Social Aware Coalition Formation with Bounded Coalition Size

Extended Abstract

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ABSTRACT

In many situations when people are assigned to coalitions the assignment must be social aware, i.e, the utility of each person is the number of friends in her coalition. Additionally, in many situations the size of each coalition should be bounded. This paper initiates the study of such coalition formation scenarios. We show that finding a partition that maximizes the utilitarian social welfare is computationally hard, and provide a polynomial-time approximation algorithm. We also investigate the existence and the complexity of finding stable partitions. Namely, we show that there always exists a Nash Stable (NS) partition and the Contractual Strict Core (CSC) is never empty, but the Strict Core (SC) of some games is empty. Finding partitions that are NS or in the CSC is computationally easy, but finding partitions that are in the SC is hard. The analysis of the core is more involved. When the coalition size is bounded by 3 the core is never empty, and we present a polynomial time algorithm for finding a member of the core. In all other cases, we provide additive and multiplicative approximations of the core. In addition, we show in simulation over 100 million games that a simple heuristic always finds a partition that is in the core.

KEYWORDS

Coalition formation; Additively separable hedonic games; Stability

ACM Reference Format:
Chaya Levinger, Amos Azaria, and Noam Hazon. 2023. Social Aware Coalition Formation with Bounded Coalition Size: Extended Abstract. In *Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), London, United Kingdom, May 29 – June 2, 2023*, IFAAMAS, 3 pages.

1 INTRODUCTION

Suppose that a group of travelers who are located at some origin, would like to reach the same destination, and later return. Each of the travelers has her own vehicle; but each traveler has a preference related to who will be with her in the vehicle. Namely, each traveler would rather share a vehicle with as many of her friends during the ride, and thus the utility of each traveler is the number of friends traveling with her. However, the vehicles have a limited capacity; this capacity can either be a physical constraint of the vehicles, or the maximal number of travelers willing to travel together. How should the travelers be assigned to vehicles in order to maximize the social welfare (the sum of all travelers' utilities)? Can the travelers be organized such that no subgroup of travelers will want to leave their current group and join together? Similar questions raise when assigning students to dormitories, colleagues to office-rooms and workers to project teams.

In this paper, we initiate the study of Additively Separable Hedonic Games (ASHGs) [8] with bounded coalition size.

The contribution of this work is being the first systematic study of additively separable hedonic games with bounded coalition size. Namely, we provide an approximation algorithm for maximizing the utilitarian social welfare and study the computational aspects of several stability concepts.

2 RELATED WORK

Dreze and Greenberg [12] initiated the study of hedonic games, in which the utility for each agent depends only on the coalition that she is a member of. Stability concepts of hedonic games were further analyzed in [5] and [9]. For more details, see the survey of Aziz et al. [2]. A special case is Additively Separable Hedonic Games (ASHGs) [8], in which each agent has a value for any other agent, and the utility she assigns to a coalition is the sum of the values she assigns to its members. The computational aspects of ASHGs are analyzed in [1, 3, 4, 6, 11, 17, 19]. None of these works imposed any restriction on the size of the coalitions.

Indeed, there are few papers that impose a restriction on the size of the coalitions. Wright and Vorobeychik [20] study a model of ASHG where there is an upper bound on the size of each coalition. Flammini et al. [14] study the online partition problem. Cseh et al. [10] require the partition to be composed of exactly k coalitions, and also assume a predefined set of size constraints. Each coalition is required to exactly match its predefined size. Bilò et al. [7] consider the same settings as Cseh et al. They analyze the existence, complexity, and efficiency of stable outcomes, and the complexity of finding a social optimum. Note that almost all other works analyzing ASHGs assume that an agent may assign a negative value to another agent. Otherwise, since they do not impose any restrictions on the coalition size, the game becomes trivial, as the grand coalition is always an optimal solution. One exception is Sless et al. [18], who, similar to our work, assume that the value each agent assigns to another agent is either 0 or 1. However, in their setting the agents must be partitioned into exactly k coalitions, without any restriction on each coalition's size.

3 PRELIMINARIES

Let $V = \{v_1, ..., v_n\}$ be a set of agents, and let $G = (V, E)$ be an undirected graph representing the social relations between the agents. A k-bounded coalition is a coalition of size at most k. A

Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), A. Ricci, W. Yeoh, N. Agmon, B. An (eds.), May 29 – June 2, 2023, London, United Kingdom.

k-bounded partition P is a partition of the agents into disjoint k-bounded coalitions. Given a coalition $S \in P$, and $v \in S$, let $N(v, S)$ be the number of immediate neighbors of $v \in V$ in S, i.e., $N(v, S) = |\{u \in S : (v, u) \in E\}|$. An *additively separable hedonic game with bounded coalition size* is a tuple (G, k), where for every k-bounded partition P, coalition $S \in P$, and $v \in S$, the agent v gets utility $N(v, S)$. We denote the utility of v given a k-bounded partition P, by $u(v, P)$. Given a tuple (G, k), the goal is to find a k-bounded partition P that satisfies efficiency or stability properties.

4 EFFICIENCY

We begin with the elementary concept of efficiency, which is to maximize the utilitarian social welfare.

