Complexity of Additive Committee Selection with Outliers

Extended Abstract

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ABSTRACT

We study the φ_f-Outliers problem, where we are given an election and are asked whether there are at most $\bar{n}$ votes whose removal leads to the existence of a k-committee of a desired quality under the voting rule φ_f. We investigate the (parameterized) complexity of φ_f-Outliers for additive k-committee selection rules, in both the general case and several special cases with respect to the incidence graphs of the given elections.

KEYWORDS

multiwinner voting; outliers; parameterized complexity; approval

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1 PRELIMINARIES

Approval-based k-committee selection rules, which aim to select k winners based on the dichotomous preferences of voters over candidates, have received a considerable amount of attention recently [1, 3, 7, 11–13] due to their significant applications in many areas.

Precisely, an *election* is a tuple (C, V) where C is a set of candidates and V is a multiset of votes. Each *vote* is a nonempty subset of C. We say a vote $v \in V$ *approves* a candidate $c \in C$ if $c \in v$. For a set S, 2^S denotes the *power set* of S. A k-*committee* is a subset of C of cardinality k. A *scoring* k-*committee selection rule* maps each election (C, V) to a k-committee. Particularly, let $f : 2^C \times 2^C \rightarrow \mathbb{Q}$ be a scoring function, where for a vote $v \subseteq C$ and a committee $w \subseteq C$, $f(v, w)$ is the *score* of w obtained from v. By a slight abuse of notation, for an election (C, V) and a committee w, let $f(V, w) = \sum_{v \in V} f(v, w)$ be the score of w in (C, V). A *minimizing* k-*committee selection rule* φ_f selects a k-committee with the minimum score, with respect to f, as the winning committee. We say that φ_f is *additive* if for every vote v and every nonempty committee w it holds that $f(v, w) = \sum_{c \in w} f(v, \{c\})$. In this paper, we study only minimizing additive rules that subject to the following constraints. First, for all $v, w, v', w' \subseteq C$ such that $|v| = |v'|$, $|w| = |w'|$, and $|v \cap w| = |v' \cap w'|$ it holds that $f(v, w) = f(v', w')$. For simplicity, we denote by $\mathbf{f}$ a function from $\{0, 1, \ldots, |C|\}^3$ to $\mathbb{Q}$ such that for every $v, w \subseteq C$ it holds that $\mathbf{f}(|v|, |v \cap w|, |w|) = f(v, w)$. We make the assumption that scoring functions f are given as oracles and, moreover, given $v, w \subseteq C$, $f(v, w)$ and $\mathbf{f}(|v|, |v \cap w|, |w|)$ can be returned in polynomial time in $\max\{|v|, |w|\}$. Second, for every vote v, it holds that $\mathbf{f}(|v|, 1, 1) < \mathbf{f}(|v|, 0, 1)$. Therefore, for feasible integers x, y, y', z such that $y > y' \geq 0$ it holds that $\mathbf{f}(x, y, z) < \mathbf{f}(x, y', z)$. All minimizing rules in Table 1 are additive rules fulfilling the above constraints [8].

Table 1: A summary of additive rules. Here, $v \subseteq C$ is a vote and $w \subseteq C$ is a committee. In NSAV, if v approves all candidates, i.e., $v = C$, we remove $\frac{|w \setminus v|}{|C|-|v|}$ (i.e., we take $\frac{|w \setminus v|}{|C|-|v|} = 0$).

voting rules	scoring functions $f(v, w)$	
	maximizing	minimizing
Approval voting (AV)	$\lvert v \cap w\rvert$	$\lvert v \setminus w\rvert + \lvert w \setminus v\rvert$, or, $\lvert w \setminus v\rvert$, or, $\lvert v \setminus w\rvert$
Net-approval voting (NAV)	$\lvert v \cap w\rvert - \lvert w \setminus v\rvert$	$\lvert w \setminus v\rvert - \lvert v \cap w\rvert$
Satisfaction approval voting (SAV)	$\frac{\lvert v \cap w\rvert}{\lvert v\rvert}$	$\frac{\lvert v \setminus w\rvert + \lvert w \setminus v\rvert}{\lvert v\rvert}$, or $\frac{\lvert v \setminus w\rvert}{\lvert v\rvert}$, or $\frac{\lvert w \setminus v\rvert}{\lvert v\rvert}$
Net-SAV (NSAV)	$\frac{\lvert v \cap w\rvert}{\lvert v\rvert} - \frac{\lvert w \setminus v\rvert}{\lvert C\rvert - \lvert v\rvert}$	$\frac{\lvert w \setminus v\rvert}{\lvert C\rvert - \lvert v\rvert} - \frac{\lvert v \cap w\rvert}{\lvert v\rvert}$

