# Enabling Imitation-Based Cooperation in Dynamic Social Networks

## JAAMAS Track

Jacques Bara
University of Warwick
Coventry, United Kingdom
jack.bara@warwick.ac.uk

Paolo Turrini
University of Warwick
Coventry, United Kingdom
p.turrini@warwick.ac.uk

Giulia Andrighetto
National Research Council
Rome, Italy
giulia.andrighetto@istc.cnr.it

## ABSTRACT

The emergence of cooperation among self-interested agents has been a key concern of the multi-agent systems community for decades. With the increased importance of network-mediated interaction, researchers have shifted the attention to the impact of social networks and their dynamics on cooperation, drawing various context-dependent and at times conflicting conclusions. In this short paper, summarising the findings in [1], we provide an evolutionary game theory framework to understand coevolutionary processes from a bottom up perspective - in particular the emergence of a cooperator-core and defector-periphery - clarifying the impact of partner selection and imitation strategies in promoting cooperative behaviour, without assuming underlying communication or reputation mechanisms. In doing so we provide a unifying framework to study imitation-based cooperation in dynamic social networks and show that disputes in the literature can in fact coexist.

## KEYWORDS

Social Network Dynamics; Evolutionary Game Theory; Cooperation

## 1 INTRODUCTION

The effects of networks on the emergence of cooperation in games have been disputed over the years. Theoretical results [5] backed by empirical evidence [6] have suggested that static networks have little effect on cooperation or contribution [20]. Other works, however, have found they improve cooperation in experiments [14], simulations [15] and theory [4, 10, 12]. Moreover it seems to be the case that dynamic/temporal networks are far more amenable to cooperation than their static counterparts [9]; a positive effect on cooperation was found experimentally in dynamic [13, 21] and temporal [8] networks with some analytic backing [11, 12, 16].

When edges (partnerships) evolve over time one must seriously consider time scales - characteristic lengths of time over which a particular process occurs. For example when edge activity is "bursty" - i.e. narrow, sudden spikes of activity - cooperation is

impeded while intermediate temporality analytically maximises cooperation [8]. Other studies have found a similar Goldilocks zone - maximal gain for an intermediate value(s) of input(s) - in the time scales [18]. The most common observation, on the other hand, is that of a threshold in time scales [7, 13] - below this threshold cooperation fails and above it flourishes - which are further backed by a slew of theoretic work [11, 12, 16].

In this extended abstract we outline the differential equations governing our Cooperative And Networked DYnamics (CANDY) Framework in Section 2, which is able to reproduce theoretical results for when agents undergo *active linking* [11] with fixed strategies in Section 3, and present simulation results that highlight the importance of timescales between partner-updates and strategy-updates in Section 4, which are finally discussed in Section 5.

## 2 THEORETICAL MODEL

For a simple graph $G = (V, E)$ of $N$ nodes playing a repeated Prisoner's Dilemma game, denote the adjacency matrix as $A = (a_{ij} : i, j \in V)$ and the strategies as a vector $\boldsymbol{s} = (s_i : i \in V)$, where the binary strategy $s$ can either be 1 (cooperate) or 0 (defect). A cooperators pays a cost $c$ per neighbour, such that each of her neighbours gains a benefit $b$. A defector, on the other hand, pays nothing and nothing happens. An agent $i$'s payoff at time $t$ is given by $\pi_i(t)$. This payoff structure was chosen to match the predominant games considered in the literature [13].

