, and allows for obtaining another given set of literals, called the objective state.

First, let us recall that a *propositional variable* is a variable that can either be true or false. A *literal* is either a propositional variable or a negated propositional variable. As usual, we use the unary logical connective $\neg$ to denote the negation. Moreover, given a set of propositional variables V, we use $Lit(V)$ to denote the set of all the possible literals that are defined using the variables in V, i.e., $Lit(V) = V \cup \{\neg p \mid p \in V\}$.

Definition 2.1 (Persuasion Frame). A *persuasion frame* is a tuple $(\mathcal{S}, \mathcal{A}, W)$ where $\mathcal{S}$ is a non empty finite set of propositional variables representing abstract statements, $\mathcal{A}$ is a finite set of arguments built over $\mathcal{S}$ and W a mapping that associates a weight (an integer) to each argument in $\mathcal{A}$. An *argument* a over $\mathcal{S}$ is an ordered pair $\langle X, C\rangle$ where $X, C \subseteq Lit(\mathcal{S})$, $X \cup C$ is consistent, $X \cap C = \emptyset$ and $C \neq \emptyset$, X is called the *support* of a and C the *conclusion* of the latter.

Given an argument a, we use $Supp(a)$ and $Conc(a)$ to denote its support and its conclusion respectively.

The weight of an argument is used to represent the cost of the latter. In particular, the weight mapping can be used as a weakness measure of the arguments regarding the impact on the persuadee agent.

Example 2.2. Let us first consider the following set of statements $\mathcal{S}$:

- ad_i: the advertisement number i convinces the considered agent.
- h: the considered agent thinks that the product p is healthy.
- eff: the considered agent thinks that the product p is effective.
- exp: the considered agent thinks that the product p is expensive.
- $pack$: the considered agent likes the packaging of p.
- buy: the considered agent is convinced that the product p has to be purchased.

In this example, we consider the following set of arguments $\mathcal{A}$: $a_1 = \langle\{ad_1\}, \{h, eff\}\rangle$, $a_2 = \langle\{ad_2\}, \{\neg exp\}\rangle$, $a_3 = \langle\{ad_3\}, \{pack\}\rangle$, $a_4 = \langle\{h, eff, \neg exp, pack\}, \{buy\}\rangle$, $a_5 = \langle\{h, eff, pack\}, \{buy\}\rangle$ and $a_6 = \langle\{pack\}, \{buy\}\rangle$, where $\langle X, C\rangle$ means that the truth of the elements of X has as a consequence the truth of the elements of C. For instance, the argument a_6 means that if the persuadee agent likes the packaging of the product p, then she/he is convinced that she/he has to buy it. Clearly, the argument a_4 is stronger than a_5 which is stronger than a_6. To represent this fact, one can use the weight mapping by setting for instance $W(a_4) = 0$, $W(a_5) = 1$ and $W(a_6) = 2$.

A persuasion frame can be developed by using a crowdsourcing-based collaborative approach. For instance, an interesting approach may consist in allowing crowd members to propose arguments and also to vote on arguments as describe in Figure 1. The votes are then exploited for defining the weights of the arguments. For the sake of illustration, one can use the following simple vote-based mapping:

$$W(a) = \#NegVotes(a) - \#PosVotes(a)$$

where $\#NegVotes(a)$ and $\#PosVotes(a)$ corresponds respectively to the number of votes against and for the argument a. This function means that more votes for a and less votes against it reduce the weight.

Definition 2.3 (Argument Path). Given a persuasion frame $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$ and a set $I \subseteq Lit(\mathcal{S})$, an *I-path* in $\mathcal{F}$ is a sequence $s = a_1, \ldots, a_k$ of distinct arguments in $\mathcal{A}$ where, for all $i \in 1..k$, $Supp(a_i) \subseteq I \cup \bigcup_{1\leq j<i} Conc(a_j)$.

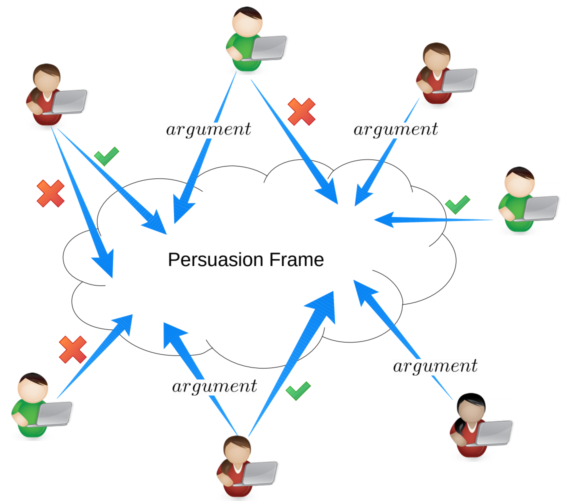

Figure 1: Building a persuasion frame

The condition on a sequence of arguments to be an I-path means only that we have to obtain the support of an argument before using it.

Given an I-path $s = a_1, \ldots, a_k$, we use $Arg(s)$, $\mathcal{L}(s)$, $\mathcal{W}(s)$ and $C(s)$ to denote respectively the set $\{a_1, \ldots, a_k\}$, the length of the sequence k, $\sum_{i=1}^{k} W(a_i)$ and $I \cup \bigcup_{i=1}^{k} Conc(a_i)$.

Definition 2.4 (Consistency). An I-path $a_1, \ldots, a_k$ is said to be *consistent* if the set of literals $I \cup \bigcup_{i=1}^{k} Conc(a_i)$ is consistent.

