

# Charging Electric Vehicles Fairly and Efficiently

## Extended Abstract

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### ABSTRACT

Motivated by electric vehicle (EV) charging, we formulate the problem of *fair* and *efficient* allocation of a divisible resource among agents that arrive and depart over time and consume the resource at different rates. The agents (EVs) derive utility from the amount of charge gained, which depends on their own charging rate as well as that of the charging outlet. The goal is to allocate charging time at different outlets among the EVs such that the final allocation is *envy-free*, *pareto optimal*, and in certain contexts, *group-strategyproof*. The differences in the charging rates of the outlets and the EVs, and a continuous time-window where the arrivals and departures occur make this a non-trivial combinatorial optimization problem. We show possibilities and impossibilities of achieving a combination of properties such as envy-freeness, pareto optimality, leximin, and group-strategyproofness under different operational settings, e.g., when the EVs have (dis)similar charging technology, or when there are one or more dissimilar charging outlets. We complement the positive existence results with polynomial-time algorithms.

### KEYWORDS

EV Charging; Divisible Resource Allocation; Fair Division

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## 1 INTRODUCTION

Climate change has pushed all nations across the globe to consider low carbon-emitting solutions for their daily routine. In the transportation sector, the *electric vehicles* (EV) have received a significant endorsement by the governments and acceptance from the consumers primarily because of their carbon-friendly behavior and the subsidies provided by the administration in promoting them. This has reflected in the growth of the EV market in various geographies, in particular, in the developing economies [2]. However, the growth of the EV market has also brought in a different challenge which is not very common in the traditional transportation sector. The current battery technology of the EVs typically requires frequent charging (at least once in a day for affordable EVs that

run continuously during the day), but each charge takes a significant amount of time (a 20 kW *fast* charger takes approximately 1.5 hours to charge a 30kWh battery). This time constraint, along with relatively smaller number of charging outlets make the allocation of EVs to the charging outlets to be an incredibly complicated combinatorial optimization problem. Ensuring this in a *fair* and *efficient* manner is an important and timely problem to consider. In this paper, we consider the problem of efficient and fair scheduling of EVs in the available charging outlets. We assume that the prices per unit of electricity is fixed (e.g., by some regulatory authority) and not part of the mechanism. However, the arrival and departure time as well as the demand of electricity are assumed to be privately known to the EVs, and needs to be elicited truthfully.

One relevant category of literature for our work is the allocation of divisible resources that involve monetary transfers. Several works in this strand consider the problem of scheduling using payments as a tool to satisfy several objectives [3, 7, 8, 14]. The minimization of cost in EV charging [10, 11, 15] and the computational complexity of battery charging algorithms with monetary payments [5] have also been addressed. The other category comes from the classical field of scheduling [12]. Porter [13] investigates strategic aspects of maximizing weighted completion in online hard real-time scheduling where tasks have weights, release times, deadlines, and durations. In the context of EV charging, Gerding et al. [6] provide several fairness results.

The problems we address in this paper are different from both these strands since we consider mechanisms without monetary transfers, and focus on group-strategyproofness and efficiency properties in addition to fairness. The paper closest to ours is [6], which considers time to be discrete and charging rates of only EV side of the market. The authors do not consider properties like leximin, pareto optimality, or group-strategyproofness. The properties like **leximin** and GSP have been investigated before but in a different domain. Particularly, Bogomolnaia and Moulin [4] and Kurokawa et al. [9] show that in matching problems and under dichotomous preferences, a **leximin** allocation satisfies PO, EF (classical notion), GSP and proportionality.