We study the problem of committee determination in the presence of outliers which models the scenario where a limited number of voters, called outliers, are needed to be removed in order to find a desired winning k-committee. The formal definition is as follows.

φ_f-Outliers	
Given:	An election (C, V), two nonnegative integers $k \leq \lvert C\rvert$ and $\bar{n} < \lvert V\rvert$, and a rational number t.
Question:	Are there $U \subseteq V$ and $w \subseteq C$ such that $\lvert w\rvert = k$, $\lvert U\rvert \leq \bar{n}$ and $f(V', w) \leq t$, where $V' = V \setminus U$?

The φ_f-Committee Determination problem (φ_f-CD) is a special case of φ_f-Outliers where $\bar{n} = 0$. Throughout this paper, let $m = |C|$, $n = |V|$, $n^* = n - \bar{n}$, and $\bar{k} = m - k$.

The φ_f-Outliers problem was first studied by Dey et al. [4] for minimizing versions of AV and NAV. We study the class of additive rules, which include many important rules not studied in [4]. Moreover, we mainly develop general theorems which hold for almost all

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additive rules. In addition, we explore the parameterized complexity of this problem with respect to some structural parameters of the incidence graphs of elections.

2 RESULTS IN THE GENERAL CASE

For φ_f-CD, polynomial-time algorithms have been developed for some concrete additive rules [2, 4]. We give a general result.

Theorem 2.1. *For an additive rule φ_f, an optimal k-committee with respect to φ_f can be calculated in polynomial time.*

Dey et al. [4] studied φ_f-Outliers for minimizing variants of AV and NAV, and showed FPT results with respect to m and n. We extend their result to all additive rules.

Theorem 2.2. *For an additive rule φ_f, φ_f-Outliers is FPT with respect to both m and n.*

As $\bar{n} + n^* = n$ and $\bar{k} + k = m$, it remains to study the combined parameters $n^* + k$, $n^* + \bar{k}$, $\bar{n} + k$, and $\bar{n} + \bar{k}$. Concerning the parameters $n^* + k$ and $\bar{n} + \bar{k}$, Dey et al. [4] showed that φ_f-Outliers for minimizing variants of AV and NAV is W[1]-hard. In particular, for $n^* + k$ the result holds even when every voter approves only two candidates. However, for the parameter $\bar{n} + \bar{k}$, their reduction does not apply to this special case.

Theorem 2.3. *Let f be an additive function such that for every $k \geq 3$ it holds that $\mathbf{f}(2, 1, k) > 0$. Then, φ_f-Outliers is W[1]-hard with respect to $n^* + k$ and $\bar{n} + \bar{k}$, even when every voter approves at most two candidates.*

The above theorem applies to all rules in Table 1 except NSAV.

Theorem 2.4. *NSAV-Outliers is W[1]-hard with respect to both $\bar{n} + \bar{k}$ and $n^* + k$.*

Regarding $n^* + \bar{k}$ and $\bar{n} + k$, in the W[1]-hardness proof for the minimizing variants of AV in [4], all voters approve the same number of candidates. In this case, the minimizing variants of SAV and AV are equivalent. However, this is not the case for NSAV.

Theorem 2.5. *NSAV-Outliers is W[1]-hard with respect to the parameters $n^* + \bar{k}$ and $\bar{n} + k$.*

Let $\triangle = \max_{v \in V}\{|v|\}$; hence, every voter approves at most $\triangle$ candidates. As shown in Theorem 2.3, φ_f-Outliers cannot be FPT even with respect to the combined parameters $n^*+k+\triangle$ and $\bar{n}+\bar{k}+\triangle$ unless FPT=W[1]. In the following, we show an FPT-algorithm for φ_f-Outliers with respect to the parameter $\bar{n} + k + \triangle$.

