after deleting an agent from one type the other agent from this type will always have zero utility and cannot be part of a profitable swap, implying that any swap-equilibrium has vertex-robustness $|V(G)|$. Thus, in the following, considering vertex-robustness, we assume that $|T_1| \geq 3$ and $|T_2| \geq 2$.

Focusing on edge-robustness for a moment, only the deletion of edges between agents of the same type has an influence on the stability of a swap-equilibrium. This is stated more precisely in the following proposition.

Proposition 4.2 (★). *Let* $\mathbf{v}$ *be a swap-equilibrium for a Schelling game on topology* G*. Let* $S \subseteq E(G)$ *be a set of edges such that* $\mathbf{v}$ *is not a swap-equilibrium on* $G - S$*. Then,*

(i) S *contains at least one edge between agents of the same type.*
(ii) $\mathbf{v}$ *is also not a swap-equilibrium on* $G - S'$*, where* $S' \subseteq S$ *is the subset of edges from* S *that connect agents of the same type.*
(iii) For every set $A \subseteq \{\{v_i, v_j\} \in E(G) \mid i, j \in T_1 \vee i, j \in T_2\}$ *of edges between agents of the same type,* $\mathbf{v}$ *is also not a swap-equilibrium on* $G - (S \cup A)$*.*

For vertex-robustness, one can similarly observe that only deleting a vertex occupied by an agent a adjacent to at least one vertex occupied by a friend of a can make a swap-equilibrium unstable.

Next, note that the utility of an agent only depends on its neighborhood. Thus, whether two agents i and j have a profitable swap in $G - S$ only depends on the edges/vertices incident/adjacent to v_i and v_j in S. Combining this with the observation that no profitable swap can involve an agent on an isolated vertex, it follows that if a swap-equilibrium cannot be made unstable by deleting $2 \cdot (\Delta(G) - 1)$ edges/vertices, then it cannot be made unstable by deleting an arbitrary number of edges/vertices:

Observation 1. *Let* $\mathbf{v}$ *be a swap-equilibrium for a Schelling game on* G*. If* $\mathbf{v}$ *is* $2 \cdot (\Delta(G) - 1)$*-edge-robust,* $\mathbf{v}$ *has edge-robustness* $|E(G)|$ *and if* $\mathbf{v}$ *is* $2 \cdot (\Delta(G) - 1)$*-vertex-robust,* $\mathbf{v}$ *has vertex-robustness* $|V(G)|$*.*

The simple fact that the utility of an agent only depends on its neighborhood leads to a polynomial-time algorithm to determine whether a given swap-equilibrium $\mathbf{v}$ has edge-robustness $r \in \mathbb{N}_0$: We simply iterate over all pairs of agents i and j and check whether we can delete at most r edges between v_i and adjacent vertices occupied by friends of i and between v_j and adjacent vertices occupied by friends of j such that the swap of i and j becomes profitable (note that the stability of $\mathbf{v}$ only depends on the number of such deleted edges in the neighborhood of each agent, not the exact subset of edges). For vertex-robustness, an analogous approach works.

Proposition 4.3 (★). *Given a Schelling game with* n *agents, a swap-equilibrium* $\mathbf{v}$*, and an integer* $r \in \mathbb{N}_0$*, one can decide in* $O(n^2 \cdot r)$ *time whether* $\mathbf{v}$ *is* r*-edge/vertex-robust.*

Note, however, that finding a swap-equilibrium whose vertex- or edge-robustness is as high as possible is NP-hard, as we have proven in Theorem 3.1 that already deciding whether a Schelling game admits some swap-equilibrium is NP-hard.

4.2 Robustness of Equilibria on Different Graph Classes

In this subsection, we analyze the influence of the topology on the robustness of swap-equilibria. We first analyze cliques where each swap-equilibrium has edge-robustness zero and vertex-robustness $|V(G)|$. Subsequently, we turn to cycles, paths, and grids and find that there exists a swap-equilibrium on all these graphs with edge-robustness and vertex-robustness zero. For paths, we observe that the difference between the edge/vertex-robustness of the most and least robust equilibrium can be arbitrarily large. Finally, with α-star-constellation graphs for $\alpha \in \mathbb{N}_0$, we present a class of graphs on which all swap-equilibria have at least edge/vertex-robustness α.