Roughly speaking, a consistent I-path is a sequence of arguments that can be used together starting from I and do not produce contradictory pieces of information. Consider again Example 2.2. The sequence a_1, a_2, a_3, a_4 is a consistent $\{ad_1, ad_2, ad_3\}$-path, but it is not a consistent $\{ad_1, ad_2, ad_3, exp\}$-path since a_2 produce $\neg exp$ which is in contradiction with exp in the initial state.

Now, we introduce the central decision problem studied in this work.

Definition 2.5 (Persuasion Satisfiability Problem). Given a persuasion frame $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$, a consistent set of literals $I \subseteq Lit(\mathcal{S})$, called the *initial state*, a non empty consistent set of literals $O \subseteq Lit(\mathcal{S})$, called the *objective state*, a length bound $k \in \mathbb{N} \cup \{\infty\}$ and a weight bound $v \in \mathbb{Z} \cup \{\infty\}$, the persuasion satisfiability problem is to check whether there exists a consistent I-path s in $\mathcal{F}$ such that $O \subseteq C(s)$, $\mathcal{L}(s) \leq k$ and $\mathcal{W}(s) \leq b$.

We denote every instance of the persuasion satisfiability problem as a tuple of the form $(\mathcal{F}, I, O, k, b)$. Further, note that ∞ is only used to formally represent the absence of a bound.

For example, if we only consider the strong approach, the initial state corresponds to knowledge, beliefs and wishes of the persudee agent and objective state to the purpose of the persuader agent. Conversely, if we only consider the weak approach, the initial state corresponds to the purpose of the persuader and the objective state to wishes of the persuadee.

Example 2.6. We here provide an example inspired from [9]. Let us consider the following statements: s_1 =being healthy, s_2 =stop smoking, s_3 =a long life, s_4 =good looking, s_5 =supporting family. Let $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$ be a persuasion frame such that $\mathcal{S} = \{s_1, s_2, s_3, s_4, s_5\}$, $\mathcal{A} = \{a_1 = \langle\{s_1\}, \{s_2\}\rangle, a_2 = \langle\{s_3\}, \{s_1\}\rangle, a_3 = \langle\{s_4\}, \{s_1\}\rangle, a_4 = \langle\{s_5\}, \{s_3\}\rangle, a_5 = \langle\{s_4\}, \{s_2\}\rangle\}$, and $W(a_1) = 3$, $W(a_2) = 2$, $W(a_3) = 2$, $W(a_4) = 1$ and $W(a_5) = 6$. In this context, we consider the instance of the persuasion satisfiability problem $P = (\mathcal{F}, \{s_5\}, \{s_2\}, 3, 6)$. Thus, using the strong approach, the aim in P is to convince the persuadee to stop smoking by using the fact that it is important for her/him to support her/his family. The $\{s_5\}$-path a_4, a_2, a_1 is a solution of P which corresponds to the sequence: supporting family → a long life → being healthy → stop smoking.

It is worth noticing that the support and the conclusion of an argument are not treated in our framework as an implication in classical logic. Consider for instance the two arguments $a = \langle\{p, \neg q\}, \{r\}\rangle$ and $a' = \langle\{q\}, \{r\}\rangle$. Clearly, a, a' and a', a are both not $\{p\}$-paths since the supports of a and a' are both not included in $\{p\}$ (see Definition 2.3). However, we have in classical logic $\{p\} \cup \{p \wedge \neg q \to r, q \to r\} \vdash r$. In fact, r is obtained from $\{p\} \cup \{p \wedge \neg q \to r, q \to r\}$ using the law of excluded middle $q \vee \neg q$ that is valid in classical logic. But in our framework, we need to know the disjunct in $q \vee \neg q$ that we have to take into account for choosing the appropriate argument to produce r: in the case where q is true, we use the argument a', otherwise, we use a. For example, consider that $p = \mathit{having\ time}$, $q = \mathit{there\ is\ an\ exam}$ and $r = \mathit{review\ lessons}$. In this case, the argument a can be used when the persuadee agent prefers reviewing her lessons if she has time to do it without the constraint of an exam, while a' can be used when she can be convinced by the constraint of an exam. In a sense, this example shows that the support and the conclusion of an argument are treated in our framework as a constructive implication. The word "constructive" is used to refer to the constructivism principles in mathematics (see e.g. [6, 24]). One of the main constructivism principles is the rejection of the law of excluded middle.

2.3 Computational Complexity

We here show that the persuasion satisfiability problem is NP-complete, even when we do not consider one of the bounds on the length and the weight. In order to show NP-hardness, we use the well-known NP-complete problem of Hamiltonian cycle.

