Parameterized Complexity of Committee Elections with Dichotomous and Trichotomous Votes

Aizhong Zhou
School of Computer Science and Technology, Shandong University
Jinan, China
azzhou@mail.sdu.edu.cn

Yongjie Yang
Central South University
Changsha, China
yyongjiecs@gmail.com

Jiong Guo
School of Computer Science and Technology, Shandong University
Jinan, China
jguo@sdu.edu.cn

ABSTRACT

We study the winner determination problem for three prevalent committee election rules: Chamberlin-Courant Approval Voting (CCA), Proportional Approval Voting (PAV), and Satisfaction Approval Voting (SAV). Axiomatic and algorithmic studies of elections under these rules have been conducted recently. It is known that the winner determination problem is NP-hard for CCA and PAV and polynomial-time solvable for SAV, if the input votes are dichotomous. Moreover, parameterized complexity of the two NP-hard cases has been examined with respect to some natural parameters such as the number of candidates or the number of votes. In this paper, we extend the above studies to committee elections with trichotomous votes and identify cases, where trichotomous votes lead to an increase of parameterized complexity. We also consider the maximin (or egalitarian) variations of the rules, where the minimum satisfaction of the voters is maximized.

KEYWORDS

Committee elections, parameterized complexity, maximin

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1 INTRODUCTION

The problem of aggregating the preferences of different agents (voters) occurs in diverse situations and plays a fundamental role in artificial intelligence and social choice [10, 17]. While the most studied setting is the single-winner case, voting can also be used to elect a fixed-size set of winners (multi-winner), called committee. Recently, we witness an increasing interest in the study of the axiomatic and algorithmic aspects of committee elections [2, 3, 5]. Multi-winner voting rules have received a considerable amount of attention, due to their significant applications in social choice [22, 29, 30, 33]. One of the most prominent voting rules for committee elections is the so-called Approving Voting (AV) rule, which is originally defined for dichotomous votes. In a dichotomous vote, the voter assigns an approval to each of her favorite candidates and all other candidates receive disapproval. Then, AV picks the top k candidates, who receive the most approvals, to form the committee. The parameter k denotes the size of the committee and these k candidates are the winners of the election. Thus, AV can be considered as one of the maxisum (or utilitarian) approaches.

Although AV demonstrates many desirable properties in the single-winner case, it becomes less favorite in the committee elections, due to the lack of egalitarianism [9]. Therefore, various variations of AV have been introduced in the literature [25], for example, the Minisum and Minimax Approval Voting introduced by Brams et al. [12–14]. Here, the dichotomous votes are represented as $\{0, 1\}$-vectors, where 0 stands for disapproval and 1 for approval, while a $\{0, 1\}$-vector with exactly k 1-entries can be used to represent a committee of size k. Minisum AV seeks for a size-k committee minimizing the total dissatisfaction of the votes. The dissatisfaction between a vote and a committee is measured as the Hamming distance between the corresponding vectors. In contrast, Minimax AV seeks for a committee minimizing the maximum Hamming distance between the committee and the votes. It is known that Minisum AV can be solved in polynomial time, while Minimax AV is NP-hard [26].

We consider here three AV variations following the maxisum approach, that is, the variations pick size-k committees maximizing the total “satisfaction score" over all voters. Under the Chamberlin-Courant Approval Voting (CCA) rule [15], the satisfaction score of a committee W with respect to a voter is set to 1, if W contains at least t favorite candidates of this voter for a given bound $t > 0$; otherwise, it is set to 0.[1] The Proportional Approval Voting (PAV) rule [5] defines the satisfaction score of a committee W with respect to a voter as an increasing function depending on the number of candidates in W, who are approved by this voter. However, the increasing rate of the satisfaction score decreases as the number of such candidates increases. The Satisfaction Approval Voting (SAV) [11] rule defines the satisfaction score of a committee W with respect to a voter based on the ratio of the candidates, who are approved by this voter and appear in W, to the total number of candidates approved by this voter. For formal definitions of these rules we refer to Section 2. It is known that the winner determination problem, which asks for a size-k committee with a total satisfaction score over a given bound, is polynomial-time solvable for SAV [32], but becomes NP-hard for CCA [23] and PAV [32].

In 2015, Baumeister and Dennisen [6] extended the study of AV and its variations to other forms of votes such as trichotomous votes, linear orders, and partial orders. A voting with trichotomous votes allows the voters in addition to approval and disapproval to abstain

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[1]Note that in the literature, CCA is usually defined with $t = 1$. We consider the general case with $t > 0$ to investigate the influence of t on the complexity of committee elections under CCA. t-CCA is a special case of the threshold rule proposed by Fishburn et al. [24].

for a candidate, which could be of particular interest in many real-world applications such as multiple referenda elections. If we use a $\{1,0\}$-vector over the candidates to denote a dichotomous vote, where 1's stand for approved candidates and 0's stand for disapproved candidates, then a trichotomous vote can be represented as a $\{1,-1,0\}$-vector over the candidates, where 1's and -1's stand for approved and disapproved candidates, and 0's stand for abstentions. Clearly, dichotomous votes represent a special case of trichotomous votes. Baumeister et al. [7] investigated the classical complexity of Minisum AV and Minimax AV with trichotomous votes, and proved polynomial-time solvability of Minisum AV and NP-hardness of Minimax AV. Alcantud and Laruelle [1] investigated a variant of AV with trichotomous votes, which shares some common features with CCA, PAV, and SAV and selects the candidates, who obtain the largest difference between the number of "positive" votes and the number of "negative" votes.

Recently, parameterized complexity of committee elections has been extensively studied with the number of votes, the number of candidates, or the committee size as parameter. Betzler et al. [8] introduced the minimax version of a CCA-related rule and provided both fixed-parameter tractability (FPT) and W[2]-hardness results for the above three parameterizations. As for the Minimax AV with dichotomous votes, Misra et al. [31] presented parameterized and kernelization algorithms. Assuming the Exponential Time Hypothesis (ETH), Cygan et al. [19] showed that the FPT-algorithm by Misra et al. [31] is essentially tight. In addition, they also developed parameterized approximation scheme for minimax AV [19]. Aziz et al. [4] considered a Borda-based egalitarian committee election and achieved both FPT and W[1]-hardness results. Liu and Guo [28] studied the parameterized complexity of the winner determination problem for AV with trichotomous votes with respect to parameters such as the size of committee, the number of votes, or the number of candidates. Yang and Wang [34] showed that the winner determination problems for CCA and PAV are FPT with respect to the number of candidates or the number of votes, but become W[2]-hard and W[1]-hard with respect to the committee size.