We start by observing that on a clique every assignment is a swap-equilibrium. From this it directly follows that every swap-equilibrium has vertex-robustness $|V(G)|$, as deleting a vertex from a clique results in another clique. In contrast, each swap-equilibrium can be made unstable by deleting one edge. Thereby, the following observation also proves that the difference between the edge- and vertex-robustness of a swap-equilibrium can be arbitrarily large:

Observation 2. *In a Schelling game on a clique* G *with* $|T_1| \geq 2$ *and* $|T_2| \geq 2$*, every swap-equilibrium* $\mathbf{v}$ *has edge-robustness zero and vertex-robustness* $|V(G)|$*.*

Proof. It remains to prove that the edge-robustness is always zero. Let $i \neq j \in T_1$, $e := \{v_i, v_j\} \in E(G)$, and $l \in T_2$. As G is a clique, it holds that $u_i^G(\mathbf{v}) = \frac{|T_1|-1}{|T_1|+|T_2|-1}$ and $u_l^G(\mathbf{v}) = \frac{|T_2|-1}{|T_1|+|T_2|-1}$. Swapping i and l is profitable in $\mathbf{v}$ on $G - \{e\}$ for both i and l, as

$$u_i^{G-\{e\}}(\mathbf{v}^{i\leftrightarrow l}) = \frac{|T_1| - 1}{|T_1| + |T_2| - 1} > \frac{|T_1| - 2}{|T_1| + |T_2| - 2} = u_i^{G-\{e\}}(\mathbf{v}) \text{ and}$$
$$u_l^{G-\{e\}}(\mathbf{v}^{i\leftrightarrow l}) = \frac{|T_2| - 1}{|T_1| + |T_2| - 2} > \frac{|T_2| - 1}{|T_1| + |T_2| - 1} = u_l^{G-\{e\}}(\mathbf{v}).$$

□

For a cycle G, we can show that in a swap-equilibrium $\mathbf{v}$, every agent is adjacent to at least one friend. Then, picking an arbitrary agent $i \in T_1$ that has utility $1/2$ in $\mathbf{v}$ and deleting i's neighbor from T_1 or the edge between i and its neighbor from T_1 makes $\mathbf{v}$ unstable.

Proposition 4.4 (★). *In a Schelling game on a cycle* G *with* $|T_1| \geq 2$ *and* $|T_2| \geq 2$*, every swap-equilibrium* $\mathbf{v}$ *has edge-robustness zero. For* $|T_1| \geq 3$ *and* $|T_2| \geq 2$*, every swap-equilibrium* $\mathbf{v}$ *has vertex-robustness zero.*

Next, we turn to paths and prove that every Schelling game on a path with sufficiently many agents from both types has an equilibrium with edge-/vertex-robustness zero and one with edge-robustness $|E(G)|$ and vertex-robustness $|V(G)|$. This puts paths in a surprisingly sharp contrast to cycles. The reason for this is that on a path, we can always position the agents such that there exists only one edge between agents of different types, yielding a swap-equilibrium with edge-robustness $|E(G)|$ and vertex-robustness $|V(G)|$. This is not possible on a cycle.

Theorem 4.5. *For a Schelling game on a path* G *with* $|T_1| \geq 4$ *and* $|T_2| \geq 2$*, there exists a swap-equilibrium* $\mathbf{v}$ *that has edge-robustness and vertex-robustness zero and a swap-equilibrium* $\mathbf{v}'$ *that has edge-robustness* $|E(G)|$ *and vertex-robustness* $|V(G)|$*.*

Proof. Let $V(G) = \{w_1, \ldots, w_n\}$ and $E(G) = \{\{w_i, w_{i+1}\} \mid i \in [n-1]\}$. In $\mathbf{v}$, vertices w_1 and w_2 are occupied by agents from T_1, vertices w_3 to $w_{|T_2|+2}$ are occupied by agents from T_2, and the remaining $|T_1| - 2 \geq 2$ vertices are occupied by agents from T_1 (see

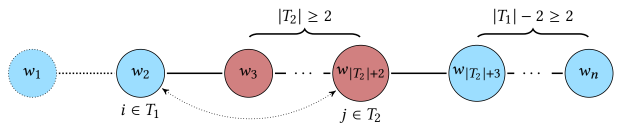

Figure 3: The swap-equilibrium with robustness zero from Theorem 4.5. After deleting $\{w_1, w_2\} \in E(G)$ or $w_1 \in V(G)$, swapping i and j is profitable.

Figure 3 for a visualization). As all agents have at most one neighbor of the other type and at least one neighbor of the same type, for each pair i, j of agents of different types it holds that $u_i(\mathbf{v}^{i\leftrightarrow j}) \leq 1/2 \leq u_i(\mathbf{v})$. Thus, $\mathbf{v}$ is a swap-equilibrium. Further, after deleting the edge between w_1 and w_2 or deleting the vertex w_1, swapping the agent on w_2 with the agent on $w_{|T_2|+2}$ is profitable. It follows that $\mathbf{v}$ has edge-robustness and vertex-robustness zero.