Theorem 2.7. *The persuasion satisfiability problem is NP-Complete.*

Proof. Given an instance $P = (\mathcal{F}, I, O, k, b)$ with $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$ and an I-path s in $\mathcal{F}$, one can check that s is a solution of P in polynomial time. Indeed, we only have to check the properties $O \subseteq C(s)$, $\mathcal{L}(s) \leq k$ and $\mathcal{W}(s) \leq b$. Moreover, one can easily see that P is satisfiable iff there exists a solution bounded by the number of abstract statement ($|\mathcal{S}|$). Indeed, this comes from the fact that one can require w.l.o.g. that each used argument has to bring at least one additional piece of information in $Lit(\mathcal{S})$. As a consequence, the persuasion satisfiability problem is in NP. To show that the latter is NP-hard, we use the well-known NP-complete problem of Hamiltonian cycle, which consists in determining if an undirected graph contains a Hamiltonian cycle. Let us recall that a Hamiltonian path in a graph is a path that visits each vertex exactly once. A Hamiltonian cycle is a Hamiltonian path that is a cycle. Let us now define our reduction of the Hamiltonian cycle problem into the persuasion satisfiability problem. Let $G = (V, E)$ be an undirected graph. Then, we associate to G the instance $P_G = (\mathcal{F}, \{v_0\}, V \cup \{v'_0\}, |V|, \infty)$ where $v_0 \in V$, v'_0 is a fresh vertex ($v'_0 \notin V$) and $\mathcal{F} = (V, \mathcal{A} = (\bigcup_{\{v,v'\} \in E}\{\langle\{v\}, \{v'\}\rangle\}) \cup (\bigcup_{\{v,v_0\} \in E}\{\langle\{v\}, \{v'_0\}\rangle\}), W)$ such that $W(a) = 0$ for every $a \in \mathcal{A}$. Assume that G admits as a Hamiltonian cycle the following path $c = \{v_0, v_1\}, \{v_1, v_2\}, \ldots, \{v_{n-1}, v_n\}\{v_n, v_0\}$. Then, the sequence $s = \langle\{v_0\}, \{v_1\}\rangle, \langle\{v_1\}, \{v_2\}\rangle, \ldots, \langle\{v_{n-1}\}, \{v_n\}\rangle, \langle\{v_n\}, \{v'_0\}\rangle$ is a $\{v_0\}$-path of $\mathcal{F}$ with $C(s) = V \cup \{v_0\}$ since c is a Hamiltonian cycle. Using again the fact that c is a Hamiltonian cycle, we get $\mathcal{L}(s) \leq |V|$. Then, s is a solution of P_G. It is easy to use a similar approach to show that every solution of P_G can be transformed into a Hamiltonian cycle of G. □

Our proof of Theorem 2.7 shows that the persuasion satisfiability problem is NP-complete, even when we do not consider the upper bound on the weight. Similarly, by using the weights of the arguments to represent the path length, we can also show that the problem remains NP-complete even when we do not consider the upper bound on the length.

Proposition 2.8. *The restriction of the persuasion satisfiability problem to the instances of the form $(\mathcal{F}, I, O, k, \infty)$ is NP-complete. Further, the restriction of the persuasion satisfiability problem to the instances of the form $(\mathcal{F}, I, O, \infty, b)$ is also NP-complete.*

2.4 A Basic Bidirectional Approach

In our framework, we do not explicitly describe how the persuader agent enters into a dialogue with the persuadee agent. However, this does not mean that we only consider a unidirectional approach for persuasion. For instance, in the same way as the asymmetric persuasion framework introduced in [16], our framework can be used in the context of a bidirectional asymmetric persuasion approach by allowing the persuadee to only accept or reject every argument.

Consider the approach described in Figure 2. First, the persuader agent uses the solution $a_1 \to \cdots \to a_l$. The persuadee agent accepts only the subsequence $a_1 \to \cdots \to a_{i-1}$ and rejects the argument a_i. Then, the persuader agent recomputes a new solution without taking into account the argument $a_1, \ldots, a_i$, since $a_1, \ldots, a_{i-1}$ are already accepted and a_i is rejected. Using the fact that the persuader agent communicated already i arguments to the persuadee agent, the length upper bound is set to $k - i$. Further, the weight upper bound has to be reduced by the weights of the accepted arguments $a_1, \ldots, a_{i-1}$. More precisely, the new weight upper bound is equal to $b - \sum_{j=1}^{i-1} W(a_j)$. Note that $W_{|\mathcal{A}'}$ corresponds to the restriction of W to $\mathcal{A}'$

3 AN ENCODING IN PARTIAL WEIGHTED MAXSAT

In this section, we employ a declarative approach for solving the persuasion satisfiability problem. Indeed, we propose an encoding of this problem in partial weighted MaxSAT, which is a well-known optimization problem within the artificial intelligence community.

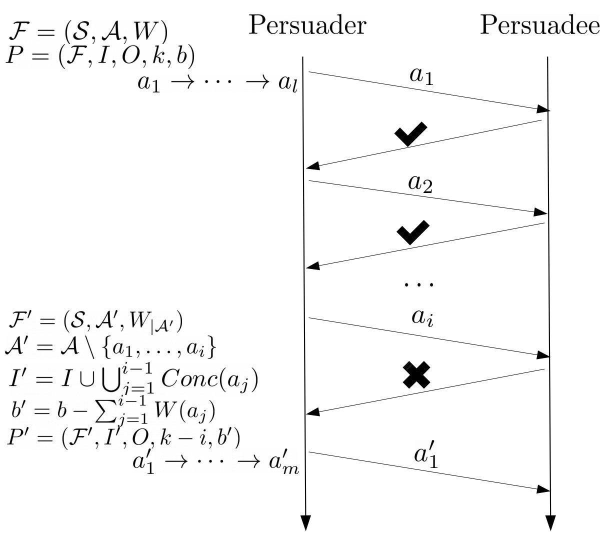

Figure 2: A basic bidirectional approach

In fact, our encoding allows for finding a solution of the smallest weight, since partial weighted MaxSAT is an optimization problem.

First, let us recall that a CNF formula is a conjunction of clauses where a *clause* is a disjunction of literals. It is well-known that every propositional formula can be translated to CNF w.r.t. the satisfiability problem using Tseitin's linear encoding [25]. A *Boolean interpretation* of a CNF formula ϕ is an assignment that associates truth values in $\{0, 1\}$ to propositional variables in $Var(\phi)$, where 0 stands for false and 1 stands for true. A Boolean interpretation is extended to CNF formulas as usual. A *model* of a CNF formula ϕ is a Boolean interpretation $\mathcal{B}$ satisfying this formula, i.e., $\mathcal{B}(\phi) = 1$. The problem of determining whether there exists a model that satisfies a given CNF formula, abbreviated as SAT, is one of the most studied NP-complete problems.