In this paper, we continue this line of research and study the parameterized complexity of the winner determination problem for CCA and PAV with trichotomous votes. In addition, we define the "maximin" versions of CCA, PAV, and SAV on dichotomous and trichotomous votes, which pick committees maximizing, instead of the total satisfaction score over all votes, the minimum satisfaction score of the votes.[2] We derive parameterized complexity results of the winner determination problem under these maximin rules. The formal definitions of the maximin rules can be found in Section 2 and Tables 1 and 2 give an overview of the results achieved for these rules.

2 PRELIMINARIES

An election is denoted as a pair $E = (C, V)$, where C is the set of candidates $C = \{c_1, c_2, \ldots, c_m\}$ and V is the multiset of votes $V = \{v_1, v_2, \ldots, v_n\}$. Each vote v_i is defined as a length-$|C|$ vector, whose j-th position is denoted as v_{ij}. Let k be a positive integer with $k \leq |C|$. A *k-committee election rule* maps an election to a subset of candidates $W \subseteq C$ such that $|W| = k$. The subset W is called a *k-committee*.

[2] Aziz et al. [4] use the term "utilitarian rules" to denote the maxisum versions we considered here, while the maximin versions are called "egalitarian rules".

2.1 Dichotomous votes

Given a dichotomous vote v_i, its j-th position v_{ij} satisfies $v_{ij} \in \{1, 0\}$, where $v_{ij} = 1$ (or 0) means that the candidate c_j is approved (or disapproved) by v_i. For a vote $v_i \in V$, let $v_i^{[1]}$ be the set of candidates approved by v_i, and $v_i^{[0]}$ the set of candidates disapproved by v_i. In the following, let t be a positive rational number.

The Chamberlin-Courant Approval Voting (CCA) rule aims at finding a k-committee that satisfies as many votes as possible. Concretely, a vote v_i is satisfied by a committee W, if and only if at least t approval candidates of v_i are contained in W. Here, t is a given bound. We define the CCA-score of W with respect to v_i as

$$\mathsf{CCA}(v_i, W) = \begin{cases} 1, & |v_i^{[1]} \cap W| \geq t, \\ 0, & \text{otherwise.} \end{cases}$$

The total CCA-score of W is then defined as

$$\mathsf{CCA}(V, W) = \sum_{v_i \in V} \mathsf{CCA}(v_i, W).$$

Given an election, the CCA rule picks a committee W maximizing $\mathsf{CCA}(V, W)$. The maximin version of the CCA rule is called the Maximin Chamberlin-Courant Approval Voting (MCCA) rule, which maps an election to a k-committee W, which satisfies every vote. We set the MCCA-score of a committee W with respect to v_i to $\mathsf{MCCA}(v_i, W) = |v_i^{[1]} \cap W|$.

The Proportional Approval Voting (PAV) rule aims at finding a k-committee W maximizing the total PAV-score. The PAV-score of W with respect to a vote v_i is set to

$$\mathsf{PAV}(v_i, W) = 1 + \frac{1}{2} + \ldots + \frac{1}{|v_i^{[1]} \cap W|}.$$

The total PAV-score of W is then

$$\mathsf{PAV}(V, W) = \sum_{v_i \in V} \mathsf{PAV}(v_i, W).$$

Similarly, we can define the maximin version of PAV, called Maximin Proportional Approval Voting (MPAV). MPAV maps an election to a k-committee W, such that for each vote v_i, we have $\mathsf{PAV}(v_i, W) \geq t$. We define the MPAV-score of W with respect to v_i as $\mathsf{MPAV}(v_i, W) = \mathsf{PAV}(v_i, W)$.

The Satisfaction Approval Voting (SAV) rule selects a k-committee W maximizing the total SAV-score. The SAV-score of a committee W with respect to a vote v_i is set to

$$\mathsf{SAV}(v_i, W) = \frac{|v_i^{[1]} \cap W|}{|v_i^{[1]}|},$$

while the total SAV-score of W is then

$$\mathsf{SAV}(V, W) = \sum_{v_i \in V} \mathsf{SAV}(v_i, W).$$

Similarly, we can define the maximin version of SAV, Maximin Satisfaction Approval Voting (MSAV). The MSAV-score of a committee W with respect to a vote v_i is defined as $\mathsf{MSAV}(v_i, W) = \mathsf{SAV}(v_i, W)$.

Table 1: Parameterized complexity results for τ-WD. The FPT results with respect to m are trivial. TSAV-WD can be solved in a similar way as SAV-WD. Our results are in bold.

τ	CCA	PAV	SAV	TCCA	TPAV	TSAV
n	FPT[34]	FPT[34]	P[11]	**FPT**[Prop. 3.3]	OPEN	P
m	FPT [34]	FPT [34]	P [11]	**FPT**	**FPT**	P
k	W[2]-h[34]	W[1]-h[34]	P[11]	**W[2]-h**[34]	**W[1]-h**[34]	P
d	FPT[34]	OPEN	P[11]	**W[2]-h**[Thm. 3.10]	**Para-NP-h**[Thm. 3.11]	P

Table 2: Parameterized complexity results for the maximin versions of τ-WD. The FPT results with respect to m are trivial. "Para-NP-h" means that the problem is NP-hard, even if the corresponding parameter is a constant. Our results in bold.

