A Polynomial-time Fragment of Epistemic Probabilistic Argumentation

Extended Abstract

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ABSTRACT

Probabilistic argumentation allows reasoning about argumentation problems in a way that is well-founded by probability theory. However, in practice, this approach can be severely limited by the fact that probabilities are computed by adding an exponential number of terms. We show that this exponential blowup can be avoided in an interesting fragment of epistemic probabilistic argumentation and that some computational problems that have been considered intractable can be solved in polynomial time.

KEYWORDS

Probabilistic Argumentation; Complexity

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1 INTRODUCTION

Abstract argumentation [5] studies the acceptability of arguments independent of their content, just based on their relationships. Several probabilistic extensions have been studied in recent years [4, 6–8, 13, 14, 18, 19, 26–28, 30, 31]. We focus on the epistemic approach to probabilistic argumentation that evolved from work in [12, 29]. In this approach, probability functions over possible worlds assign degrees of beliefs to arguments. Semantical constraints restrict the possible degrees of beliefs based on the relationships between arguments. For example, the probability of an argument can be bounded from above based on the probabilities of its attackers or bounded from below by the probability of its supporters. This is conceptually similar to weighted argumentation frameworks, where attack relations are supposed to decrease the strength of arguments, whereas support relations are supposed to increase the strength [1, 2, 20, 22, 25].

Two basic computational problems have been introduced in [16]. The *satisfiability problem* asks whether a given set of semantical constraints over an argumentation graph can be satisfied. The *entailment problem* is to answer queries about the probability of arguments. Based on their close relationship to problems considered in probabilistic reasoning, it has been conjectured that these problems are intractable. However, as we will explain, both problems can be solved in polynomial time. The reason is that they consider only atomic probability statements. Therefore, reasoning with probability functions over possible worlds turns out to be equivalent to reasoning with functions that assign probabilities to arguments directly. We call these functions probability labellings as they can be seen as generalizations of labellings in classical abstract argumentation [3] that, intuitively, label arguments as rejected (probability 0), accepted (probability 1) or undecided (probability 0.5).

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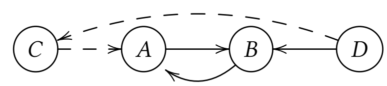

Figure 1: A simple example BAF.

2 BACKGROUND AND MAIN RESULTS

We consider bipolar argumentation frameworks (BAFs) $(\mathcal{A}, \mathcal{R}, \mathcal{S})$ consisting of a set of arguments $\mathcal{A}$, an attack relation $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ and a support relation $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$. $\mathrm{Att}(A) = \{B \in \mathcal{A} \mid (B, A) \in \mathcal{R}\}$ denotes the set of attackers of an argument A and $\mathrm{Sup}(A) = \{B \in \mathcal{A} \mid (B, A) \in \mathcal{S}\}$ denotes its supporters. We visualize BAFs by graphs, where nodes denote arguments, solid edges denote attacks and dashed edges denote supports. Figure 1 shows an example.

A *possible world* is a subset of arguments $w \subseteq \mathcal{A}$. Intuitively, w contains the arguments that are accepted. $2^{\mathcal{A}}$ denotes the set of all subsets of $\mathcal{A}$, that is, the set of all possible worlds. In order to talk about agents' beliefs in arguments, we consider probability functions $P : 2^{\mathcal{A}} \rightarrow [0, 1]$ such that $\sum_{w \in 2^{\mathcal{A}}} P(w) = 1$. $\mathcal{P}_{\mathcal{A}}$ denotes the set of all probability functions over $\mathcal{A}$. The probability of an argument $A \in \mathcal{A}$ under P is defined by adding the probabilities of all worlds in which A is accepted, that is, $P(A) = \sum_{w \in 2^{\mathcal{A}}, A \in w} P(w)$.

Semantics are given to attack and support relations via semantical constraints. For the satisfiability and entailment problem in [16], the following constraints have been considered (for attack-only graphs).