In $\mathbf{v}'$, the agents from T_1 occupy the first $|T_1|$ vertices and agents from T_2 the remaining vertices. Let $S \subseteq E(G)$ or $S \subseteq V(G)$ and consider $G - S$. As for $j \in [|T_1| - 1] \cup [|T_1| + 2, n]$, in $G - S$, the agent on w_j got deleted, has no neighbor, or is only adjacent to friends, it can never be involved in a profitable swap. Further, swapping the agent on $w_{|T_1|}$ and the agent on $w_{|T_1|+1}$ can also never be profitable, since after the swap none of the two is adjacent to a friend. Thus, $\mathbf{v}'$ is a swap-equilibrium on $G - S$. □

If $\max\{|T_1|, |T_2|\} \leq 3$, which is not covered by Theorem 4.5, then in every swap-equilibrium the path is split into two subpaths and agents from T_1 occupy one subpath and agents from T_2 occupy the other subpath. As argued in the proof of Theorem 4.5, such an assignment has edge-robustness $|E(G)|$ and vertex-robustness $|V(G)|$.

Turning to grids, which besides paths form the class which has been most often considered in the context of Schelling's segregation model, for both vertex- and edge-robustness, we show using some more involved arguments that every swap-equilibrium has either robustness one or zero and that there exists an infinite class of Schelling games on grids admitting a swap-equilibrium with robustness zero and one with robustness one.

Theorem 4.6 (★). *(1) In a Schelling game with $|T_1| \geq 2$ and $|T_2| \geq 2$ on an $(x \times y)$-grid with $x \geq 2$, $y \geq 2$, the edge-robustness of a swap-equilibrium is at most one.*
(2) In a Schelling game with $|T_1| \geq 4$ and $|T_2| \geq 4$ on an $(x \times y)$-grid with $x \geq 3$ and $y \geq 3$, the vertex-robustness of a swap-equilibrium is at most one.
(3) In a Schelling game with $|T_1| = |T_2|$ on an $(x \times y)$-grid with even $x \geq 4$ and $y \geq 2$, there exists a swap-equilibrium $\mathbf{v}$ with edge- and vertex-robustness zero and a swap-equilibrium $\mathbf{v}'$ with edge- and vertex-robustness one.

Lastly, motivated by the observation that on all previously considered graph classes there exist swap-equilibria with zero edge-robustness and on all considered graph classes except cliques there exist swap-equilibria with zero vertex-robustness, we investigate α-star-constellation graphs, a generalization of stars and α-caterpillars. We prove that every swap-equilibrium in a Schelling game on an α-star-constellation graph is α-vertex-robust and α-edge-robust. We also show that a swap-equilibrium on an α-star-constellation graph may fail to exist but that we can precisely characterize swap-equilibria on such graphs. Using this characterization, we design a polynomial-time algorithm for S-Eq on α-star-constellation graphs and show that there always exists a swap-equilibrium on an α-caterpillar, that is, an α-star-constellation graph which restricted to non-degree-one vertices forms a path.

Theorem 4.7. *In a Schelling game on an α-star-constellation graph for $\alpha \in \mathbb{N}_0$, every swap-equilibrium $\mathbf{v}$ is α-edge and α-vertex-robust.*

Proof. Let $\mathbf{v}$ be a swap-equilibrium on an α-star-constellation graph G for some $\alpha \in \mathbb{N}_0$. We make a case distinction based on whether or not there exists an agent i on a degree-one vertex adjacent to an agent j of the other type in $\mathbf{v}$. If this is the case, then assume without loss of generality that $i \in T_1$ and $j \in T_2$ and observe that it needs to hold that all agents $j' \in T_2 \setminus \{j\}$ are only adjacent to friends, as otherwise j' and i have a profitable swap. Now consider the topology $G - S$ for some subset $S \subseteq E(G)$ or some subset $S \subseteq V(G)$. Then, for all $j' \in T_2 \setminus \{j\}$, agent j' cannot be involved in a profitable swap in $G - S$, as j' got deleted, is only adjacent to friends, or placed on an isolated vertex. Moreover, there also cannot exist a profitable swap for j, as no agent from T_1 is adjacent to an agent from $T_2 \setminus \{j\}$. Hence, $\mathbf{v}$ is $|E(G)|$-edge-robust and $|V(G)|$-vertex-robust.