τ	MCCA	MPAV	MSAV	MTCCA	MTPAV	MTSAV
n	**FPT** [Prop. 3.2]	**FPT** [Thm. 3.4]	**FPT** [Prop. 3.2]	**FPT** [Thm. 3.1]	**FPT** [Thm. 3.4]	**FPT** [Thm. 3.1]
m	**FPT**	**FPT**	**FPT**	**FPT**	**FPT**	**FPT**
k	**W[2]-h** [Thm. 3.5]	**W[2]-h** [Thm. 3.5]	**W[2]-h** [Thm. 3.9]	**W[2]-h** [Prop. 3.6]	**W[2]-h** [Prop. 3.6]	**W[2]-h** [Thm. 3.9]
t	**Para-NP-h** [Thm.3.5]	**Para-NP-h** [Thm.3.5]	**Para-NP-h** [Thm.3.7]	**Para-NP-h** [Prop. 3.6]	**Para-NP-h** [Prop. 3.6]	**Para-NP-h** [Prop. 3.8]

2.2 Trichotomous votes

Given a trichotomous vote v_i, its j-th position v_{ij} satisfies $v_{ij} \in \{1, 0, -1\}$, where $v_{ij} = 1$ (or $v_{ij} = -1$) means that the candidate c_j is approved (or disapproved) by v_i and $v_{ij} = 0$ means that v_i abstains with respect to c_j. For a vote $v_i \in V$, let $v_i^{[1]}$ be the set of candidates approved by v_i, $v_i^{[-1]}$ the set of candidates disapproved by v_i, and $v_i^{[0]}$ the set of candidates to whom v_i abstains. Dichotomous votes are clearly special trichotomous votes with no abstention.

We define the variation of CCA for trichotomous votes, called Trichotomous Chamberlin-Courant Approval Voting (TCCA), which aims at finding a k-committee W that satisfies as many votes as possible. We define TCCA-score of W with respect to a vote v_i as

$$\mathsf{TCCA}(v_i, W) = \begin{cases} 1, & |v_i^{[1]} \cap W| - |v_i^{[-1]} \cap W| \geq t, \\ 0, & \text{otherwise.} \end{cases}$$

The total TCCA-score of W is set to

$$\mathsf{TCCA}(V, W) = \sum_{v_i \in V} \mathsf{TCCA}(v_i, W).$$

The maximin version of TCCA is called Maximin Trichotomous Chamberlin-Courant Approval Voting (MTCCA). MTCCA finds a k-committee W such that for each vote v_i, we have $|v_i^{[1]} \cap W| - |v_i^{[-1]} \cap W| \geq t$. Thus, the MTCCA-score of W with respect to a vote v_i is set to

$$\mathsf{MTCCA}(v_i, W) = |v_i^{[1]} \cap W| - |v_i^{[-1]} \cap W|.$$

For the definition of the version of PAV for trichotomous votes, called Trichotomous Proportional Approval Voting (TPAV), we set the TPAV-score of a committee W with respect to a vote v_i as

$$\mathsf{TPAV}(v_i, W) = \mathsf{PAV}(v_i^{[1]}, W) - \mathsf{PAV}(v_i^{[-1]}, W).$$

The total TPAV-score of W is

$$\mathsf{TPAV}(V, W) = \sum_{v_i \in V} \mathsf{TPAV}(v_i, W).$$

Thus, TPAV returns a k-committee W maximizing $\mathsf{TPAV}(V, W)$. Similarly, we call the maximin version of TPAV with trichotomous votes as Maximin Trichotomous Proportional Approval Voting (MTPAV). MTPAV finds a k-committee W such that for each vote v_i we have $\mathsf{TPAV}(v_i, W) \geq t$. The MTPAV-score of W with respect to v_i is set to $\mathsf{MTPAV}(v_i, W) = \mathsf{TPAV}(v_i, W)$.

Finally, we define the version of SAV with trichotomous votes, called Trichotomous Satisfaction Approval Voting (TSAV), which aims at selecting a k-committee maximizing the total TSAV-score. The TSAV-score of a committee W with respect to a vote v_i is set to

$$\mathsf{TSAV}(v_i, W) = \frac{|v_i^{[1]} \cap W|}{|v_i^{[1]}|} - \frac{|v_i^{[-1]} \cap W|}{|v_i^{[-1]}|}.$$

The total TSAV-score of W is then

$$\mathsf{TSAV}(V, W) = \sum_{v_i \in V} \mathsf{TSAV}(v_i, W).$$

Similarly, we define the maximin version of TSAV, Maximin Trichotomous Satisfaction Approval Voting (MTSAV). MTSAV finds a k-committee W such that for each vote v_i, we have $\mathsf{TSAV}(v_i, W) \geq t$. The MTSAV-score of W with respect to a vote v_i is set to $\mathsf{MTSAV}(v_i, W) = \mathsf{TSAV}(v_i, W)$.

2.3 Winner determination

We have now all tools to define the central problem of this paper, called Winner Determination for τ (τ-WD), where τ denotes the rules. Here, we distinguish between the maxisum and maximin rules.

We first define τ-WD for the maxisum rules, namely, $\tau \in$ {CCA, PAV, SAV, TCCA, TPAV, TSAV}.

> **Winner Determination for τ (τ-WD)**
> **Input**: An election $E = (C, V)$, one positive integers $k \leq |C|$, and two positive rational numbers t and d.
> **Question**: Is there a committee $W \subseteq C$ with $|W| = k$ satisfying $\tau(V, W) \geq d$?

Next, we define τ'-WD for $\tau' \in$ {MCCA, MPAV, MSAV, MTCCA, MTPAV, MTSAV}.

> **Winner Determination for τ' (τ'-WD)**
> **Input**: An election $E = (C, V)$, a positive integer $k \leq |C|$, and one positive rational number t.
> **Question**: Is there a committee $W \subseteq C$ with $|W| = k$ satisfying $\tau'(v_i, W) \geq t$ for each $v_i \in V$?

In this paper, we consider the following parameters: $m = |C|$, $n = |V|$, k, d (called the total satisfaction bound), t (called the individual satisfaction bound). Note that the parameterization with d is invalid for the maximin rules. Accordingly, we do not consider the parameterization with t for the maxisum rules.

2.4 Parameterized Complexity

Parameterized complexity allows to give a more refined analysis of computational problems and in particular, can provide a deep exploration of the connection between the problem complexity and various problem specific parameters. Misra et al. [31] initialized the study of parameterized complexity of approval voting. Yang and Wang [34] showed that CCA-WD and PAV-WD are fixed-parameter tractable (FPT) with respect to the number of candidates m or the number of votes n, but become W[2]-hard and W[1]-hard with respect to the committee size k. An FPT problem admits an $O(f(k) \cdot |I|^{O(1)})$-time algorithm, where I denotes the whole input instance, k is the parameter, and f can be any computable function. Fixed-parameter intractability problems can be classified into many complexity classes, where the most popular ones are W[1]-hard and W[2]-hard. For more details on parameterized complexity, we refer to [18, 21].

3 OUR RESULTS

3.1 Parameter: n

We first consider the parameterization with respect to the number of votes n. Note that CCA-WD and PAV-WD are FPT with this parameterization [34].