- **COH:** P is called *coherent* if for all $A, B \in \mathcal{A}$ with $(A, B) \in \mathcal{R}$, we have $P(B) \leq 1 - P(A)$.
- **SFOU:** P is called *semi-founded* if $P(A) \geq 0.5$ for all $A \in \mathcal{A}$ with $\mathrm{Att}(A) = \emptyset$.
- **FOU:** P is called *founded* if $P(A) = 1$ for all $A \in \mathcal{A}$ with $\mathrm{Att}(A) = \emptyset$.
- **SOPT:** P is called *semi-optimistic* if $P(A) \geq 1 - \sum_{B \in \mathrm{Att}(A)} P(B)$ for all $A \in \mathcal{A}$ with $\mathrm{Att}(A) \neq \emptyset$.
- **OPT:** P is called *optimistic* if $P(A) \geq 1 - \sum_{B \in \mathrm{Att}(A)} P(B)$.
- **JUS:** P is called *justifiable* if P is coherent and optimistic.

The intuition for these constraints comes from the idea that probability 0.5 represents indifference, whereas probabilities smaller (larger) than 0.5 tend towards rejectance (acceptance). Coherence imposes an upper bound on the beliefs in arguments based on the beliefs in their attackers. Semi-Foundedness says that an agent should not reject an unattacked argument. Foundedness even demands that such an argument should be fully accepted. Semi-optimistic and Optimistic give lower bounds on the beliefs in an argument based on the beliefs in its attackers. Usually, not all constraints are employed, but a subset is selected that seems reasonable for a particular application.

Example 2.1. If we demand COH and FOU for the BAF in Figure 1, we get $P(C) = 1$ and $P(D) = 1$ from FOU. From COH, we get $P(B) \leq 1 - P(D)$. Together, this also implies $P(B) = 0$.

We can define dual constraints for support-only graphs by replacing $\mathcal{R}$ with $\mathcal{S}$, probability $(1 - p)$ with p, $\leq$ with $\geq$ and vice versa. The following two constraints are dual to COH and OPT.

- **S-COH:** P is called *s-coherent* if for all $A, B \in \mathcal{A}$ with $(A, B) \in \mathcal{S}$, we have $P(B) \geq P(A)$.
- **PES:** P is called *pessimistic* if $P(A) \leq \sum_{B \in \mathrm{Sup}(A)} P(B)$.

Example 2.2. If we add S-COH to our previous example, we get $P(C) \geq P(D)$ and $P(A) \geq P(C)$. Since we already know that $P(C) = 1$, we can conclude $P(A) = 1$.

If both attack and support relations are present, one may want to consider more flexible constraints that take account of both attackers and supporters simultaneously. In order to do so, a general constraint language has been introduced recently that incorporates all of the previous examples [15]. We consider only a simple fragment of this language. Our fragment still captures the previous examples and, in particular, gives us polynomial performance guarantees.

Definition 2.3 (Linear Atomic Constraint, Satisfiability). A *linear atomic constraint* is an expression of the form $\sum_{i=1}^{n} c_i \cdot \pi(A_i) \leq c_0$, where $A_i \in \mathcal{A}$ and $c_i \in \mathbb{Q}$. A probability function P satisfies a linear atomic constraint iff $\sum_{i=1}^{n} c_i \cdot P(A_i) \leq c_0$. P satisfies a set of linear atomic constraints C, denoted as $P \models C$, iff it satisfies all $l \in C$.

Note that $\geq$ and $=$ can be expressed as well. For $\geq$, just note that $\sum_{i=1}^{n} c_i \cdot \pi(A_i) \leq c_0$ is equivalent to $\sum_{i=1}^{n} -c_i \cdot \pi(A_i) \geq -c_0$. For $=$, note that $\sum_{i=1}^{n} c_i \cdot \pi(A_i) \leq c_0$ and $\sum_{i=1}^{n} c_i \cdot \pi(A_i) \geq c_0$ together are equivalent to $\sum_{i=1}^{n} c_i \cdot \pi(A_i) = c_0$. We merely restrict our language to constraints with $\leq$ in order to keep the notation simple. All semantical constraints that we mentioned before are indeed linear atomic constraints.