In the continuum, we can instead consider $\tilde{a}_{ij} \equiv \mathbb{P}(a_{ij} = 1)$ and $\tilde{s}_i \equiv \mathbb{P}(s_i = 1)$ as the probabilities for edge $(i, j)$ to exist and for node $i$ to cooperate at time $t$, respectively. We then assume such probabilities evolve due to two independent processes - a vector field acting on the adjacency matrix, $f_g$, and one acting on the strategy vector $f_s$. In other words in the joint probability space $[0, 1]^{N^2} \times [0, 1]^N$ where a point represents an entire state, this point moves due to the 'velocities' $f_g$ and $f_s$, each of which have characteristic timescales $\tau_g$ and $\tau_s$.

$$\frac{d\tilde{a}_{ij}}{dt} = f_{ij}^g(A, \boldsymbol{s}, t; \tau_g), \qquad \frac{d\tilde{s}_i}{dt} = f_i^s(A, \boldsymbol{s}, t; \tau_s)$$

## 3 ACTIVE LINKING FOR FIXED STRATEGIES

$$
\begin{aligned}
f_{ij}^g = {} & s_i s_j \left[ \alpha_C^2 (1 - \tilde{a}_{ij}) - \beta_{CC} \tilde{a}_{ij} \right] \\
& + (s_i + s_j - 2 s_i s_j) \left[ \alpha_C \alpha_D (1 - \tilde{a}_{ij}) - \beta_{CD} \tilde{a}_{ij} \right] \\
& + (1 - s_i)(1 - s_j) \left[ \alpha_D^2 (1 - \tilde{a}_{ij}) - \beta_{DD} \tilde{a}_{ij} \right]
\end{aligned}
\tag{1}
$$

We consider the active linking model [11] with fixed strategies ($f_i^s = 0$) to illustrate how our framework can recover their results.
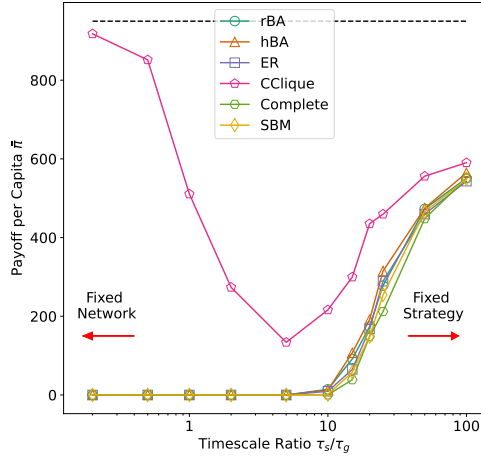
**Figure 1: Final ensemble-averaged payoff-per-capita versus timescale ratio. Colour denote initial condition; the black dashed line is a baseline of a complete graph of cooperators. Note towards the fixed strategy limit the rate of cooperation is bounded-from-above by the initial level of $C = 15$.**

In active linking, cooperators and defectors form edges at some constant rate $\alpha_C$ and $\alpha_B$ and that different edge types decay away at rates $\beta_{CC}, \beta_{CD}$ and $\beta_{DD}$ for $CC, CD$ and $DD$ edges respectively.

By taking the sum of each of the three terms in Eq. 1 we recover exactly the evolutionary equations for the number of CC, CD and DD edges ($X, Y$ and $Z$ resp.) laid out originally in [11] and moreover can find the steady states $X_*, Y_*, Z_*$, in terms of $X_m, Y_m, Z_m$ the maximum edge-set sizes.

$$X_* = \frac{\alpha_C^2 X_m}{\alpha_C^2 + \beta_{CC}}, \quad Y_* = \frac{\alpha_C \alpha_D Y_m}{\alpha_C \alpha_D + \beta_{CD}}, \quad Z_* = \frac{\alpha_D^2 Z_m}{\alpha_D^2 + \beta_{DD}}$$

In a realistic setting cooperators rarely broke from each other [13] so that $\beta_{CC} \approx 0$ while defectors are regularly unfriended by everyone [3] $\beta_{DD} \gg \alpha_D^2$. We see therefore that $X_* \approx X_m$, $Z_* \ll Z_m$ and $Y_* \in [0, Y_m]$. In other words we see a very clear core-periphery emerging, wherein cooperators form a core and defectors are typically isolated in the periphery, regardless of initial condition. This structure has been seen elsewhere in the literature, such as in multidimensional opinion spaces with only edge-breaking [17].

