For One and All: Individual and Group Fairness in the Allocation of Indivisible Goods*

Extended Abstract

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ABSTRACT

Fair allocation of indivisible goods is a well-explored problem. Traditionally, research focused on *individual fairness* — are individual agents satisfied with their allotted share? — and *group fairness* — are groups of agents treated fairly? In this paper, we explore the coexistence of individual envy-freeness (i-EF) and its group counterpart, group weighted envy-freeness (g-WEF), in the allocation of indivisible goods. We propose several polynomial-time algorithms that provably achieve i-EF and g-WEF simultaneously in various degrees of approximation under three different conditions: (i) when agents have identical additive valuation functions, i-EFX and g-WEF1 can be achieved simultaneously; (ii) when agents within a group share a common valuation function, an allocation satisfying both i-EF1 and g-WEF1 exists; and (iii) when agents' valuations for goods within a group differ, we show that while maintaining i-EF1, we can achieve a $\frac{1}{3}$-approximation to a notion termed ex-ante g-WEF1. Our results thus provide a first step towards connecting individual and group fairness in the allocation of indivisible goods, in the hopes of its useful application to domains requiring the reconciliation of diversity with individual demands.

KEYWORDS

Fair Allocation; Weighted Envy-Freeness; Social Choice

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1 INTRODUCTION

Fairly allocating indivisible goods is a fundamental problem at the intersection of computer science and economics [6, 13, 32, 36]. For instance, a classic problem in fair allocation involves the allocation of courses to students [15, 16, 27]. Courses have limited capacity, and slots are often allocated via a centralized mechanism. More broadly, analogous problems also surface in numerous areas of social concern, such as the distribution of vaccines to hospitals [35], the allocation of educational resources, and access to infrastructure such as transport, water, and electricity [17, 39].

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Several recent works have explored a variety of *distributive justice criteria*. These broadly fall into two categories – *individual* (e.g., that individual students are not envious of their peers), and *group* (e.g. that students of certain ethnic, gender or professional groups are treated fairly overall). While both individual and group fairness have been studied extensively in recent works, to our knowledge, there have been no works proposing algorithms that ensure both concurrently in the setting of indivisible goods with fixed groups. In this work, we explore

> *efficient algorithms that concurrently ensure approximate individual and group fairness, for certain classes of agent valuation functions.*

The tension between individual and group fairness exists in a variety of allocation scenarios studied in the literature; for example, when allocating public resources (such as housing, slots in public schools, or scheduling problems in general) [1–4, 10, 24, 34, 38] — it is important to maintain fairness towards individual recipients, as well as groups (such as ethnic or socioeconomic groups). Another example is the allocation of reviewers (who, in this metaphor, are the goods) to papers [22, 37], it is important to balance the individual papers' satisfaction with their allotted reviewers, and the overall quality of reviewers assigned to tracks (e.g. ensuring that the overall reviewer quality for the *Game Theory* track is commensurate with that of reviewers for the *Machine Learning* track).

In this paper, we address the question of whether individually envy-free and group weighted envy-free allocations can co-exist when allocating indivisible goods. We present algorithms that compute approximately individually envy-free (EF) and group weighted envy-free (WEF) allocations, where the approximation quality depends on the class of agents' valuation functions.

1.1 Our Contributions

We design algorithms that (approximately) reconcile individual and group envy-freeness in the allocation of indivisible goods. The strength of our results naturally depends on the generality of the valuation classes we consider, with more general valuation classes yielding worse approximation guarantees.

We show that when agents have identical valuation functions, individual envy-freeness up to any good (i-EFX) can be achieved in conjunction with group weighted envy-freeness up to one good (g-WEF1). Additionally, when agents within each group have common valuation functions, then envy-freeness up to one good (i-EF1) can be satisfied together with g-WEF1. Finally, when valuation functions are distinct, we show that together with i-EF1, we can obtain a constant factor $\frac{1}{3}$ approximation to a notion termed g-WEF1 ex-ante (see Definition 3).

1.2 Related Work

Envy-freeness (EF) is a particularly important individual fairness notion when deciding how to fairly allocate indivisible goods [31, 33]. The existence of approximate EF allocations in conjunction with other individual fairness notions and welfare measures (such as proportionality [7], pareto-optimality [18], maximin share [14]) have been studied extensively.

Conitzer et al. [23] and Aziz and Rey [8] introduce the notion of group fairness (applied to every partition of agents within the population), with both offering the "up to one good" relaxation of removing one good per agent. A similar concept was also considered in the economics literature [11]. Benabbou et al. [9] explore the relationship between metrics such as utilitarian social welfare in connection with group-wise fairness via an optimization approach.

Several works also suggest notions of group envy-freeness [5, 25, 28, 38]. We focus on a popular notion called weighted envy-freeness (WEF) [19–21, 30], which focuses on group fairness with fixed groups, allowing us to study guarantees with the removal of a single good per group. This was also raised as an open question in Conitzer et al. [23].

2 PRELIMINARIES

In the problem of allocating indivisible goods, we are given a set of *agents* $N = \{p_1, \ldots, p_n\}$ and *goods* $G = \{g_1, \ldots, g_m\}$. Subsets of goods in G are referred to as *bundles*. Agents belong to predefined *groups* (or *types*) $\mathcal{T} = \{T_1, \ldots, T_\ell\}$. We assume that $\bigcup_{k=1}^{\ell} T_k = N$, and that no two groups intersect. Furthermore, each group T_k has a *weight* w_k, corresponding to its size, i.e. $w_k = |T_k|$. Each agent $p_i \in N$ has a non-negative *valuation function* over bundles of goods: $v_i : 2^G \to \mathbb{R}_+$. We assume that v_i is additive, which is a common assumption in the fair division literature [12, 18, 23], i.e, that $v_i(S) = \sum_{g \in S} v_i(\{g\})$. When all agents have the same valuation, we denote their *common valuation* by v.