The following two computational problems generalize the problems from [16] to arbitrary linear atomic constraints over bipolar argumentation frameworks.

- **PArgAtSAT:** Given a finite set of linear atomic constraints C, decide whether C is satisfiable.
- **PArgAtENT:** Given a finite set of satisfiable linear atomic constraints C and an argument A, compute tight lower and upper bounds on the probability of A. Formally, solve the two optimization problems: $\min_{P \in \mathcal{P}_{\mathcal{A}}} / \max_{P \in \mathcal{P}_{\mathcal{A}}} \{P(A) \mid P \models C\}$.

In our naming scheme, PArg stands for probabilistic argumentation, At for the restriction to linear atomic constraints and SAT and ENT stand for satisfiability and entailment, respectively. The computational problems from [15] also allowed partial probability assignments (fixing the probability of some arguments), but these partial assignments can just be seen as simple linear atomic constraints.

Example 2.4. Consider the BAF in Figure 1. Say our partial probability assignment assigns probability 1 to B and 0 to C. These assignments correspond to the two linear constraints $\pi(B) = 1$ and $\pi(C) = 0$. Say we also impose COH. Then, we additionally have the constraints $\pi(A) + \pi(B) \leq 1$ and $\pi(B) + \pi(D) \leq 1$. Taken together, these constraints imply that every probability function P that satisfies all constraints, must satisfy $P(B) = 1$, $P(C) = 0$ (partial assignment constraints), $P(A) = 0$ and $P(D) = 0$ (follow with coherence constraints). Note that when also adding the foundedness constraints $\pi(C) = 1$ and $\pi(D) = 1$, the set of constraints becomes unsatisfiable.

It has been conjectured that PArgAtSAT and PArgAtENT are intractable because they are similar to intractable probabilistic reasoning problems studied in [11]. However, both problems can actually be solved in polynomial time.

Theorem 2.5. *PArgAtSAT and PArgAtENT can be solved in polynomial time.*

The algorithms and proofs can be found in [23]. What makes PArgAtSAT and PArgAtENT easier than the probabilistic reasoning problems in [11] is that they talk only about atomic beliefs and not about beliefs in complex formulas. This allows us to replace probability functions over an exponential number of possible worlds equivalently with *probability labellings* $L : \mathcal{A} \to [0, 1]$ that assign probabilities to arguments directly. Formally, this equivalence can be shown by defining an equivalence relation over probability functions and defining a bijective mapping from equivalence classes of probability functions to probability labellings. The technical details can be found in [23]. Conceptually similar ideas have been considered in probabilistic-logical reasoning [9, 10, 17, 21]. However, in this area, equivalence relations are introduced over possible worlds and identifying compact representatives for these equivalence classes remains intractable in general [24].

How far can our fragment be extended without loosing polynomial runtime guarantees? Complexity results in [23] show that the fragment cannot be extended very far. It is natural to suspect that k-th order labellings $L : \mathcal{A}^k \to [0, 1]$ that assign probabilities to k-tuples of arguments could be used to work with complex formulas over k arguments efficiently. However, assuming, $P \neq NP$, this idea cannot even be successful for $k = 2$. Intuitively, the labelling approach cannot be applied for $k > 1$ efficiently because an exponential number of marginal consistency constraints is needed in order to establish a useful relationship between k-th order labellings and probability functions, see [23] for the details.

Implementations for solving PArgAtSAT and PArgAtENT can be found in the Java-library ProBabble.[1] Problems with thousands of arguments can usually be solved within a few hundred milliseconds. Without the labelling approach, the same amount of time would be needed for 10-15 arguments already because the number of possible worlds grows exponentially.

[1] https://sourceforge.net/projects/probabble/

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