Strategic Location and Network Formation Games

Doctoral Consortium

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ABSTRACT

Game theory is a powerful approach to analyze settings which result from selfish behavior. This research aims to investigate models from different game theory areas: Network Creation Games, Schellings Segregation and Strategic Facility Location. I propose extensions of the classical simple models with more realistic assumptions and to analyze these models with respect to the existence of stable states, the Price of Anarchy and convergence dynamics.

KEYWORDS

Location Analysis, Facility Location Games, Approximate Pure Subgame Perfect Equilibria, Agent-based Simulation, Dynamics

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1 INTRODUCTION

Many real-world phenomena result from individual behavior which is not coordinated by a central authority. My thesis is inspired by the following examples of such phenomena: (i) The Internet was not centrally designed and optimized but it emerged from the interaction of many selfish agents, e.g. ISPs forming the AS level network via bilateral peering agreements [14]. (ii) Residential segregation as observed in many US urban areas [1] where residential patterns emerge via selfish location choices by the city residents. (iii) Co-Location of competing facilities in spatial markets. Competing firms selfishly decide on a profitable location for opening their facilities and this results in the so called principle of minimum differentiation [6], which means that firms tend to co-locate. In all of these settings selfish agents interact to optimize their individual situation. The combination of the strategic choices of the involved agents then defines the observed state in the real world.
A common concept modeling outcomes is the pure Nash Equilibrium. There no player can unilaterally improve on her current situation by a strategy change. Hence, these states are stable and yield a good prediction for the outcome of complex social interaction.

An important assumption is that agents act according to consistent and stable preferences and choose their individual best alternative from a set of feasible options. The interests and goals of an agent are modeled via a cost or utility function which evaluates the current state of the game from the agent's perspective.

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Game theory offers a wide range of useful models and methods to analyze the above settings [13]. Especially non-cooperative game theory provides general mathematical techniques for analyzing situations in which two or more rational and selfish agents make decisions that will influence the welfare of all agents. "Rational" agents act in a consistent way according to their own interests. "Selfish" emphasizes that an agent acts egoistic and primarily to achieve her own goals. Core questions in the field, which I plan to answer in my thesis, are: (i) Quantifying the quality of the achieved outcomes compared with a centrally coordinated solution, measured by the so called Price of Anarchy which denotes the impact of selfish behavior and the lack of central coordination on the underlying optimization problem. (ii) Analyzing the process of reaching an equilibrium state, i.e. analyzing the game dynamics. Here the main interest is to determine if a potential function exists which implies that the game reaches a stable state by repeated strategy changes from the agents. (iii) Determining whether pure Nash equilibria exist at all. In many strategic games it can be shown that pure Nash equilibria do not exist. In this case another solution concept is needed for predicting the agents' behavior. A natural candidate for this are approximate pure Nash equilibria, where agents can only improve by a tiny margin by unilaterally changing their strategies.

1.1 Current Research

Network Creation Games. Game-theoretic models for network creation, where agents are associated to nodes of a network and choose their neighbors selfishly, yield interesting insights into the structure and evolution of complex networks. Based on the original model by Fabrikant et al. [7] many variants have been introduced.

Inspired by social networks where nodes have different levels of popularity proportional to their degree, we assume that establishing a link to a popular high degree node is more expensive than to an unimportant low degree node. So the cost of an edge is proportional to the degree of the connected node. However an agent wants to be central in the network. Hence, the cost function of an agent in the network consists of the sum of edge costs for all edges owned by the agent and her centrality measured by the distance cost.

We show in [2] that the Price of Anarchy is constant, but also that the game does not always converge to an equilibrium and may never reach a stable state, even though there always exists a Nash equilibrium. Variations of this game where agents can only buy edges in their local 2-neighborhood or are not allowed to delete edges change the properties, so that for instance the upper bound for the Price of Anarchy is the diameter of the reached network.

Schelling Segregation. A different agent-based approach is that the structure of the network is fixed and instead of choosing the neighbors an agent decides for an neighborhood she wants to live in.

The landmark model in sociology for this setting is the segregation model by Schelling [16]. In this very simple and elegant agent-based model two types of agents are placed on a line or a grid which models some residential area. Each agent is aware of her neighboring agents and is content with her current residential position if at least a τ fraction of agents in her neighborhood is of the same type, for some $0 \leq \tau \leq 1$. If this condition is not met, then the agent becomes discontent with her current position and tries to find a randomly chosen new spot. Schelling's model shows the counter-intuitive phenomenon that residential segregation between individuals of different groups can emerge even if all involved individuals are tolerant, i.e., for $\tau \leq 0.5$. Although the model is widely studied, no pure game-theoretic version existed.