Theorem 2.6. *φ_f-Outliers for additive rules φ_f is FPT with respect to the combined parameter $\bar{n} + k + \triangle$.*

3 RESTRICTED INCIDENCE GRAPHS

For an election $E = (C, V)$, its *incidence graph* $G_E = (C \cup V, A)$ is a bipartite graph with the vertex bipartition (C, V) and edge set $A = \{(c, v) \mid c \in C, v \in V, c \in v\}$. We study some structural parameters of G_E. First, we have the following result.

Theorem 3.1. *For an additive rule φ_f, φ_f-Outliers is FPT with respect to the size of the maximum matching of the incidence graph of the given election.*

Now we identify several special classes of incidence graphs restricted to which φ_f-Outliers is polynomial-time solvable. For two graphs H and H', we say that H is *H'-free* if H contains no induced subgraph that is isomorphic to H'. For two graph classes $\mathcal{H}$ and $\mathcal{H}'$, $\mathcal{H}$ is *$\mathcal{H}'$-free* if none of $\mathcal{H}$ has an induced subgraph isomorphic to some graph in $\mathcal{H}'$. A *star* with r leaves is denoted by $K_{1,r}$, and a *path* of length r (the number of vertices in it) is denoted by P_r.

Theorem 3.2. *φ_f-Outliers is polynomial-time solvable if the incidence graph of the given election is $K_{1,3}$-free.*

One may wonder whether we can extend the above result to $K_{1,r}$-free incidence graphs for some other constants $r \geq 4$. Unfortunately, we show that this is not the case.

Theorem 3.3. *For an additive function f such that $\mathbf{f}(2, 1, k) > 0$ for every $k \geq 3$, φ_f-Outliers is NP-hard even if the incidence graph of the given election is $K_{1,4}$-free.*

Based on the following lemma, we show that φ_f-Outliers for many additive rules is polynomial-time solvable when the incidence graph is P_5-free. A *non-trivial connected component* is a connected component with at least two vertices.

Lemma 3.4. *Let $G = (C \cup V, A)$ be a P_5-free bipartite graph, and H a non-trivial connected component of G. Then, there is an order $(c_1, \ldots, c_x)$ of the candidates in H such that for each $v \in V$ there is a positive integer $\tau(v) \leq x$ such that v is adjacent to c_i if and only if $1 \leq i \leq \tau(v)$. Moreover, such an order can be found in polynomial time.*

In fact, the above lemma shows that any election whose incidence graph is P_5-free is a special case of the so-called *Candidate Interval* (CI) election studied in the literature [5, 6, 9, 10].

Theorem 3.5. *φ_f-Outliers is polynomial-time solvable if the incidence graph is P_5-free.*

4 A GENERAL FPT RESULT

For a graph class $\mathcal{H}$, let $\mathrm{F}_{\mathcal{H}}$ be the class of all $\mathcal{H}$-free graphs. For an integer $r > 0$, let $\mathrm{F}^{r+}_{\mathcal{H}}$ be the class of graphs which include at most r induced subgraphs isomorphic to graphs in $\mathcal{H}$ in total. φ_f-Outliers restricted to a graph class $\mathcal{G}$ means that the incidence graphs of the given elections are in $\mathcal{G}$.

Theorem 4.1. *Let $\mathcal{H}$ be a set consisting of a constant number of graphs each of which contains at least one edge and at most d vertices, where $d \geq 2$ is a constant. If φ_f-Outliers restricted to $\mathrm{F}_{\mathcal{H}}$ is polynomial-time solvable for all additive rules φ_f, then, φ_f-Outliers restricted to $\mathrm{F}^{r+}_{\mathcal{H}}$ is FPT for all additive rules φ_f, with respect to r.*

Due to Theorems 3.2, 3.5, and 4.1, we have the following result.

Corollary 4.2. *For an additive rule φ_f, φ_f-Outliers is FPT with respect to the number of induced claw/P_5 in the incidence graph of the given election.*

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