Theorem 3.1. *MTCCA-WD and MTSAV-WD are FPT with respect to n.*

Proof. Recall that MTCCA-WD seeks for a size-k subset $W \subseteq C$ such that for each vote it holds $|v_i^{[1]} \cap W| - |v_i^{[-1]} \cap W| \geq t$.

We use an $n \times m$-matrix M to represent all votes, where each of the m columns can be considered as an element in $\{1, 0, -1\}^n$. Thus, there are at most 3^n different column types. Let H denote the set of all column types, and for each type $h \in H$, let n_h be the number of columns in M of type h. Further, let $\chi_{h,i} \in \{1, 0, -1\}$ denote the value at position i of a given column type h.

We transform the given MTCCA-WD instance into an integer linear program (ILP). The instance of ILP is defined over a set of $|H| \leq 3^n$ variables, one variable x_h for each column type $h \in H$. Hereby, $x_h = c$ for an integer $c \geq 0$ means that the k-committee W sought for contains c candidates, whose corresponding columns in M are of type h. The ILP instance then consists of the following constraints:

$$\begin{aligned}
\sum_{h \in H} \chi_{h,i} \cdot x_h &\geq t, && \forall 1 \leq i \leq n,\\
x_h &\leq n_h, && \forall h \in H,\\
\sum_{h \in H} x_h &= k,\\
x_h &\in \{0, 1, 2, \ldots, k\}, && \forall h \in H.
\end{aligned}$$

The equivalence between the above ILP instance and the original MTCCA-WD instance can be proven by the following argumentation. Given a solution of ILP, for each variable x_h, we add x_h many candidates to the committee W, whose corresponding columns in M are of type h. The first inequality guarantees that for each vote v_i, the committee W contains at least t more $v_i^{[1]}$-candidates than $v_i^{[-1]}$-candidates. The second inequality means that the number of candidates in W, whose corresponding columns in M are of type h, is upper-bounded by the number of type-h columns in M. And, the third equality makes sure that W contains exactly k candidates. The direction of constructing a solution for the ILP instance from a solution of the MTCCA-WD instance can be shown in a similar manner.

Similarly, we can construct an ILP instance with $|H| \leq 3^n$ variables for each MTSAV-WD instance. Note that MTSAV-WD seeks a size-k subset $W \subseteq C$ such that for each vote v_i, it holds $\mathsf{TSAV}(v_i, W) \geq t$. For each $h \in H$ and $1 \leq i \leq n$, set

$$S(h, i) = \begin{cases} \frac{1}{|v_i^{[1]}|}, & \text{if } \chi_{h,i} = 1,\\ -\frac{1}{|v_i^{[-1]}|}, & \text{if } \chi_{h,i} = -1,\\ 0, & \text{if } \chi_{h,i} = 0. \end{cases}$$

The value of $S(h, i)$ means that if the committee W contains a candidate with the corresponding column in M being of type h, then this candidate contributes $S(h, i)$ to the TSAV-score of W with respect to vote v_i. We construct the following ILP instance.

$$\begin{aligned}
\sum_{h \in H} S(h, i) \cdot x_h &\geq t, && \forall 1 \leq i \leq n,\\
x_h &\leq n_h, && \forall h \in H,\\
\sum_{h \in H} x_h &= k,\\
x_h &\in \{0, 1, 2, \ldots, n\}, && \forall h \in H,
\end{aligned}$$

Given the meaning of $S(h, i)$, it is easy to observe that the first inequality guarantees $\mathsf{TSAV}(v_i, W) \geq t$ for each vote v_i. Therefore, in a similar way, we can show that the ILP instance has a feasible solution, if and only if the original MTSAV-WD instance has a solution. Then, by the result of Lenstra [27], MTCCA-WD and MTSAV-WD are FPT with respect to n. □

Since dichotomous votes represent a special case of trichotomous votes, the above theorem directly implies the following proposition.

Proposition 3.2. *MCCA-WD and MSAV-WD are FPT with respect to n.*

By slightly modifying the above ILP instance for MTCCA-WD, we can show the following proposition.

Proposition 3.3. *TCCA-WD is FPT with respect to n.*

By similar approaches, we can show FPT results for MPAV-WD and MTPAV-WD with parameter n as well.

Theorem 3.4. *MPAV-WD and MTPAV-WD are FPT with respect to n.*

3.2 Parameter: t

In the following, we consider the case with t as parameter. Here we can show NP-hardness with t being a constant.

Theorem 3.5. *For every positive constant t, MCCA-WD and MPAV-WD are NP-hard and W[2]-hard with k as parameter.*

Proof. We prove the claim only for $0 < t \leq 1$. For the case with greater values of t, we can show the NP-hardness by a reduction from the case with $0 < t \leq 1$ with some additional candidates.

With $0 < t \leq 1$, a committee W satisfies a vote v_i with $|v_i^{[1]} \cap W| \geq t$ if and only if the inequality $\mathsf{PAV}(v_i^{[1]}, W) \geq t$ holds. Thus, if $0 < t \leq 1$, then MCCA-WD is equivalent to MPAV-WD. In the following, we show the claim only for MCCA-WD. We prove the theorem by reducing Dominating Set to MCCA-WD. A dominating set of an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a set $D \subseteq \mathcal{V}$ such that every vertex $v_i' \in \mathcal{V}$ is adjacent to a vertex of D or $v_i' \in D$. Given an instance $(\mathcal{G} = (\mathcal{V} = \{v_1', v_2', ..., v_n'\}, \mathcal{E}), k')$, Dominating Set asks for a dominating set D with $|D| \leq k'$. Dominating Set is a well-known NP-hard problem and W[2]-hard with k' as parameter [20]. We construct an MCCA-WD instance $E = (C, V, k, t)$ as follows.

For each $v_i' \in \mathcal{V}$, we add one candidate c_i to C and one vote v_i to V. If there is an edge between v_i' and v_j', then the corresponding votes v_i and v_j satisfy $v_{ij} = v_{ji} = 1$. In addition, we set $v_{ii} = v_{jj} = 1$. All other positions of the votes are set to 0. In $\mathcal{G}$, let $N(v_i')$ denote the set of vertices that are adjacent to v_i'. We call the vertices in $N(v_i')$ the neighbors of the v_i'. Finally, set $k := k'$. In the following, we show that the Dominating Set instance has a size-k' dominating set, if and only if there is a size-k subset $W \subseteq C$ such that all votes v_i satisfy $|v_i^{[1]} \cap W| \geq t$.