We closed this gap by introducing and analyzing a generalized game-theoretic model [3] where agents choose strategically their position via swapping with another agent or via jumping to an empty spot. An agent's high priority goal is to find a location where she is happy, i.e. where she has at least a τ fraction of similar neighbors. One extension of this is that agents additionally also have a desire to be close to a certain location. We introduced and explored the influence of such individual location preferences.

We investigate the model via agent-based simulation and theoretical analysis. Our simulations showed that Schelling's model with strategic agents yields similar results than the original version, i.e., they also show the emergence of residential segregation. We analyze the convergence properties of many variants of our model and show that if agents are tolerant, i.e. $\tau \leq 0.5$, are only allowed to swap and do not have individual location preferences in the network, convergence is guaranteed. Moreover, also the swap game, where agents actually care about their favorite spots behaves nicely on regular networks. In contrast, versions where agents jump to empty spots behave different, since improving response cycles exist and therefore it is not guaranteed that the game ends up in a stable placement, i.e. a pure Nash equilibrium. Furthermore, there exist stable placements for many variations. It is possible, that in a stable placement all agents of one type are unhappy with their current neighborhood but have no opportunity to improve their situation.

Strategic Facility Location. Besides the similarity to her neighbors an agent can have different reasons why she chooses a certain position in a network. Another aspect is how profitable the current position is. The Hotelling-Downs model [4, 11] has been applied to many problems: spatial competition, competition via product differentiation, political competition via selecting a political agenda along a linear spectrum. It assumes a market where infinitely many customers are distributed along the line. Finitely many firms strategically choose shop locations. Assume the products of the firms are equal and customers always go to the closer shop. The utility of the firms is proportional to the number of clients visiting their facility. This setting allows a prediction of real-world phenomena like the principle of minimum differentiation. It states that similar competing firms tend to co-locate their shops instead of spreading them evenly along the market. This can often be observed, e.g. stores of different fast-food chains are located right next to each other.
The work of Hotelling and Downs encouraged a number of extensions. It is obvious that a multitude of factors influence the choice of a location. Real clients would not only evaluate distances but also how congested a facility is. Many other clients visiting the same store induce a higher waiting time. This more realistic variant was proposed by Kohlberg [12]. We [9] answer the question how a client social optimum looks like and how tolerant the facility agents have to be, to accept the social optimum placement for the customers.

However, pure equilibria exist only in particular cases and if and only if there is an even number of facilities [15]. But even if no equilibrium is achievable the concept of approximate pure subgame perfect equilibria is appropriate to study how much an agent can improve by changing her strategy. In the real world it is not true, that actors radically change their current strategy even if they can improve only by a tiny margin. Hence, also approximate equilibria can make a valid prediction for real world scenarios. We showed analytically and by extensive agent-based simulations that for every cost function which is a linear combination of distances and congestion, there exist approximate subgame perfect equilibria which respect the principle of minimum differentiation and where each firm can only increase by a very small multiplicative factor. This is in contrast to results for pure equilibria which indicates that studying approximate equilibria may yield more realistic results than solely focusing on exact equilibria and may lead to new insights for other models in the realm of Location Analysis. Furthermore these placements are also socially beneficial for all clients.

1.2 Future Research

Since the current models are somewhat artificial, I want to augment them with more realistic assumptions about the agents' behavior and analyze this influence on the properties of the predicted outcomes. Moreover, combining these models establishes new possibilities. My methodological plan to achieve this is via a combination of theoretical analysis and agent-based simulations. The following ideas are interesting research directions.

In Schelling's segregation model the underlying network is fixed. A more realistic assumption is to pick up the idea of [10] where agents have a small aversion from being connected to others who are dissimilar to themselves and therefore are able to swap their edges. To investigate this model from a network creation game perspective and combine network creation games with Schelling's segregation model could yield to interesting new insights.

Related to Schelling's segregation, there exists a voter system [5] where an agent examines her neighbors and if a certain threshold is of another type she changes her type. In this model segregation is visible and could be a promising idea for further research.

As already mentioned, in reality clients choose a location due to multiple factors. For instance, clients may disavow all facilities if no facility is close enough to the client preferences [8]. If there are multiple candidates which are close enough, clients can evaluate distances and how congested the facility is or can have a probability distribution over the facilities. Inspired by Schelling's segregation, clients may choose a facility which is also chosen by other clients which are similar. Another approach how we can continue this work is to refrain from the continuous version and the line and analyze our model in the discrete version and on general networks, respectively. It also may not hold in reality that clients have complete information about the facilities, hence this should be incorporated in the model.

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