In partial weighted MAX-SAT, each clause is either relaxable (soft) with an associate cost or non-relaxable (hard). The objective is to find a Boolean interpretation that satisfies all the hard clauses and minimize the cost of the falsified soft clauses. Given a satisfiable partial weighted MAX-SAT instance ϕ and a solution $\mathcal{B}$ of ϕ, we use $cost(\mathcal{B})$ to denote the cost of $\mathcal{B}$.

Consider for instance the partial weighted MAX-SAT instance $\phi = \phi_h \wedge \phi_s$ where $\phi_h = (p \vee q) \wedge (p \vee r)$ is the hard part and $\phi_s = (3 : \neg p) \wedge (1 : \neg q) \wedge (1 : \neg r)$ is the soft part. Clearly, it is not possible to satisfy at the same time all the hard and soft clauses. The Boolean interpretation $\mathcal{B}$ defined by $\mathcal{B}(p) = 0$ and $\mathcal{B}(q) = \mathcal{B}(r) = 1$ is a solution of the instance ϕ. Indeed, the cost of $\mathcal{B}$ is equal to 2 and the costs of all the other models of the hard part ϕ_h are greater than or equal to 3.

Let $P = (\mathcal{F}, I, O, k, b)$ be an instance of the persuasion satisfiability problem, where $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$. In order to define our encoding for the instance P, we need some syntactic elements. We first associate to each literal $l \in Lit(\mathcal{S})$ a set of $k + 1$ distinct propositional variables denoted $p_l^0, \ldots, p_l^k$. The propositional variable p_l^i is used to express whether or not the literal l is produced at the step number i. In particular, p_l^0 is true if and only if l occurs in the initial state I. Moreover, given a subset of literals $X \subseteq Lit(\mathcal{S})$ and $0 \leq i \leq k$, we use $R(X, i)$ to denote the set of variables $\{p_l^i \mid l \in X\}$. We also associate to each argument $a \in \mathcal{A}$ a distinct propositional variable denoted q_a and a set of k distinct propositional variables denoted $r_a^1, \ldots, r_a^k$. The propositional variable q_a is used to express whether or not the argument a is used in the solution, and similarly r_a^i is used to express whether or not the argument a is used in the solution at the step number i. Further, given a literal $l \in Lit(\mathcal{S})$, we use $Arg(l, \mathcal{A})$ to denote the set of the arguments $\{\langle X, C\rangle \in \mathcal{A} \mid l \in C\}$. We generalize this notation to the sets of literals as follows: $Arg(X, \mathcal{A}) = \{\langle X, C\rangle \in \mathcal{A} \mid X \cap C \neq \emptyset\}$.

In the following, we describe our partial weighted MaxSAT encoding. We first define the hard part of this encoding. In this context, the following formula expresses that I corresponds to the initial state:

$$\bigwedge_{l \in I} p_l^0 \wedge \bigwedge_{l' \in Lit(\mathcal{S}) \setminus I} \neg p_{l'}^0 \tag{1}$$

Then, we introduce a formula that allows us to require a consistent I-path:

$$\bigwedge_{e \in \mathcal{S}} \bigwedge_{i=0}^{k} \bigwedge_{j=0}^{k} (\neg p_e^i \vee \neg p_{\neg e}^j) \tag{2}$$

The following formula is used to relate the truth of every variable of the form p_l^i to the use of at least one argument containing l in its conclusion at the step i:

$$\bigwedge_{l \in Lit(\mathcal{S})} \bigwedge_{i=1}^{k} (p_l^i \rightarrow \bigvee_{a \in Arg(l, \mathcal{A})} r_a^i) \tag{3}$$

The next formula expresses that the use of an argument at the step i requires the truth of its support before this step:

$$\bigwedge_{a = \langle X, C\rangle \in \mathcal{A}} \bigwedge_{i=1}^{k} (r_a^i \rightarrow (\bigwedge_{l \in X} (\bigvee_{j=0}^{i-1} p_l^j))) \tag{4}$$

Then, we propose a formula that states that if an argument is used at the step i, then the literals of its conclusion are true at this step:

$$\bigwedge_{a = \langle X, C\rangle \in \mathcal{A}} \bigwedge_{i=1}^{k} (r_a^i \rightarrow (\bigwedge R(C, i))) \tag{5}$$

where $\bigwedge R(C, i)$ represents the conjunction of the literals occurring in $R(C, i)$.

We now introduce the formula expressing the fact that each argument is used at most once:

$$\bigwedge_{a \in \mathcal{A}} \sum_{i=1}^{k} r_a^i \leq 1 \tag{6}$$

Similarly, the following formula expresses the fact that in each step we use at most one argument:

$$\bigwedge_{i=1}^{k} \sum_{a \in \mathcal{A}} r_a^i \leq 1 \tag{7}$$

Essentially, the next conjunction of clauses states that we have to obtain the objective state:

$$(\bigwedge_{l \in O \setminus I} (\bigvee_{i=1}^{k} p_l^i)) \tag{8}$$

The following formula is only used to associate the truth of q_a to the use of the argument a:

$$\bigwedge_{a \in \mathcal{A}} ((\bigvee_{i=1}^{k} r_a^i) \leftrightarrow q_a) \tag{9}$$

The relaxable part contains only the following soft clauses:

$$W(a) : \neg q_a \ for\ every\ a \in \mathcal{A} \tag{10}$$

Every soft clause allows for associating each argument to its weight.