## 2 PRELIMINARIES

Consider a set of EVs  $N = \{1, 2, \dots, n\}$  and a set of charging outlets  $M = \{1, 2, \dots, m\}$ . The maximum charging rates of the EVs and the outlets are  $r_i^{EV}, i \in N$  and  $r_k^{Ch}, k \in M$  respectively. The rate at which EV  $i$  will charge when plugged into an outlet  $k$  is given by  $r_{ik} = \min\{r_i^{EV}, r_k^{Ch}\}$ . We consider the EVs as *agents* and an aggregator (manages the charging outlets) in a region as the *planner*. We assume that the planner is non-strategic and its objective is to assign the outlets to the EVs satisfying certain desirable goals. Agent



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Table 1: Summary of results

Properties	Identical cars		Non-identical cars	
	Single	Multiple outlet(s)	Single outlet	Multiple outlets
Envy-freeness+Max-Delivered	✓		✗	✗
Envy-freeness+Pareto-Optimality	✓		✓	?
Envy-freeness+Pareto-Optimality+Group-strategyproofness	✓		?	?
Leximin+Group-strategyproofness	✓		✓	✓
Leximin+Envy-freeness+Group-strategyproofness	✓		✗	✗

$i \in N$  comes with type  $\theta_i$  denoted by the triplet  $(a_i, d_i, c_i)$ , where  $a_i$  and  $d_i$  denote the arrival and departure times of the EV within a given time horizon (e.g., a day) and  $c_i$  is her demand of electricity. Note that  $\theta_i$  is agent  $i$ 's private information and the planner needs to elicit this information. The type profile is denoted by  $\theta$  and the set of all feasible type profiles is denoted by  $\Theta$ . When asked about their types, agent  $i \in N$  reveals  $\hat{\theta}_i = (\hat{a}_i, \hat{d}_i, \hat{c}_i)$ , which may be different from  $\theta_i$ , her true type. Based on the reported types  $\hat{\theta}$ , we divide the time horizon into a set of non-overlapping and exhaustive time intervals that cover the earliest arrival and the latest departure time in the following way. From  $\hat{\theta}$ , the time *checkpoints* are identified where an agent either arrives or departs. Let  $t_{\text{start}} = \min\{\hat{a}_i : i \in N\}$  and  $t_{\text{end}} = \max\{\hat{d}_i : i \in N\}$  be the earliest arrival and the latest departure times respectively. Let the (ascending) sorted order of the time checkpoints except for  $t_{\text{start}}$  and  $t_{\text{end}}$  be denoted by  $t_1, t_2, \dots, t_k(\hat{\theta})$  such that  $\exists i \in N, \exists t_\ell = a_i$  or  $d_i, \forall \ell = \{1, 2, \dots, k(\hat{\theta})\}$ . We denote the collection of intervals  $\{[t_{\text{start}}, t_1], [t_1, t_2], \dots, [t_k(\hat{\theta}), t_{\text{end}}]\}$  by  $I(\hat{\theta})$  where the active agents remain the same in any given interval. We use the index  $j$  to denote an interval in  $I(\hat{\theta})$  and the set of such indices by  $J(\hat{\theta})$ . A member of  $I(\hat{\theta})$  will be denoted as  $I_j$  for  $j \in J(\hat{\theta})$ . When clear from the context, we will use the shorthand  $J$  for  $J(\hat{\theta})$ . Therefore, the indices of the *active intervals* of agent  $i \in N$  are denoted by  $J_i := \{j \in J(\hat{\theta}) : I_j \cap (a_i, d_i) \neq \emptyset\}$ .

An *allocation* is specified by the three-dimensional matrix  $x = [x_{ijk}, i \in N, j \in J, k \in M]$ , where  $x_{ijk}$  denotes the time allocated to agent  $i$  in interval  $j$  at charging outlet  $k$ . An allocation is said to be feasible if all  $x_{ijk}$  are non-negative, agents are allocated at most their demand and only in their active intervals, and the total time allocated to (a) all agents at a given (outlet, interval) is at most the interval's duration (b) each agent across all outlets at a given interval is at most the interval's duration. We denote the allocation to agent  $i$  by  $x_i := (x_{ijk})_{j \in J, k \in M}$ , the complete feasible allocation by  $x = (x_i)_{i \in N}$ , and the set of all feasible allocations by  $X$ .

The utility function of agent  $i \in N$  is given by  $u_i : X \rightarrow \mathbb{R}_{\geq 0}$ . Formally,  $u_i(x) = u_i(x_i) = \min\{c_i, \sum_{j \in J_i} \sum_{k \in M} x_{ijk} r_{ik}\}$ ,  $\forall i \in N$ . More generally, we define the utility of an agent  $i \in N$  for any agent  $h$ 's allocation  $x_h$  as  $u_i(x_h) = \min\{c_i, \sum_{j \in J_i} \sum_{k \in M} x_{hjk} r_{ik}\}$ . Note that this is computed at  $i$ 's charging rate and active interval.