"$\Longrightarrow$": Suppose that there exists a size-k' dominating set D in $\mathcal{G}$. Each vertex $v_i' \in \mathcal{V}$ is dominated by a vertex $v_j' \in D$, that is, $v_i' = v_j'$ or $v_i' \in N(v_j')$. We set W to be the set of candidates, who correspond to the vertices in D. It means that for each vertex $v_i' \in \mathcal{V}$, its corresponding candidate c_i or the candidate c_j corresponding to v_j' is in W. Since for each vote v_i, the set $v_i^{[1]}$ contains only the candidates corresponding to the vertices in $N(v_i) \cup \{v_i\}$, we can conclude that each vote v_i satisfies $|v_i^{[1]} \cap W| \geq 1 \geq t$, and W is a solution of MCCA-WD.

"$\Longleftarrow$": Suppose that there exists a k-committee W such that all votes v_i satisfy $|v_i^{[1]} \cap W| \geq t$. We set D to be the set of the vertices, which correspond to the candidates in W. Since the set $v_i^{[1]}$ of a vote v_i contains only the candidates corresponding to the vertices in $N(v_i) \cup \{v_i\}$, it holds for each vertex $v_i' \in \mathcal{V}$ that $v_i' \in D$ or one neighbor of v_i' is in D. Thus, D is a size-k' dominating set of $\mathcal{G}$. □

The hardness for the case of trichotomous votes follows directly.

Proposition 3.6. *For every positive constant t, MTCCA-WD and MTPAV-WD are NP-hard and W[2]-hard with k as parameter.*

Note that MSAV-WD is polynomial-time solvable for $t \geq 1$. However, it becomes NP-hard for every rational number $0 < t < 1$.

Theorem 3.7. *MSAV-WD is NP-hard for every constant rational number* $0 < t < 1$.

Proof. The reduction is based on the same idea as the one in the proof of Theorem 3.5, but requires much more effort to deal with the constant $t = \frac{\alpha}{\beta}$. Here, α and β are two given positive integers with $\alpha < \beta$. Given a Dominating Set instance $(\mathcal{G} = (\mathcal{V} = \{v_1', v_2', ..., v_n'\}, \mathcal{E}), k')$, we also create a candidate c_i and a vote v_i for each vertex $v_i' \in \mathcal{V}$ as in the proof of Theorem 3.5. However, we need additional candidates and votes to guarantee that for each vote v_i, the committee W satisfies that $\frac{|W \cap v_i^{[1]}|}{|v_i^{[1]}|} \geq t$. To this end, we add for each vertex v_i' some new candidates. Let $\mathsf{deg}(v_i')$ denote the degree of v_i' in $\mathcal{G}$, and γ_i be the minimum integer such that $(\beta - \alpha) \times \gamma_i \geq \mathsf{deg}(v_i') + 1$. Then, we add $x_i^1, \ldots, x_i^{\delta_i}$ many new candidates to C with $\delta_i = \beta \times \gamma_i - \mathsf{deg}(v_i') - 1$. Note that we have totally $|\mathcal{V}| + \sum_{1 \leq i \leq |\mathcal{V}|} \delta_i$ many candidates in C.

Then, the vote v_i is a vector with $|C|$ positions, where the first $|\mathcal{V}|$ positions represent the vertices in $\mathcal{V}$ and the remaining positions correspond to the new candidates. In vote v_i, we set $v_{ij} := 1$ for each vertices v_j' in $N(v_i') \cup \{v_i'\}$ and in addition set $v_{ij} := 1$, where the j-th position corresponds to a new candidate x_i^r created for v_i'. The other positions of v_i are set to 0. Then, we create some new votes. More specifically, we add, for each vertex $v_i' \in \mathcal{V}$, $\theta_i = \gamma_i \times \alpha - 1$ many new votes $v_i^1, \ldots, v_i^{\theta_i}$. Note that $\theta_i < \delta_i$. Each of these votes v_i^j with $1 \leq j \leq \theta_i$ has only one position set to 1, namely, the position corresponding to the candidate x_i^j. The construction of v_i can be formally described by the following function:

$$f(v_i, c_{i'}) = \begin{cases} 1, & v_i' = v_{i'}' \text{ or } v_{i'}' \in N(v_i'), \\ 0, & \text{otherwise.} \end{cases}$$

$$f(v_i^j, c_{i'}) = \begin{cases} 1, & c_{i'} = x_i^j, \\ 0, & \text{otherwise.} \end{cases}$$

Then, $f(v_i, c_{i'}) = 1$ means that the candidate $c_{i'}$ is approved by the vote v_i, and $f(v_i, c_{i'}) = 0$ means that v_i is disapproved by $c_{i'}$. Finally, we set $k := k' + \sum_{1 \leq i \leq |\mathcal{V}|} \theta_i$. Next, we prove the equivalence between the instances.

"$\Longrightarrow$": Given a size-k' dominating set D of $\mathcal{G}$, we set $W := \{c_i | v_i' \in D\} \cup \{x_i^j | 1 \leq i \leq |\mathcal{V}|, 1 \leq j \leq \theta_i\}$. Clearly, $|W| = k + \sum_{1 \leq i \leq |\mathcal{V}|} \theta_i$. The votes $v_i^1, \ldots, v_i^{\theta_i}$ with $1 \leq i \leq |\mathcal{V}|$ are clearly satisfied by W, since $\mathsf{MSAV}(v_i^j, W) = 1 > t$ for all $1 \leq i \leq |\mathcal{V}|$ and $1 \leq j \leq \theta_i$. By the construction of the votes v_i, we know $f(v_i, c_{i'}) = 1$, if $v_i' = v_{i'}'$ or $v_{i'}' \in N(v_i')$. Thus, the following must hold:

$$(v_i^{[1]} \setminus \{x_i^1, \ldots, x_i^{\theta_i}\}) \cap W \neq \emptyset,$$

$$\mathsf{MSAV}(v_i, W) \geq \frac{|v_i^{[1]} \cap W|}{|v_i^{[1]}|} \geq \frac{1 + \theta_i}{\delta_i + \mathsf{deg}(v_i') + 1} = \frac{\gamma_i \alpha}{\gamma_i \beta} = t.$$

We can then conclude that W is a k-committee.