The formulas (6) and (7) involve the well-known at-most-one constraint. There are several linear encodings of this constraint as CNF formulas w.r.t. the propositional satisfiability problem (e.g. see [23]). Furthermore, for the sake of clarity, we do not use the conjunctive normal form for describing the hard part of our encoding. However, it is easy to transform the hard part into an equi-satisfiable CNF formula by using Tseitin's linear encoding [25]. The main idea of this encoding consists in using De Morgan's laws with other valid equivalence rules, and associating fresh propositional variables to subformulas. Consider for instance the formula (4). We first associate a fresh variable s_l^i for every subformula of the form $\bigvee_{j=0}^{i-1} p_l^i$. Then, we replace (4) with the conjunction of clauses $\bigwedge_{a=\langle X,C\rangle \in \mathcal{A}} \bigwedge_{i=1}^{k} \bigwedge_{l \in X} (\neg r_a^i \vee s_l^i)$ and conjunctively add implications of the form $s_X^i \rightarrow \bigvee_{j=0}^{i-1} p_l^i$, which can be easily replaced by simple clauses. It is worth mentioning that we do not need to use the equivalence connective for relating the fresh variables to their associated subformulas, since the implication connective is sufficient to preserve satisfiability.

From now on, we use $\mathcal{E}(P)$ to denote the encoding that corresponds to the partial weighted MaxSAT instance (1)$\wedge \cdots \wedge$(10).

Proposition 3.1 (Soundness). *Let $P = (\mathcal{F}, I, O, k, b)$ be an instance of the persuasion satisfiability problem. Then, P admits a solution iff $\mathcal{E}(P)$ admits a solution $\mathcal{B}$ s.t. $cost(\mathcal{B}) \leq b$.*

Proof.

Part $\Rightarrow$. Assume that P admits as a solution the I-path $s = a_1, \ldots, a_m$. Then, we associate to s a Boolean interpretation $\mathcal{B}$ as follows. First, for all $1 \leq i \leq m$, $\mathcal{B}(q_{a_i}) = 1$, and for all $a \notin \{a_1, \ldots, a_m\}$, $\mathcal{B}(q_a) = 0$. Then, for all $1 \leq i \leq m$, $\mathcal{B}(r_{a_i}^i) = 1$, and for all $r_a^j \notin \{r_{a_i}^i \mid 1 \leq i \leq m\}$, $\mathcal{B}(r_a^j) = 0$. Clearly, the previous definitions implies that $\mathcal{B}$ satisfies the formulas (6), (7) and (9). The following property on $\mathcal{B}$ shows that the latter satisfies (1): for all $l \in Lit(\mathcal{S})$, if $l \in I$ then $\mathcal{B}(p_l^0) = 1$, otherwise $\mathcal{B}(p_l^0) = 0$. Further, we define the truth values of the variables of the form p_l^i for $i \in 1..k$ by using the following property: $\mathcal{B}(p_l^i) = 1$ iff $l \in Conc(a_i)$. Using the previous property and the fact that s is an I-path, we know that $\mathcal{B}$ satisfies the formulas (3), (4) and (5). Moreover, using the fact that s is consistent, $\mathcal{B}$ satisfies also the formula (2). Then, using the property $O \subseteq C(s)$, $\mathcal{B}$ satisfies (8). Finally, using the fact $\mathcal{W}(s) \leq b$, $cost(\mathcal{B}) \leq b$ holds. As a consequence, the partial MaxSAT instance $\mathcal{E}(P)$ admits a solution $\mathcal{B}'$ s.t. $cost(\mathcal{B}') \leq b$.

Part $\Leftarrow$. Assume that $\mathcal{E}(P)$ admits a solution $\mathcal{B}$ s.t. $cost(\mathcal{B}) \leq b$. Using the formula (7), we know that there is at most one true variable of the form r_a^i for every $1 \leq i \leq k$. Then, there is a unique sequence of argument $s = a_{i_1}, \ldots, a_{i_m}$ s.t. $i_n < i_{n'}$ for every $n < n'$, $\mathcal{B}(r_{a_{i_j}}^j) = 1$ for every $1 \leq j \leq m$, and $\mathcal{B}(r_a^{j'}) = 0$ for every $r_a^{j'} \notin \{r_{a_{i_j}}^j \mid 1 \leq j \leq m\}$. Further, using the formula (6), we know that the arguments in s are pairwise distinct. Then, using the formulas (1), (3), (4) and (5), $Supp(a_{i_j}) \subseteq I \cup \bigcup_{1 \leq j' < j} Conc(a_{i_{j'}})$ holds for every $1 \leq j \leq m$. As a consequence, s is an I-path. Further, using the formula (2), we know that s is consistent. The formula (8) allows us to obtain $O \subseteq C(s)$. Using the formula (9) and the soft part (10), $cost(\mathcal{B}) = \mathcal{W}(s)$ holds. Thus, knowing that $cost(\mathcal{B}) \leq b$, we obtain $\mathcal{W}(s) \leq b$. Therefore, s is a solution of P. □

An improvement of our encoding can be accomplished by removing the formula (6). Indeed, this formula is used to express that each argument is applied at most once, but one can easily see that it is not problematic to have arguments that occur more than once in a solution respecting the considered upper-bounds. In fact, we only need to keep the first application of each argument to obtain a solution where every argument occurs at most once.