Definition 4.1 (MaxUtil problem). Given a coalition size limit k and a graph G, find a MaxUtil k-bounded partition.

The MaxUtil problem when $k = 2$ is equivalent to the maximum matching problem, and thus it can be computed in polynomial time [13]. However, our problem becomes intractable when $k \geq 3$.

Theorem 4.2. *The decision variant of the MaxUtil problem is in NP-Complete.*

Since we showed that the MaxUtil problem is in NP-Complete, we provide the Match and Merge (MnM) algorithm (Algorithm 1), which is a polynomial-time approximation algorithm for any $k \geq 3$.

Algorithm 1: Match and Merge (MnM)

Input: A graph $G(V, E)$ and a limit k
Result: A k-bounded partition P of V.
$G_1(V_1, E_1) \leftarrow G(V, E)$
for $l \leftarrow 1$ *to* $k - 1$ **do**
  $M_l \leftarrow$ maximum matching in G_l
  $G_{l+1} = (V_{l+1}, E_{l+1}) \leftarrow$ an empty graph
  $V_{l+1} \leftarrow V_l$
  for every $(v_{i_1,\dots,i_l}, v_j) \in M_l$ **do**
    add vertex $v_{i_1,\dots,i_l,j}$ to V_{l+1}
    remove $v_{i_1,\dots,i_l}, v_j$ from V_{l+1}
  for every $v_{i_1,\dots,i_{l+1}} \in V_{l+1}$ **do**
    for every $v_q \in V_{l+1}$ **do**
      if $(v_{i_1,\dots,i_l}, v_q) \in E_l$ or $(v_{i_{l+1}}, v_q) \in E_l$ **then**
        add $(v_{i_1,\dots,i_{l+1}}, v_q)$ to E_{l+1}
$P \leftarrow$ an empty partition
for every $v_{i_1,\dots,i_j} \in G_k$ **do**
  add the set $\{v_{i_1}, \dots, v_{i_j}\}$ to P
return P

Theorem 4.3. *The MnM algorithm provides a solution for the MaxUtil problem with an approximation ratio of $\frac{1}{k-1}$ for every $k \geq 3$, and this ratio is tight.*

5 STABILITY

When considering a stability concept c, we analyze the following two problems:

- Existence: determine whether for any (G, k) there exists a partition that satisfies c.
- Finding: given (G, k), decide if there exists a partition that satisfies c and if so, find such a partition.

Theorem 5.1. *There always exists a k-bounded Nash stable partition, and it can be found in polynomial time.*

The analysis of the core is more involved. First, we show that for $k = 3$ the core is never empty and can be found in polynomial time. Specifically, we use an algorithm that begins with all agents in singletons and iteratively considers for each 3-bounded coalition whether it strongly blocks the current partition.

Theorem 5.2. *There always exists a 3-bounded partition in the core, and it can be found in polynomial time.*

For $k > 3$ it is unclear whether the core can be empty, and how to find a partition in the core. Therefore, we now investigate additive and multiplicative approximations of the core.

Definition 5.3 (Additive approximation). A k-bounded coalition S is said to *ϵ_a-strongly block* a k-bounded partition P if it improves the utility of each of its members by more than an additive factor of ϵ_a. That is, for every $v \in S$, $N(v, S) > u(v, P) + \epsilon_a$. A k-bounded partition P is in the *ϵ_a-core* if it does not have any ϵ_a-strongly blocking k-bounded coalitions.

The *ϵ_m-core*, which is the multiplicative approximation of the core, is defined similarly. That is, a k-bounded coalition S is said to *ϵ_m-strongly block* a k-bounded partition P if for every $v \in S$, $N(v, S) > \epsilon_m \cdot u(v, P)$.

Theorem 5.4. *For $\epsilon_a = \lfloor \frac{k}{2} \rfloor - 1$, there always exists a k-bounded partition in the ϵ_a-core, and it can be found in polynomial time.*

Theorem 5.5. *For $\epsilon_m = 2$, there always exists a k-bounded partition in the ϵ_m-core, and it can be found in polynomial time.*

Next, we show in simulation that a simple heuristic always finds a partition that is in the core. We test our heuristic function for $k = 5$ over more than 100 million random graphs of different types. Our heuristic always found a k-bounded partition that is in the core.

We now show that for every size limit, k, there is at least one graph where there is no k-bounded partition in the strict core. Furthermore, even verifying the existence of the strict core is a hard problem.

Definition 5.6 (SC existence problem). Given a coalition size limit k and a graph G, decide whether a k-bounded partition exists that is in the strict core.

Theorem 5.7. *The SC existence problem is in NP-hard.*

Finally, we analyze the Contractual Strict Core (CSC) and prove the following theorem.

Theorem 5.8. *There always exists a k-bounded partition in the CSC, and it can be found in polynomial time.*

6 FUTURE WORK

In future work, we intend to analyze social aware coalition formation with bounded coalition size in weighted graphs.

7 ACKNOWLEDGMENTS

This research was supported in part by the Ministry of Science, Technology & Space, Israel. This work is part of the "Human-Centric Ridesharing" project [15]. Some of the results in this paper have appeared in [16].

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