"$\Longleftarrow$": Suppose that there is a k-committee W. Observe that each k-committee has to contain all candidates in $\{x_i^j | 1 \leq i \leq |\mathcal{V}|, 1 \leq j \leq \theta_i\}$, because each of these candidates is the only approved candidate of v_i^j with $1 \leq j \leq \theta_i$. Moreover, none of other new candidates x_i^j with $\theta_i < j \leq \delta_i$ can be in W, because each of them is approved in only one vote v_i and v_i approves at least one candidate corresponding to a vertex in $\mathcal{V}$. Therefore, W needs to contain k' candidates corresponding to vertices in $\mathcal{V}$. Let W' be the set containing these k' candidates, $W' \subseteq W$ and $|W'| = k'$. Since for each vote v_i corresponding to a vertex $v'_i \in \mathcal{V}$, we have $|v_i^{[1]}| = \beta \times \gamma_i$ and $\frac{\theta_i}{\beta \times \gamma_i} < \frac{\alpha}{\beta}$, the set $v_i^{[1]} \cap W$ should contain at least one of the candidates in W'. This implies that the vertices corresponding to the candidates in W' form a dominating set of $\mathcal{G}$. □

As in the case of dichotomous votes, if $t \leq -1$ or $t \geq 1$, then MTSAV-WD can be solved in polynomial time. NP-hardness can be shown for $-1 < t < 1$.

Proposition 3.8. *MTSAV-WD is NP-hard for every rational number $-1 < t < 1$.*

3.3 Parameter: k

For the parameterization with k, we already proved fixed-parameter intractability results for MCCA-WD and MPAV-WD (Theorem 3.5) and their trichotomous versions (Proposition 3.6). As shown in the following theorem, MSAV-WD is also W[2]-hard with this parameterization.

Theorem 3.9. *MSAV-WD and MTSAV-WD are W[2]-hard with respect to k.*

Proof. We prove the theorem by reducing Dominating Set to MSAV-WD. Given a Dominating Set instance $(\mathcal{G} = (\mathcal{V}, \mathcal{E}), k')$, where $\mathcal{V} = \{v'_1, v'_2, ..., v'_n\}$, we construct an MSAV-WD instance $E = (C, V, k, t)$ in the same way as the one in the proof of Theorem 3.5, except the parameter $t := \frac{1}{n}$ here. Since $|v_i^{[1]}| \leq n$, $\mathsf{MSAV}(v_i, W) \geq t = \frac{1}{n}$ if and only if $|v_i^{[1]} \cap W| \geq 1$. This means that with $t = \frac{1}{n}$, solving MSAV-WD on $E = (C, V, k, t)$ is equivalent to solving MCCA-WD on $E = (C, V, k, t)$. Therefore, MSAV-WD is W[2]-hard with respect to k. The case with trichotomous votes follows directly. □

3.4 Parameter: d

The next two theorems are dedicated to the parameterization with d. The first result concerns with TCCA-WD. In contrast to the case of dichotomous votes, where CCA-WD is FPT with respect to d, the case with trichotomous votes turns out to be intractable.

Theorem 3.10. *TCCA-WD is W[1]-hard with respect to the combined parameter of d and k.*

Proof. We prove the theorem by reducing Clique to TCCA-WD. A clique $\mathcal{K}$ in an undirected graph $\mathcal{G}$ is a subset of vertices, which form a complete graph. In other words, the vertices in $\mathcal{K}$ are pairwise adjacent in $\mathcal{G}$. Given a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and an integer k', Clique asks for a size-k' clique. Clique is W[1]-complete with respect to k' [20]. We construct a TCCA-WD instance $E = (C, V, k, d)$ from $(\mathcal{G} = (\mathcal{V}, \mathcal{E}), k')$ as follows.

For each edge $e_u = \{v'_j, v'_l\} \in \mathcal{E}$ (assume $j < l$), we create two candidates c_{2u-1} and c_{2u}. To ease the presentation, we also use c^{jl} to refer to c_{2u-1} and c^{lj} to refer to c_{2u}, denoted as $c_{2u-1} \tilde{=} c^{jl}$ and $c_{2u} \tilde{=} c^{lj}$. And let $C_1 = \bigcup_{u=1}^{|\mathcal{E}|} \{c_{2u-1}, c_{2u}\}$. We add another set of dummy candidates: $C_2 = \{c_p | 2|\mathcal{E}| < p \leq 2|\mathcal{E}| + k'(k'-1) - 3(k'-1) + 1\}$. Set $C := C_1 \cup C_2$. Further, we set $k := 2k'(k'-1) - 3(k'-1) + 1$, $d := k'$, and $t := 1$. To give a formal description of the construction of the votes, we define the following function:

$$f(v_i, c_s) = \begin{cases} 1, & c_s \in C_2, \\ 1, & c_s \in C_1, c_s \tilde{=} c^{jl}, i = j, \\ 0, & c_s \in C_1, c_s \tilde{=} c^{lj}, i = j, \\ -1, & \text{otherwise.} \end{cases}$$

Then, $f(v_i, c_s) = 1$ means that the candidate c_s is approved by the vote v_i, $f(v_i, c_s) = -1$ means that the candidate c_s is disapproved by the vote v_i, and $f(v_i, c_s) = 0$ means that v_i abstains with respect to c_s. We create n votes v_i in V, one-to-one corresponding to the vertices $v'_i \in \mathcal{V}$. The votes v_i are then constructed according to the definition of $f(v_i, c_s)$. We show the equivalence between the instances in the following.