4 WEAK CONSISTENCY AND FLEXIBILITY

In this section, we introduce a property on paths, called weak consistency, that formalizes the fact that the persuder agent may hide parts of conclusions of used arguments in order to present a consistent path.

To illustrate our motivation, consider a persuadee agent that has the following set of wishes $I = \{being\ healthy, \neg\ playing\ sports\}$, which means that this agent wants to be healthy without playing sports. Moreover, consider a persuader agent that has only the argument $a = \langle\{being\ healthy\}, \{stop\ smoking, playing\ sports\}\rangle$, that is, being healthy requires to stop smoking and to play sports. Clearly, the I-path a is not consistent. However, in certain cases, it would be interesting to hide literals in the conclusions of the considered arguments, as for instance, hiding the literal *playing sports* in the case of the previous argument. To this end, we introduce here the notion of weak inconsistency, which states that we only have to inspect the consistency of the supports of the considered arguments in a path.

Definition 4.1 (Weak Consistency). An I-path $a_1, \ldots, a_k$ is said to be *weak consistent* if the set of literals $I \cup \bigcup_{i=1}^{k} Supp(a_i)$ is consistent.

In the following proposition, we formally show that weak consistency is a weaker version of consistency.

Proposition 4.2. *Given a set of literals I, if an I-path is consistent, then it is also weak consistent.*

Proof. Let $s = a_1, \ldots, a_k$ be an I-path. Assume that s is not weak consistent. Then, the set of literals $I \cup \bigcup_{i=1}^{k} Supp(a_i)$ is not consistent. Further, using the definition of an I-path, we know that $Supp(a_i) \subseteq I \cup \bigcup_{1 \leq j < i} Conc(a_j)$ for every $i \in 1..k$. Thus, $I \cup \bigcup_{i=1}^{k} Supp(a_i) \subseteq I \cup \bigcup_{i=1}^{k} Conc(a_i)$ holds. As a consequence,

$I \cup \bigcup_{i=1}^{k} Conc(a_i)$ is not consistent and we deduce that s is not consistent. □

Consider for instance the persuasion frame $\mathcal{F} = (\{a, b, c, d\}, \{a_1 = \langle\{a\}, \{b, c\}\rangle, a_2 = \langle\{d\}, \{\neg c\}\rangle\}, W)$ where $W(a_1) = 1$ and $W(a_2) = 1$, and the instance of the persuasion satisfiability problem $P = (\mathcal{F}, \{a, d\}, \{b, \neg c\}, 2, 2)$. Clearly, there is no consistent $\{a, d\}$-path that satisfies P. However, a_1, a_2 is a weak consistent $\{a, d\}$-path that allows for obtaining $\{b, \neg c\}$ from $\{a, b\}$.

In order to use a partial weighted MaxSAT encoding for solving the persuasion satisfiability problem where we use weak consistency instead of consistency, we just need to remove the formula (5) from the encoding described in Section 3. Indeed, the formula (5) is used to impose the satisfaction of the propositional variables representing the conclusion of the applied arguments. It is noteworthy that the satisfaction of the variables representing the supports of the applied arguments comes particularly from the formulas (2) and (4).

In general, it would be interesting to allow the persuader agent to select the literals that can be hided to avoid inconsistency. For instance, the persuader agent may decide to hide only the negations of the literals that occur in the objective state and/or the initial state. To adapt our partial weighted MaxSAT encoding, we only have to use the following formula instead of (5):

$$\bigwedge_{a=\langle X,C\rangle\in\mathcal{A}} \bigwedge_{i=1}^{k} (r_a^i \to (\bigwedge R(C \setminus T, i))) \tag{11}$$

where T is the set containing the literals that can be hided. This formula states that if an argument is used at the step i, then the literals of its conclusion are true at this step except the literals that can be hided.

The previous variant show the flexibility of the proposed framework and our solution based on the use of encodings in partial weighted MaxSAT. In this context, one can easily define other variants to take into account other aspects. In order to better illustrate this point, we introduce a variant where the persuader agent considers conflicts between the available arguments. More precisely, given a persuasion frame $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$, we assume that the persuader agent has a conflict graph $G = (\mathcal{A}, E)$, which is an undirected graph where the set of vertices is $\mathcal{A}$, and having an edge $\{a, a'\}$ in E means that the arguments a and a' cannot be used together in any solution. For this variant, our encoding can be adapted by adding the following formula to the hard part:

$$\bigwedge_{\{a,a'\}\in E} (\sum_{i=1}^{k} r_a^i) + (\sum_{j=1}^{k} r_{a'}^j) \leq 1 \tag{12}$$

Indeed, assume w.l.o.g. that there exists $i \in 1..k$ such that r_a^i is assigned to 1. Then, $\sum_{i=1}^{k} r_a^i = 1$ holds, and consequently, we obtain $\sum_{j=1}^{k} r_{a'}^j = 0$, which means that a' is not used in the found solution.

Several other variants can be defined by reasoning on different other aspects, such as the opening argument, the last argument, etc.

5 PARETO OPTIMALITY

In the persuasion satisfiability problem, there are explicite upper bounds on the length and on the weight of the solution. The choice of these bounds may be arbitrary without specific knowledge about the considered case, which can be seen as a real drawback of our approach. Thus, to avoid the use of bounds, we here consider the notion of Pareto optimality.