Now, assume that all agents on a degree-one vertex are only adjacent to friends in $\mathbf{v}$ and consider the topology $G - S$ for $S \subseteq E(G)$ or $S \subseteq V(G)$ with $|S| \leq \alpha$. Note that, in $G - S$, only agents $i \in T_1$ and $j \in T_2$ with $\deg_G(v_i) > 1$ and $\deg_G(v_j) > 1$ and $\deg_{G-S}(v_i) \geq 1$ and $\deg_{G-S}(v_j) \geq 1$ can be involved in a profitable swap, since all other agents either occupy an isolated vertex or are only adjacent to friends in $G - S$. For vertex-robustness, it additionally needs to hold that $v_i, v_j \notin S$. Since G is an α-star-constellation graph and we delete at most α edges or α vertices, it holds that both v_i and v_j are adjacent to at least as many degree-one vertices as non-degree-one vertices in $G - S$. By our assumption, the agents on degree-one vertices adjacent to v_i are friends of i and the agents on degree-one vertices adjacent to v_j are friends of j. Hence, swapping i and j cannot be profitable, as $u_k^{G-S}(\mathbf{v}) \geq 1/2$ and $u_k^{G-S}(\mathbf{v}^{i\leftrightarrow j}) \leq 1/2$ for $k \in \{i, j\}$. □

Theorem 4.7 has no implications for the existence of swap-equilibria on α-star-constellation graphs. Indeed, we observe that there is no swap-equilibrium in a Schelling game with $|T_1| = 5$ and $|T_2| = 7$ on the 1-star-constellation graph depicted in Figure 4. Notably, to the best of our knowledge the graph from Figure 4 is the first known graph without a swap-equilibrium that is not a tree.

Proposition 4.8. *A Schelling game on an α-star-constellation graph G may fail to admit a swap-equilibrium, even if G is a split graph, that is, the vertices of the graph can be partitioned into a clique and an independent set.*

Proof. Consider the Schelling game with $|T_1| = 5$ many agents of type T_1 and $|T_2| = 7$ many agents of type T_2 on the graph G from Figure 4, which consists of three 3-stars whose central vertices form a clique. Observe that as all stars in G consist of four vertices and neither $|T_1| = 5$ nor $|T_2| = 7$ are divisible by four, in any assignment $\mathbf{v}$, there exists a degree-one vertex occupied by an agent $i \in T_l$ such that the adjacent central vertex is occupied by an

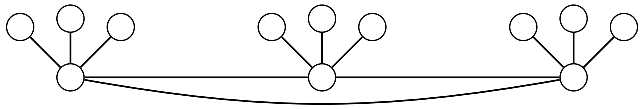

Figure 4: There is no swap-equilibrium in a Schelling game with $|T_1| = 5$ and $|T_2| = 7$ on this 1-star-constellation graph.

agent $j \in T_{l'}$ of the other type with $l \neq l'$. Let $v \neq v'$ be the other two central vertices. We make a case distinction based on whether the agents on the degree-one vertices adjacent to v and v' have the same type as their respective neighbor on the central vertex. If this is the case, then since we have $|T_1| < 8$ and $|T_2| < 8$, the vertices v and v' cannot be occupied by agents of the same type. Assume by symmetry, without loss of generality, that an agent $j' \in T_{l'}$ occupies vertex v and an agent $i' \in T_l$ occupies vertex v'. Then, we have $u_{j'}(\mathbf{v}) < 1$ and swapping i and j' is profitable, as it holds that $u_i(\mathbf{v}) = 0 < u_i(\mathbf{v}^{i\leftrightarrow j'})$ and $u_{j'}(\mathbf{v}) < 1 = u_{j'}(\mathbf{v}^{i\leftrightarrow j'})$.

Otherwise, there is an agent on a degree-one vertex that has a different type than the agent on the adjacent central vertex v or v'. This implies that there is an agent $j' \neq j$ from $T_{l'}$ with $u_{j'}(\mathbf{v}) < 1$. Then, similarly to the case above, swapping i and j' is profitable. □

On the positive side, we can precisely characterize swap-equilibria in Schelling games on α-star-constellation graphs.