Given the reported type profile  $\hat{\theta}$ , the planner decides the allocation which is given by the function  $f : \Theta \rightarrow X$ . We next formalize the desirable properties of this function.

**Definition 1** (Pareto Optimality (PO)). An allocation  $x \in X$  is *Pareto optimal* if there does not exist  $y \in X$  such that,  $u_i(y) \geq u_i(x), \forall i \in$

$N$  and  $u_{i'}(y) > u_{i'}(x)$ , for some  $i' \in N$ . An allocation function  $f$  is PO if for every  $\theta \in \Theta$  the allocation  $f(\theta)$  is Pareto optimal.

**Definition 2** (Max-Delivered (MD)). An allocation  $x'$  is *Max-Delivered (MD)* if  $x' \in \operatorname{argmax}_{x \in X} \sum_{i \in N} \sum_{j \in J_i} \sum_{k \in M} x_{ijk} r_{ik}$ . In other words, maximum resource is delivered to agents. An allocation function  $f$  is MD if for every  $\theta \in \Theta$ ,  $f(\theta)$  satisfies MD.

**Definition 3** (Manipulability). An allocation function  $f$  is (a) *manipulable* if there exists  $\theta \in \Theta$  and  $i \in N$ , s.t.  $u_i(f(\theta'_i, \theta_{-i})) > u_i(f(\theta_i, \theta_{-i}))$  for some  $\theta'_i$ , and (b) *group manipulable* if there exists  $\theta \in \Theta$  and  $S \in 2^N \setminus \emptyset$ , s.t. for every  $i \in S$ ,  $u_i(f(\theta'_S, \theta_{-S})) > u_i(f(\theta_S, \theta_{-S}))$ , for some  $\theta'_S$ .

We call an allocation function *strategyproof* (SP) if it is not manipulable, and *group strategyproof* (GSP) if it is not group manipulable. Note that, since manipulability implies group manipulability, group strategyproofness implies strategyproofness.

On the fairness front, we want our allocation to be EF among the agents. We say an agent  $i$  envies another agent  $i'$ 's bundle  $(x_{i'})$  if its valuation for  $x_{i'}$  is greater than  $x_i$  when both bundles are evaluated at  $i$ 's charging rate and active interval.

**Definition 4** (Envy-freeness (EF)). An allocation  $x$  is *envy-free* (EF) if  $u_i(x_i) \geq u_i(x_{i'})$  for every  $i, i' \in N$ , where  $x_i$  and  $x_{i'}$  are the allocations of agents  $i$  and  $i'$  respectively. An allocation function  $f$  is EF if  $f(\theta)$  is EF for every  $\theta \in \Theta$ .

**Definition 5** (Leximin). An allocation is **leximin** if it maximizes the minimum utility that any agent receives; and subject to this, maximizes the second least utility, and so on. Formally, let  $u^{(1)}(x), u^{(2)}(x), \dots, u^{(n)}(x)$  denote the non-decreasing order of agent utilities for an allocation  $x \in X$ . Then,  $x$  is **leximin** if it maximizes the above utilities in the lexicographic order. An allocation function  $f$  is **leximin** if  $f(\theta)$  is **leximin** for every  $\theta \in \Theta$ .

### 3 RESULTS AND CONCLUSIONS

Our results are summarized in Table 1 and also the open questions for future investigation. For complete details on the proofs and algorithms refer to full version [1].

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## ETHICS STATEMENT

This paper adheres to the principles of research ethics, research integrity, and social responsibility. The research conducted in this paper is purely theoretical, and no human or animal subjects were involved in the research. Therefore, there were no ethical concerns related to the treatment of participants or the use of personal data. Also, designing fair and efficient protocols in a resource allocation setting, e.g., allocating spectrum, electricity, etc., without money is a standard practice both in computer science and economics. Therefore the results presented in this paper do not consider anything that adversely impacts the society.

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