We define a *PO-instance* as a triple of the form $P = (\mathcal{F}, I, O)$ where $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$ is a persuasion frame, $I \subseteq Lit(\mathcal{S})$ a consistent set of literals, called the *initial state*, and $O \subseteq Lit(\mathcal{S})$ a non empty consistent set of literals, called the *objective state*. Clearly, a PO-instance can be seen as an instance of the persuasion satisfiability problem without the upper bounds on the length and on the weight of the solution. From this angle, we say that a sequence of arguments s is a solution of PO-instance $P = (\mathcal{F}, I, O)$ if it is a solution of the instance of the persuasion satisfiability problem $P = (\mathcal{F}, I, O, \infty, \infty)$.

Definition 5.1 (Pareto-Optimal Solution). A *Pareto-optimal solution* of a PO-instance P is a solution s of P where, for all solution s' of P, (*i*) if $\mathcal{L}(s') < \mathcal{L}(s)$ then $\mathcal{W}(s) < \mathcal{W}(s')$, and (*ii*) if $\mathcal{W}(s') < \mathcal{W}(s)$ then $\mathcal{L}(s) < \mathcal{L}(s')$.

Given two solutions s and s' of a PO-instance, we say that s dominates s' if at least one the following properties is satisfied:

- $\mathcal{L}(s) < \mathcal{L}(s')$ and $\mathcal{W}(s) \leq \mathcal{W}(s')$;
- $\mathcal{W}(s) < \mathcal{W}(s')$ and $\mathcal{L}(s) \leq \mathcal{L}(s')$.

In other words, s dominates s' if s is better than s' on one criterion (length or weight) and s is not worse than s' on the other criterion. Note that a solution is Pareto-optimal if and only if it is not dominated by any other solution.

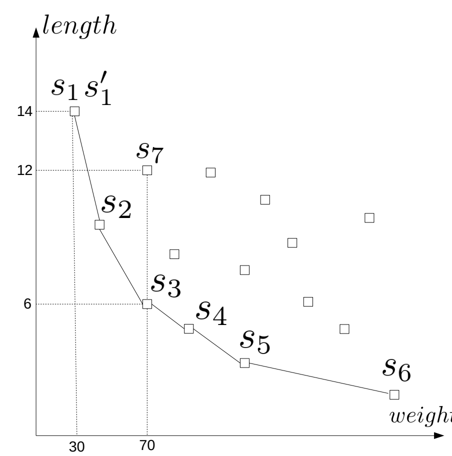

Figure 3: Pareto Front

For instance, in Figure 3, we show all the solutions of a given PO-instance. The x-axis and y-axis represent respectively the weights and the lengths of these solutions. It is important to note that a point can represent more than one solution (e.g. the point that represents the solutions s_1 and s_1'), since there may exist several solutions with both the same length and the same weight. The solutions s_1, s_1' and s_3 are Pareto-optimal, but s_7 is not because it is dominated by s_3 ($\mathcal{L}(s_3) = 6 < \mathcal{L}(s_7) = 12$ and $\mathcal{W}(s_3) = \mathcal{W}(s_7) = 70$). Moreover, s_1 does not dominate s_3 because of $\mathcal{L}(s_3) = 6 < \mathcal{L}(s_1) = 14$, and s_3 does not dominate s_1 because of $\mathcal{W}(s_1) = 30 < \mathcal{W}(s_3) = 70$.

We call *Pareto frontier* of a given PO-instance the set of all its optimal solutions. For instance, the Pareto frontier in the example described in Figure 3 is the set $\{s_1, s_1', s_2, s_3, s_4, s_5, s_6\}$.

Our partial weighted MaxSAT encoding for solving the persuasion satisfiability problem described in Section 3 allows for computing a solution that satisfies the length bound constraint and has the smallest weight. More precisely, given an instance $P = (\mathcal{F}, I, O, k, b)$ of the persuasion satisfiability problem, every solution of the encoding $\mathcal{E}(P)$ corresponds to a solution s of P where $\mathcal{L}(s) \leq k$ and $\mathcal{W}(s) \leq \mathcal{W}(s')$ for every solution s' of P of length smaller than or equal to k. Thus, given a solution s of P which is obtained using $\mathcal{E}(P)$, we know that there exists a Pareto-optimal s' of the PO-instance $(\mathcal{F}, I, O)$ such that $\mathcal{W}(s') = \mathcal{W}(s)$.

Proposition 5.2. *Given a PO-instance $P = (\mathcal{F}, I, O)$ with $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$, the length of every Pareto-optimal solution is smaller than or equal to $min(|\mathcal{S}| - |I|, |\mathcal{A}|)$, where min stands for the minimum.*

Proof. Let $s = a_1, \ldots, a_n$ be a Pareto-optimal solution of P. It is trivial that $n \leq |\mathcal{A}|$ since the arguments used in s are pairwise distinct and belong to $\mathcal{A}$. Furthermore, we have $I \cup \bigcup_{i=1}^{j} Conc(a_i) \subset I \cup \bigcup_{i=1}^{j+1} Conc(a_i)$ for every $j \in 1..(n-1)$. Indeed, if there exists $j \in 1..(n-1)$, $I \cup \bigcup_{i=1}^{j} Conc(a_i) = I \cup \bigcup_{i=1}^{j+1} Conc(a_i)$, then the use of a_{j+1} is useless and we get a contradiction since s is Pareto-optimal. Thus, using the fact that $|I \cup \bigcup_{i=1}^{n} Conc(a_i)| \leq |\mathcal{S}|$, $n \leq |\mathcal{S}| - |I|$ holds. □

Given a PO-instance $P = (\mathcal{F}, I, O)$ with $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$, we use $LB(P)$ to denote the value $min(|\mathcal{S}| - |I|, |\mathcal{A}|)$. Furthermore, using Proposition 5.2, one can easily obtain a weight upper bound. Indeed, this bound can be defined as the sum of the weights of the arguments occurring in a set of $LB(P)$ greatest weights. More precisely, the weight of every Pareto-optimal solution of P is smaller than or equal to $\sum_{a \in \mathcal{A}'} W(a)$, where $\mathcal{A}' \subseteq \mathcal{A}$, $|\mathcal{A}'| = LB(P)$ and, for all $a' \in \mathcal{A} \setminus \mathcal{A}'$, $W(a') \leq min\{W(a) \mid a \in \mathcal{A}'\}$. We use $WB(P)$ to denote the bound $\sum_{a \in \mathcal{A}'} W(a)$.