## 4 COEVOLUTIONARY PROCESS

Following evidence cooperators are more popular [2, 13, 21], we take the extreme limit where only cooperators are befriended and defectors are entirely unfriended. That is at each time step (discrete round) a pair of nodes $i, j \neq i \in V$ is randomly chosen. If the alter $j$ is a cooperator, $s_j = 1$, then the ego $i$ will unilaterally form an edge with $j$ if none previously existed. Otherwise if $j$ is a defector, $s_j = 0$, $i$ will unilaterally break ties with $j$ if the edge already existed.

$$f_{ij}^g = \frac{1}{\tau_g}\left[\frac{s_i + s_j}{2} - \tilde{a}_{ij}\right]$$

Here we consider discretely payoff-dependent strategies updated by pure imitation, that is imitation occurs if and only if the proposed alter has a higher payoff. Every $\zeta$ time-steps, $\eta$ existing *edges* are picked randomly. One node per edge is then chosen randomly to be the ego $i$, who will imitate the strategy of their partner $j$ iff $\pi_j > \pi_i$.

$$f_i^s = \frac{1}{\tau_s}(s_k - s_i), \qquad k = \arg\max_{j \in V}(a_{ij}\pi_j) \text{ s.t. } \pi_j > \pi_i$$

We run agent-based simulations to explore the coevolutionary process for $N = 20$ agents, running 100 runs of a Prisoner's Dilemma with $(b, c) = (100, 50)$ to be in line with the experiments of [13]. Each simulation is ran for $5T = 950$ time-steps, where $T$ is the number of dyads $T = N(N-1)/2 = 190$. In this way, as 1 edge is updated at each time-step, then on average in $T$ time-steps then all node pairs (dyads) are updated once.

Moreover we use a variety of initial network configurations (i.e. 100 runs per network type), namely those generated by the following models: Erdös-Rènyi (**ER**) with $p = 0.2$; Barabási-Albert with $m = 3$, wherein cooperators are either assigned randomly (**rBA**) or to nodes with the highest degree (**hBA**); complete graphs (**Complete**); stochastic block model (**SBM**) with 2 communities, an in-group edge probability of $p = 0.8$ and out-group probability $q = 0.2$; cooperator clique (**CClique**) where all cooperators form a complete subgraph and all defectors have 0 degree.

## 5 DISCUSSION

For comparison, in experiments where a fraction $v$ of subject pairs - such as in [21] and [19] - are picked at random to update every round, $\tau_s/\tau_g = v$. In much of the experimental literature [13, 14, 19], there are 3 typical values for $v$: the fixed ($v = 0\%$), viscous ($v = 10\%$) and fluid ($v = 30\%$) conditions. They all found that higher $v$ have higher cooperative levels. We replicate these results across the different initial conditions (see Fig. 1) and show how other unseen phenomena may also occur.

As seen in Fig. 1, there is a separation of time scales in most realistic graphs; after such a long period of time most of these behave near identically with mass defection occurring consistently for $\tau_s/\tau_g < 10$ and non-zero levels of cooperation persisting above this threshold. Individually, this occurs as defection is optimal and preferential. Moreover note that for all realistic graphs when the network is static ($\tau_s/\tau_g = 0$), we reproduce the qualitative result, seen empirically in [5, 6] and theoretically in [20], that static networks do not promote cooperation. The fact that the fraction of cooperators is higher in more dynamic networks has previously been experimentally observed [21].

For the CClique initial graph, however, playing under the same rules a different phenomenon occurs. When strategies update rapidly, the core is large enough to convert defectors quicker than they can infiltrate the core. As the edges update faster, more and more defectors can attach themselves to the core quickly enough to start converting the cooperators. After some point strategies become too slow for everyone to defect hence the payoff-per-capita rises again in the limit of fixed strategy. These two competing factors thus produce a reverse Goldilocks zone, where cooperation is minimised, not maximised, at intermediate ratios.

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