In our framework, we consider the direct allocation of goods to agents, whilst taking into consideration agents' group affiliation, and in the process achieving both individual and group envy-freeness. Thus, the group allocation is not explicitly determined in the allocation process, but is induced from the individual allocations $\mathcal{A} = (A_1, ..., A_n)$ instead. We denote $\mathrm{Grp}_k(\mathcal{A}) = \bigcup_{i: p_i \in T_k} A_i$ as the induced *group bundle* for T_k. To keep our notations simple, for any group $T_k \in \mathcal{T}$, we will let $B_k = \mathrm{Grp}_k(\mathcal{A})$ denote this induced group bundle. We also let the group utility for T_k be $v_{T_k}(B_k) = \sum_{i: p_i \in T_k} v_i(A_i)$.

Envy-freeness was introduced by Foley [26] (see also Brandt et al. [13], and Lipton et al. [29]). However, complete, envy-free allocations with indivisible goods cannot always be guaranteed (e.g. with two agents and one good, the agent who did not get the good will always envy the other). Thus, we make use of two popular relaxations of EF as introduced by Lipton et al. [29], Budish [14], and Caragiannis et al. [18].

An allocation $\mathcal{A} = (A_1, \ldots, A_n)$ is individually *envy-free up to any good* (EFX) if, for every pair of agents $p_i, p_{i'} \in N$, and for all goods $g \in A_{i'}$, $v_i(A_i) \geq v_i(A_{i'} \setminus \{g\})$. Similarly, an allocation $\mathcal{A}$ is individually *envy-free up to one good* (EF1) if, for every pair of agents $p_i, p_{i'} \in N$, there is *some* good $g \in A_{i'}$ such that $v_i(A_i) \geq v_i(A_{i'} \setminus \{g\})$.

Chakraborty et al. [19] recently introduced an extension of the EF notion to the weighted setting, known as *weighted envy-freeness* (WEF). In this setting, we treat each "agent" as a group, where each group has a fixed weight (representing its size). We use this notion to capture inter-group envy. Similarly, we consider two relaxed notions of WEF. The definitions below rely on the assumption that the groups' valuations of a bundle are the same regardless of how goods are internally allocated according to $\mathcal{A}$; this is a valid assumption if we assume that valuation functions of agents within a group cannot differ. Definition 3 details an extension of the WEF notion to deal with the more general case. An allocation $\mathcal{A} = (A_1, \ldots, A_n)$ is said to be *weighted envy-free up to one good* (WEF1) if for every two groups $T_k, T_{k'} \in \mathcal{T}$, there exists some good $g \in B_{k'}$ such that $\frac{v_{T_k}(B_k)}{w_k} \geq \frac{v_{T_k}(B_{k'} \setminus \{g\})}{w_{k'}}$. It is weighted envy-free up to *any* good (WEFX) if this inequality holds for *any* $g \in B_{k'}$.

Note that envy-freeness and weighted envy-freeness are referred to as EF and WEF respectively in the literature, but we refer to them as i-EF and g-WEF henceforth, to highlight that the former is an individual fairness concept, and the latter is a group fairness concept. We begin with an example to illustrate these fairness concepts.

3 RESULTS

Below we state several results regarding the existence of approximate individual EF (i-EF) and group WEF (g-WEF) allocations under three classes of valuation functions. We refer the reader to the full version of our paper for more details.

Theorem 1. *Under all-common, additive valuation functions, an allocation satisfying i-EFX and g-WEF1 can be computed in polynomial time.*

Theorem 2. *Under group-common, additive valuation functions, an allocation satisfying i-EF1 and g-WEF1 can be computed in polynomial time.*

As for general additive valuations, we consider approximate g-WEF with respect to the bundle values of agents obtained in a randomized allocation, which we term *ex-ante* g-WEF1. Intuitively, instead of assuming that items are allocated to all agents by some allocation procedure, we consider what the *average* utility would be if we were to allocate each item to a *uniformly random* agent.

Definition 3 (Ex-ante g-WEF1). *An allocation $\mathcal{A} = (A_1, \ldots, A_n)$ is ex-ante weighted envy-free up to one good (ex-ante g-WEF1) if, for every $T_k, T_{k'} \in \mathcal{T}$, there exists some good $g \in B_{k'}$ such that $\frac{v_{T_k}(B_k)}{w_k} \geq \frac{\bar{v}_{T_k}(B_{k'} \setminus \{g\})}{w_{k'}}$, where $\bar{v}_{T_k}(B_{k'}) = \frac{1}{w_k} \sum_{i: p_i \in T_k} \left(\sum_{g' \in B_{k'}} v_i(g') \right)$.*

Further relaxing this notion in a standard manner, we say that an allocation is ex-ante g-WEF1 *up to a factor of* $\frac{1}{\gamma}$ for some constant γ when the condition in Definition 3 is replaced by $\frac{v_{T_k}(B_k)}{w_k} \geq \frac{1}{\gamma} \cdot \frac{\bar{v}_{T_k}(B_{k'} \setminus \{g\})}{w_{k'}}$. Then, we have the following result.

Theorem 4. *Under general, additive valuation, an allocation satisfying i-EF1 and ex-ante g-WEF1 up to a factor of $\frac{1}{3}$ can be computed in polynomial time.*

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