Strategic Cost Selection in Participatory Budgeting

Extended Abstract

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ABSTRACT

We study strategic behavior of project proposers in the context of participatory budgeting. We assume that the votes are fixed and known and the proposers want to set as high project prices as possible, provided that their projects get selected and the prices are not below the minimum costs of their delivery. We study the existence of Nash equilibria in such games. Furthermore, we report an experimental study of the games we propose.

KEYWORDS

Participatory Budgeting; Equilibria; Cost Selection; Game Theory

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1 INTRODUCTION

Consider a certain city that wants to use participatory budgeting [2, 3, 7] to let its inhabitants decide what improvements to implement. The city council fixed the available budget and asked people to submit their ideas. Specifically, each citizen could submit a project where he or she would outline the type of action to take as well as the cost of carrying it out. The citizens quickly seized the opportunity and came up with a number of proposals. However, they also realized that choosing the costs of the projects is not obvious. For example, to find the cost of building a bike path one might ask a construction company for a quote, but one would get a whole range of costs, depending on the width of the path, the materials used, the possible adaptations of the surrounding area, and so on. Indeed, the more expensive a project is, the better it fulfills its goals, but also the less likely it is to be funded (for example, due to a limited budget). Our approach is to analyze the strategic nature of project cost selection under various participatory budgeting rules.

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The Game. We assume that the sets of projects and of voters, who indicate which projects they approve, are fixed. Each project is controlled by a different proposer choosing its cost so that it is as high as possible while remaining selected. However, each project also has the lowest cost under which it can be reasonably implemented and the proposers prefer costs that are at least as high. Importantly, whether a voter approves a project or not, does not depend on its cost. The projects are chosen according to a given rule. In other words, we consider a game where project proposers (or, for simplicity, the projects) are the players, project costs are their strategies, and costs of selected projects (minus their delivery costs) are their payoffs. We analyze whether these games have pure Nash equilibria and, if so, what costs are reported under these equilibria.

2 PRELIMINARIES

Participatory Budgeting. We define a *PB instance* as a tuple $E = (P, V, B, cost)$, where $P = \{p_1, \dots, p_m\}$ is a set of projects, $V = \{v_1, \dots, v_n\}$ is a set of voters, $B \in \mathbb{R}_+$ is the available *budget*, and $cost\colon P \to \mathbb{R}_+$ is a function specifying the *cost* of each project. Each voter v_i casts a nonempty *approval ballot* $A(v_i) \subseteq P$. Also, $A(p_i)$ is the set of voters that approve p_i. Then, $|A(p_i)|$ is the *approval score* of p_i. We assume that $|A(p_i)| \geq 1$. Given a subset of projects P', we let $cost(P') = \sum_{p' \in P'} cost(p')$. Further, each PB instance comes with an implicit tie-breaking order $>$ over the projects.

Participatory Budgeting Rules. A *PB rule* is a function f that for a PB instance $E = (P, V, B, cost)$ outputs a set $f(E) \subseteq 2^P$ of projects, with total cost not exceeding the budget. We focus on the following:

BasicAV. It starts with $W = \emptyset$ and considers all the projects following their nonincreasing approval scores (with ties broken using $>$), inserting a considered project p into W if $cost(W \cup \{p\}) \leq B$.
AV/Cost. It is like BasicAV, but orders projects by $|A(p)|/cost(p)$.
Phragmén [1, 4]. Phragmén starts with $W = \emptyset$. Initially, the voters have empty accounts and they continuously earn money at the same pace. When there is a project p whose voters have $cost(p)$ funds and is within the remaining budget, p is included in W, the accounts of voters in $A(p)$ are set to zero, and p is removed from consideration. If $cost(W \cup \{p\}) > B$, then p is removed from consideration. At a time, the best such project in $>$ is taken. The rule outputs W when all projects are removed from consideration.
Method of Equal Shares (MES-Cost) [5, 6]. First, each voter receives $B/|V|$ amount of money. Then, we let $W = \emptyset$. Within

Table 1: By "$d \equiv 0$" and "*arb. d*" we mean zero and arbitrary delivery costs, while ■ and ▒ indicate that an NE always exists for, respectively, every or some $\succ$. Symbol □ marks cases admitting a game without NE for all $\succ$ and ◎ means that there is a game and a $\succ$ with no NE. Symbols "?" and † indicate results that are, respectively, conjectured or that hold also if projects' delivery costs are sufficiently low.

ballots	Plurality		Party-List		Unrestricted	
	$d \equiv 0$	arb. d	$d \equiv 0$	arb. d	$d \equiv 0$	arb. d
BasicAV	■	■	■	■	■	■
AV/Cost	■	▒ ◎	■†	▒ ◎	■†	▒ ◎
Phragmén	■†	▒ ◎	■	? ◎	□	□
MES-Cost	■	■	■	■	■	■

each iteration, for each project p not in W we compute its affordability coefficient α_p as the smallest number such that the following holds (b_i is the money that voter v_i currently has): $\sum_{v_i \in A(p)} \min(b_i, \alpha_p \cdot cost(p)) = cost(p)$. If no such value exists, then we set $\alpha_p = \infty$. If $\alpha_p = \infty$ for all projects not in W, then we output W. Otherwise, we choose a project p' with the lowest $\alpha_{p'}$ (highest in $\succ$), include p' in W, and take $\alpha_{p'} \cdot cost(p')$ money from each voter in $A(p')$ (or all remaining funds, if the voter had less than $\alpha_{p'} \cdot cost(p')$).

Special Approval Profiles. We study *plurality* profiles, where each voter approves exactly one project, and *party-list* profiles, where the projects are grouped into "parties" approved by the same voters.