In order to introduce our approach for computing certain Pareto-optimal solutions, we propose a partial weighted MaxSAT encoding that allows for computing a solution of a Given PO-instance with the smallest length. Let $P = (\mathcal{F}, I, O)$ be a PO-instance with $\mathcal{F} = (\mathcal{S}, \mathcal{A}, W)$. We use $\mathcal{E}_{length}(P)$ to denote the encoding that allows for computing a solution of smallest length. The hard part of this encoding is exactly the same as the hard part of $\mathcal{E}(P')$ with $P' = (\mathcal{F}, I, O, LB(P), \infty)$. The relaxable part is defined as follows:

$$1 : \neg q_a \ for\ every\ a \in \mathcal{A} \tag{13}$$

The soundness of $\mathcal{E}_{length}(P)$ can be obtained in the same way as the soundness of the encoding of the persuasion satisfiability problem. Indeed, knowing that the hard part is the same as that of $\mathcal{E}(P')$, we know that every solution of $\mathcal{E}_{length}(P)$ corresponds to a solution of P. Moreover, using Proposition 5.2, the hard part of $\mathcal{E}_{length}(P)$ is satisfiable if and only if P admits a solution. In addition, one can easily see that the relaxable part allows clearly for reducing the number of used arguments.

A simple approach for finding a Pareto-optimal solution of a given PO-instance P can be defined by solving two partial weighted MaxSAT encodings. Indeed, we first compute a solution s_0 using the encoding $\mathcal{E}_{length}(P)$. Then, every solution of $\mathcal{E}(P')$ with $P' = (\mathcal{F}, I, O, \mathcal{L}(s_0), \infty)$ is a Pareto-optimal solution of P. Indeed, let s be a solution of P obtained from $\mathcal{E}(P')$. Then, for every solution s' with $\mathcal{L}(s') \leq \mathcal{L}(s_0)$, $\mathcal{W}(s') \leq \mathcal{W}(s)$ holds. Moreover, we know that $\mathcal{L}(s_0)$ is the smallest length of the solutions of P. As a consequence, s is not dominated by any other solution of P.

A similar approach for finding a Pareto-optimal solution can be defined by computing first the smallest weight that can be obtained from our encoding $\mathcal{E}(P)$, and then, we use an encoding that allows for finding one of the shortest paths with respect to the previous weight. Indeed, let $P = (\mathcal{F}, I, O)$ be a PO-instance. First, a solution s_0 for $\mathcal{E}(P')$ is computed where $P' = (\mathcal{F}, I, O, LB(P), \infty)$. Using Proposition 5.2, we know that $\mathcal{W}(s_0)$ is the smallest weight of the solutions of P. Then, we use a partial weighted MaxSAT encoding $\mathcal{E}_{weight}(P, W(s_0))$ to find a Pareto-optimal solution. The encoding $\mathcal{E}_{weight}(P, \mathcal{W}(s_0))$ is obtained by only adding the following hard pseudo-Boolean constraint to $\mathcal{E}_{length}(P)$:

$$\sum_{a \in \mathcal{A}} W(a) * q_a \leq \mathcal{W}(s_0) \tag{14}$$

Clearly, this constraint allows for guaranteeing that every solution of $\mathcal{E}_{weight}(P, \mathcal{W}(s_0))$ has the smallest weight, i.e., it is also a solution of $\mathcal{E}(P')$. Moreover, for every solution s of $\mathcal{E}_{weight}(P, \mathcal{W}(s_0))$, there is no solution of P that has the weight $\mathcal{W}(s_0)$ and is also shorter than s. As a consequence, every solution of the encoding $\mathcal{E}_{weight}(P, \mathcal{W}(s_0))$ is Pareto-optimal.

It is worth mentioning that there are several efficient polynomial encodings of the pseudo-Boolean constraints as CNF formulas in the literature (e.g. see [5, 11]).

6 CONCLUSION AND PERSPECTIVES

In this paper, we have presented an argument-centric persuasion framework. This contribution proposes a decision problem, called persuasion satisfiability, that allows for dealing with computational persuasion in a simple and intuitive way. From the computational complexity point of view, we showed that this decision problem is NP-complete. We have proposed an encoding in partial weighted MaxSAT for solving this problem. We also showed the flexibility of our framework and the approach based on the use of partial weighted MaxSAT. Finally, in order to avoid the use of explicit upper bound constraints, which can be seen as a drawback of our framework in the absence of specific knowledge about the considered case, we have proposed an approach that allows for finding optimal solutions in Pareto sense.

In our future work, we intend first to improve the proposed framework following two directions: (1) allowing the persuadee to use different kinds of counterarguments; and (2) defining an updating method for persuasion frames to take into account the responses of the persuadee. We also plan to implement the proposed solving methods based on partial weighted MaxSAT to provide an experimental study on the use of the proposed framework.

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