"$\Longrightarrow$": Suppose that there exists a size-k' clique $\mathcal{K}$ in $\mathcal{G}$. We add to W all candidates in C_2 and the candidates c_s in C_1, who correspond to the edges $e_s = \{v'_j, v'_l\}$ with $v'_j \in \mathcal{K}$ and $v'_l \in \mathcal{K}$. Thus, we can get:

$$\begin{aligned} &|W| = k'(k'-1) - 3(k'-1) + 1 + k'(k'-1) \\ =&2k'(k'-1) - 3(k'-1) + 1 = k. \end{aligned}$$

According to the definition of $f(v_i, c_s)$, for each vote $v_i \in V$ with the corresponding vertex $v'_i \in \mathcal{K}$, there are $k-1$ candidates $c_s \in (C_1 \cap W)$ satisfying $f(v_i, c_s) = 1$. Similarly, there are $k-1$ candidates $c_s \in (C_1 \cap W)$ with $f(v_i, c_s) = 0$ and all $k'(k'-1) - 3(k'-1) + 1$ candidates $c_s \in V_2$ satisfying $f(v_i, c_s) = 1$. Thus, for a vote v_i with $v'_i \in \mathcal{K}$, we have:

$$\begin{aligned} &|v_i^{[1]} \cap W| = k'(k'-1) - 2(k'-1) + 1, \\ &|v_i^{[-1]} \cap W| = k'(k'-1) - 2(k'-1), \\ &|v_i^{[1]} \cap W| - |v_i^{[-1]} \cap W| = 1 \geq t. \end{aligned}$$

In contrast, for a vote v_i with $v'_i \notin \mathcal{K}$, we have:

$$\begin{aligned} &|v_i^{[1]} \cap W| = k'(k'-1) - 3(k'-1) + 1, \\ &|v_i^{[-1]} \cap W| = k'(k'-1), \\ &|v_i^{[1]} \cap W| - |v_i^{[-1]} \cap W| < t. \end{aligned}$$

Therefore, the total TCCA-score of W is $\mathsf{TCCA}(V, W) = k'$.

"$\Longleftarrow$": Since the candidates c in C_2 are approved by all votes, there is always a k-committee W containing all candidates in C_2. Thus, $|W \cap C_1| = k'(k'-1)$. Then, for each vote $v_i \in V$, we have:

$$\begin{aligned} &|v_i^{[1]} \cap (C_2 \cap W)| - |v_i^{[-1]} \cap (C_2 \cap W)| \\ =&k'(k'-1) - 3(k'-1) + 1. \end{aligned}$$

Observe that each candidate $c \in C_1$ can be in $v_i^{[1]}$ for exactly one vote $v_i \in C_1$ and in $v_j^{[0]}$ for exactly one vote $v_j \in C_1$ with $i \neq j$.

Thus, for any size-k' subset V' of V, we have:

$$\sum_{v_i \in V'} |v_i^{[1]} \cap (C_1 \cap W)| \leq k'(k'-1),$$
$$\sum_{v_i \in V'} |v_i^{[0]} \cap (C_1 \cap W)| \leq k'(k'-1),$$
$$\sum_{v_i \in V'} |v_i^{[-1]} \cap (C_1 \cap W)| \geq k' \cdot k' \cdot (k'-1) - 2k' \cdot (k'-1) = k'(k'-1)(k'-2).$$

This means that there is a vote $v_i \in V'$ satisfying:

$$|v_i^{[1]} \cap (C_1 \cap W)| - |v_i^{[-1]} \cap (C_1 \cap W)| \leq -(k'-1)(k'-3).$$

However, since W is a k-committee, there must be a set V'' of $d = k'$ votes v_i, each of which satisfies:

$$|v_i^{[1]} \cap W| - |v_i^{[-1]} \cap W| \geq t = 1.$$

This implies that for each $v_i \in V''$, we have:

$$|v_i^{[1]} \cap (C_1 \cap W)| - |v_i^{[-1]} \cap (C_1 \cap W)| \geq -(k'-1)(k'-3).$$

Thus, for each $v_i \in V''$ it satisfies that:

$$|v_i^{[1]} \cap (C_1 \cap W)| - |v_i^{[-1]} \cap (C_1 \cap W)| = -(k'-1)(k'-3).$$

We can then conclude:

$$\sum_{v_i \in V''} |v_i^{[-1]} \cap (C_1 \cap W)| = k'(k'-1)(k'-2),$$
$$\sum_{v_i \in V''} |v_i^{[1]} \cap (C_1 \cap W)| = k'(k'-1),$$
$$\sum_{v_i \in V''} |v_i^{[0]} \cap (C_1 \cap W)| = k'(k'-1).$$

Therefore, the vertices corresponding to the votes in V'' must form a clique with the edges corresponding to the candidates in $C_1 \cap W$, which completes the proof. □

Finally, we prove the hardness of TPAV-WD with d being a constant.

Theorem 3.11. *TPAV-WD is NP-hard and W[1]-hard with k as parameter, even if $d = 0$.*

Proof. We prove the theorem by reducing the Independent Set on D-Regular Graphs (ISRG) problem to TPAV-WD. A D-regular graph is an undirected graph where all vertices have the same degree D. An independent set of a graph is a set of vertices, no two of which are adjacent. Given a D-regular graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and an integer k', ISRG asks for an independent set of size at least k'. ISRG is known to be NP-hard [16] and it is easy to show that if D is not a constant, then ISRG is W[1]-hard with respect to k'. We can construct a TPAV-WD instance $E = (C, V, k, d)$ from $(\mathcal{G} = (\mathcal{V}, \mathcal{E}), k')$ as follows.

We have three sets of candidates: $|C_1| = |\mathcal{V}|$, that is, the candidates in C_1 one-to-one correspond to the vertices in $\mathcal{V}$, $|C_2| = k'-1$, and $|C_3| = 1$. There are also three sets of votes: $|V_1| = |\mathcal{E}|$, that is, the votes in V_1 correspond to the edges in $\mathcal{E}$, $|V_2| = \lfloor D \cdot \frac{k'(k'+1)^2}{2k'+1} \rfloor + 1$, and $|V_3| = D \cdot k'$. Then, $C = C_1 \cup C_2 \cup C_3$ and $V = V_1 \cup V_2 \cup V_3$. Further, set $k := 2k'$ and $d := 0$. Finally, we define the following function, which indicates the construction of the votes. In the following, a vote $v_i \in V_1$ corresponds to an edge $e_i \in \mathcal{E}$ and v'_j denotes the vertex corresponding to the candidate $c_j \in C_1$. We use $v'_j \in e_i$ to denote that a vertex v'_j is an endpoint of an edge e_i.

$$g(v_i, c_j) = \begin{cases} 1, & v_i \in V_1, c_j \in C_1, v'_j \in e_i, \\ 0, & v_i \in V_1, c_j \in C_1, v'_j \notin e_i, \\ 0, & v_i \in V_1, c_j \in (C_2 \cup C_3), \\ -1, & v_i \in V_2, c_j \in C_1, \\ 1, & v_i \in V_2, c_j \in (C_2 \cup C_3), \\ 0, & v_i \in V_3, c_j \in (C_1 \cup C_2), \\ -1, & v_i \in V_3, c_j \in C_3. \end{cases}$$

The votes are constructed according to $g(v_i, c_j)$. More specifically, $g(v_i, c_j) = 1$ means that the candidate $c_j \in v_i^{[1]}$, $g(v_i, c_j) = -1$ means $c_j \in v_i^{[-1]}$, and $g(v_i, c_j) = 0$ means $c_j \in v_i^{[0]}$. In the following, we show that $\mathcal{G}$ has a size-k' independent set, if and only if there is a k-committee W satisfying $\mathsf{TPAV}(V, W) \geq d = 0$.