PB Games. A *participatory budgeting cost game* (*PB game*) is a tuple (P, V, B, d), where P is a set of projects, V is a set of voters with approval preferences over P, B is the available budget, and $d\colon P \to \mathbb{R}_+$ assigns projects their minimal *delivery* costs. In this game, the projects report their costs. So, a *strategy profile* is a tuple $\mathbf{c} = (c_1, \dots, c_n)$, with a cost $c_i \in \mathbb{R}_+$ for each project p_i, and $\mathbf{c}(p_i)$ is the cost reported by p_i in $\mathbf{c}$. Let us fix a PB rule f and a PB game. For a profile $\mathbf{c}$, the associated PB instance is $E(\mathbf{c}) = (P, V, B, \mathbf{c})$ and the *payoff* of each project $p_i \in P$, denoted by $u_i(\mathbf{c})$, is $\mathbf{c}(p_i) - d(p_i)$ if $p_i \in f\big(E(\mathbf{c})\big)$, and 0 otherwise. Given a rule f and a PB game, we are interested in whether it has pure Nash equilibria (NE).

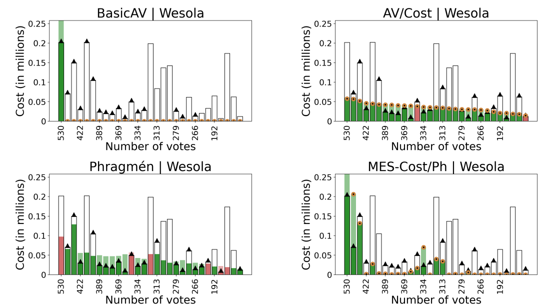
3 RESULTS

We provide a theoretical and an experimental study of our games.

Theoretical Results. We investigated, for a given voting rule, if an NE exists for every PB game. While BasicAV and, surprisingly, MES-Cost always admit an equilibrium, it is not true for the other rules we consider. Our results are dependent on the delivery costs and on specific tie-breaking orders. As such, for AV/Cost or Phragmén with plurality ballots, an NE is guaranteed for sufficiently low delivery costs, but for higher ones, it ceases not to exist for some tie-breaking orders. In fact, for Phragmén, an NE might not exist for arbitrary $\succ$. Our theoretical results are shown in Table 1.

Experimental Results. Our goal was to compute (approximate) equilibria under our rules. While for BasicAV or AV/Cost we could compute the equilibria using our theoretical results, this would not be possible for Phragmén. Hence, we simulate the following

Figure 1: Strategy profiles after 10 000 iterations of our dynamics. Bars represent the projects (in the order of their approval score, depicted on the x axis). The green bars show the final costs of the winning projects (the brighter part emphasizes the increase, as compared to the original cost), while the red shows the final costs. Black outlines denote the original costs. Triangles mark the original winners. Brown circles denote the equilibrium costs (if we can compute it).



dynamics (where we could compute an NE directly our simulations gave nearly identical results, so we expect that the results are also meaningful for the other rules). First, proposers report the cost that was originally chosen for their project. Then, in each iteration, one of them, selected uniformly at random, increases or decreases their project's cost. Specifically, the proposer chooses a number x in $[0, cost/10]$ uniformly at random (where $cost$ is the current project's cost) and changes their project's cost by x if it increases their utility. We expect to converge to an NE if one exists.

In Figure 1 we show the results of the dynamics after 10 000 iterations. Under BasicAV, as expected, all the budget goes to the most popular project. Under AV/Cost, every project submits a cost proportional to its support. For BasicAV, AV/Cost, and MES-Cost/Ph most of the projects "reached" the costs predicted by the NE.

Our conclusion from this experiment is that under AV/Cost or Phragmén the proposers are incentivized to use costs that reflect their approval score. Under MES-Cost/Ph, most strongly supported projects and those that are supported by many voters disapproving more popular projects can request higher costs. So, in an NE, MES-Cost/Ph funds fewer projects than AV/Cost or Phragmén.

4 CONCLUSION

We have introduced a game-theoretic model capturing strategic cost selection in PB. In future work, one could, e.g., consider dropping the assumption that project proposers have full knowledge of ballots, or study the complexity of checking the existence of an NE.

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REFERENCES

[1] M. Brill, R. Freeman, S. Janson, and M. Lackner. 2017. Phragmén's Voting Methods and Justified Representation. In *Proceedings of AAAI-2017*. 406–413.

[2] Y. Cabannes. 2004. Participatory Budgeting: A Significant Contribution to Participatory Democracy. *Environment and Urbanization* 16, 1 (2004), 27–46.

[3] A. Goel, A. Krishnaswamy, S. Sakshuwong, and T. Aitamurto. 2019. Knapsack Voting for Participatory Budgeting. *ACM Transactions on Economics and Computation* 7, 2 (2019), 1–27.

[4] M. Los, Z. Christoff, and D. Grossi. 2022. Proportional Budget Allocations: Towards a Systematization. In *Proceedings of IJCAI-2022*. 398–404.

[5] D. Peters, G. Pierczynski, and P. Skowron. 2021. Proportional Participatory Budgeting with Additive Utilities. In *Proceedings of NeurIPS-2021*. 12726–12737.

[6] D. Peters and P. Skowron. 2020. Proportionality and the Limits of Welfarism. In *Proceedings of EC-2020*. 793–794.

[7] S. Rey and J. Maly. 2023. *The (Computational) Social Choice Take on Indivisible Participatory Budgeting*. Technical Report arXiv.2303.00621. arXiv.