Theorem 4.9 (★). *Let G be an α-star-constellation graph with $\alpha \in \mathbb{N}_0$ and let $\mathbf{v}$ be an assignment in some Schelling game on G. The assignment $\mathbf{v}$ is a swap-equilibrium if and only if at least one of the following two conditions holds.*

1. *Every vertex $v \in V(G)$ with $\deg_G(v) = 1$ is occupied by an agent from the same type as the only adjacent agent in $\mathbf{v}$.*
2. *There exists an agent $i \in T_l$ for some $l \in \{1, 2\}$ such that all other agents $i' \in T_l \setminus \{i\}$ are only adjacent to friends in $\mathbf{v}$.*

Using Theorem 4.9, we now argue that there is a subclass of α-star-constellation graphs, namely α-caterpillars, on which a swap-equilibrium always exists. Consider a Schelling game on an α-caterpillar G with $w_1, \dots, w_\ell$ being the non-degree-one vertices forming the central path ($\{\{w_i, w_{i+1}\} \mid i \in [\ell - 1]\} \subseteq E(G)$). It is easy to construct a swap-equilibrium $\mathbf{v}$ on G by assigning for each $i \in \{1, \dots, \ell\}$ agents from T_1 to w_i and to adjacent degree-one vertices, until all agents from T_1 have been assigned; in which case the remaining vertices are filled with agents from T_2. As $\mathbf{v}$ fulfills Condition 2 from Theorem 4.9, $\mathbf{v}$ is a swap-equilibrium and it is easy to see that $\mathbf{v}$ has edge-robustness $|E(G)|$ and vertex-robustness $|V(G)|$. Notably, this assignment somewhat resembles the swap-equilibrium with edge-robustness $|E(G)|$ and vertex-robustness $|V(G)|$ on a path from Theorem 4.5. In contrast, extending the swap-equilibrium with robustness zero on a path from Theorem 4.5 such that all agents on degree-one vertices are of the same type as their only neighbor, in some Schelling games on α-caterpillars, it is possible to create a swap-equilibrium with edge- and vertex-robustness only α:

Proposition 4.10 (★). *For a Schelling game on an α-caterpillar with $\alpha \in \mathbb{N}_0$, there is a swap-equilibrium with edge-robustness $|E(G)|$ and vertex-robustness $|V(G)|$. For every $\alpha \in \mathbb{N}_0$, there is a Schelling game on an α-caterpillar with a swap-equilibrium with edge-robustness and vertex-robustness α.*

The characterization of swap-equilibria from Theorem 4.9 also yields a polynomial-time algorithm (using dynamic programming for Subset Sum) to decide for a Schelling game on an α-star-constellation graph whether it admits a swap-equilibrium.

Corollary 4.11 (★). *For a Schelling game on an α-star-constellation graph with $\alpha \in \mathbb{N}_0$, one can decide in polynomial time whether a swap-equilibrium exists.*

5 CONCLUSION

We proved that even in the simplest variant of Schelling games where all agents want to maximize the fraction of agents of their type in their occupied neighborhood, deciding the existence of a swap- or jump-equilibrium is NP-complete. Moreover, we introduced a notion for the robustness of an equilibrium under vertex or edge deletions and proved that the robustness of different swap-equilibria on the same topology can vary significantly. In addition, we found that the minimum and the maximum robustness of swap-equilibria vary depending on the underlying topology.

There are multiple possible directions for future research. First, independent of properties of the given graph, in our reduction showing the NP-hardness of deciding the existence of a swap- or jump-equilibrium, we construct a graph that is non-planar and which has a non-constant maximum degree. The same holds for graphs constructed in the reductions from Agarwal et al. [1] for showing NP-hardness in the presence of stubborn agents. Thus, the computational complexity of deciding the existence of equilibria on planar or constant-degree graphs (properties that typically occur in the real world) in Schelling games with our without stubborn agents is open. Second, Bilò et al. [5] recently introduced the notions of local swap (jump)-equilibria where only adjacent agents are allowed to swap places (agents are only allowed to jump to adjacent vertices). To the best of our knowledge, the computational complexity of deciding the existence of a local swap- or jump-equilibrium is unknown even if we allow for stubborn agents. Third, while we showed that on most considered graphs swap-equilibria can be very non-robust, it might be interesting to search for graphs guaranteeing a higher equilibrium robustness; here, graphs with a high minimum degree and/or high connectivity seem to be promising candidates. Fourth, besides looking at the robustness of equilibria with respect to the deletion of edges or vertices, one may also study adding or contracting edges or vertices. Fifth, instead of analyzing the robustness of a specific equilibrium, one could also investigate the robustness of a topology regarding the existence of an equilibrium. Lastly, for an equilibrium, it would also be interesting to analyze empirically or theoretically how many reallocations of agents take place on average after a certain change has been performed until an equilibrium is reached again.

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