"$\Longrightarrow$": Suppose that there exists a size-k' independent set I in graph $\mathcal{G}$. Set $W := I' \cup C_2 \cup C_3$, where I' contains the candidates in C_1 corresponding to the vertices in I. Thus, we have:

$$|W| = k' + (k'-1) + 1 = 2k' = k.$$

For each $v_i \in V_1$, with the degree of each vertex being D, we have:

$$|v_i^{[1]} \cap W| \leq 1, \; v_i^{[-1]} = \emptyset, \text{ and}$$
$$\sum_{v_i \in V_1} \mathsf{TPAV}(v_i, W) = D \cdot k'.$$

For each $v_i \in V_2$, it holds:

$$|v_i^{[-1]} \cap W| = |v_i^{[1]} \cap W| = k', \text{ and}$$
$$\sum_{v_i \in V_2} \mathsf{TPAV}(v_i, W) = 0.$$

For each $v_i \in V_3$, we have:

$$|v_i^{[-1]} \cap W| = 1, |v_i^{[1]} \cap W| = 0, \text{ and}$$
$$\sum_{v_i \in V_3} \mathsf{TPAV}(v_i, W) = -D \cdot k'.$$

Altogether, we have $\sum_{v_i \in V} \mathsf{TPAV}(v_i, W) = D \cdot k' + 0 - D \cdot k' = d = 0$. Thus, there exists a k-committee W satisfying $\mathsf{TPAV}(V, W) \geq d = 0$.

"$\Longleftarrow$": The key observation for this direction is that every k-committee W satisfies $C_2 \cup C_3 \subseteq W$. Suppose that this is not true. Let W be a k-committee with $(C_2 \cup C_3) \setminus W \neq \emptyset$. Thus, $|(C_2 \cup C_3) \setminus W| = l$ and $|C_1 \cap W| = k' + l$ for an integer $l > 0$. Therefore, we have

$$\mathsf{TPAV}(V_1, W) = \sum_{v \in V_1} \mathsf{TPAV}(v, W) \leq D(k' + l),$$

since $\mathcal{G}$ is a D-regular graph. Moreover, we have

$$\mathsf{TPAV}(V_2, W) = \sum_{v \in V_2} \mathsf{TPAV}(v, W)$$
$$= -|V_2|(\frac{1}{k'-l+1} + \frac{1}{k'-l+2} + \cdots + \frac{1}{k'+l})$$
$$< -(D\frac{k'(k'+1)^2}{2k'+1})(\frac{1}{k'-l+1} + \cdots + \frac{1}{k'+l}).$$

Further, we have $\mathsf{TPAV}(V_3, W) = \sum_{v \in V_3} \mathsf{TPAV}(v, W) \leq 0$. Thus, the total TPAV-score of W is

$$\mathsf{TPAV}(V, W) = \mathsf{TPAV}(V_1, W) + \mathsf{TPAV}(V_2, W) + \mathsf{TPAV}(V_3, W)$$
$$< D(k' + l) - (D\frac{k'(k'+1)^2}{2k'+1})(\frac{1}{k' - l + 1} + \cdots + \frac{1}{k' + l}).$$

The right side of the inequality is clearly less than 0, resulting in a contradiction.

With $C_2 \cup C_3 \subseteq W$, we know $|W \cap C_1| = k'$. Then, it is true that $\mathsf{TPAV}(V_2, W) = 0$ and $\mathsf{TPAV}(V_3, W) = -Dk'$. Since W is a committee, $\mathsf{TPAV}(V_1, W) \geq Dk'$. Due to $(C_2 \cup C_3) \cap v_i^{[1]} = \emptyset$ for each $v_i \in V_1$ and the fact that each candidate in $C_1 \cap W$ can be in $v_i^{[1]}$ for exactly D votes v_i in V_1, we can conclude that there is no vote $v_i \in V_1$ with $|v_i^{[1]} \cap W| > 1$. This means that no edge in $\mathcal{G}$ is between the vertices, which correspond to the votes in $C_1 \cap W$. Clearly, these vertices form an independent set of size k'. □

4 CONCLUDING REMARKS

In this paper, we studied the parameterized complexity of the winner determination problem of committee elections under three variations of the classical AV rule, namely, Chamberlin-Courant Approval Voting (CCA), Proportional Approval Voting (PAV), and Satisfaction Approval Voting (SAV). Hereby, we considered both dichotomous and trichotomous votes, complementing and extending the previous works on parameterized complexity of AV-based elections [28, 31, 34]. We also investigated the maximin version of CCA, PAV, and SAV, where the committee has to maximize the minimal satisfaction score of all voters. A collection of fixed-parameter tractable and intractable results has been achieved with respect to four natural parameters, that is, the number of votes n, the number of candidates m, the size of committee k, and the lower bound on the total/individual satisfaction bound d/t. We observe that in the cases of dichotomous and trichotomous votes, the winner determination problem admits almost the same parameterized complexity behaviour. This holds for all three AV-variations. The only exception we can identify is CCA with the total satisfaction d as parameter. There exists a parameterized algorithm for CCA with dichotomous votes, while the case of trichotomous votes leads to an intractability result.

There are two problems left open in Table 1. First, the complexity of computing a k-committee in an election with PAV and dichotomous votes remains unsolved for the total dissatisfaction parameter d. The trichotomous case is NP-hard even with $d = 0$. Second, we conjecture that the maximin version of PAV with trichotomous votes is fixed-parameter tractable with respect to the number of votes. Here, one might need a more involved ILP formulation. Finally, we leave it for future work to extend the study with trichotomous votes to other multiple winner rules.

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