Fair Scheduling of Indivisible Chores

Extended Abstract

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ABSTRACT

We study the problem of fairly assigning a set of discrete tasks (or chores) among a set of agents with additive valuations. Each chore is associated with an arrival time and a deadline, and each agent can perform at most one chore at any given time. The goal is to find a fair and efficient schedule of the chores, where fairness pertains to satisfying envy-freeness up to one chore (EF1) and efficiency pertains to maximality (i.e., no unallocated chore can be feasibly assigned to any agent). Our main result is a polynomial-time algorithm for computing an EF1 and maximal schedule for two agents under monotone valuations when the conflict constraints constitute an arbitrary interval graph. The algorithm uses a coloring technique in interval graphs that may be of independent interest. For an arbitrary number of agents, we provide an algorithm for finding a fair schedule under identical dichotomous valuations when the constraints constitute a path graph. We also rule out the existence of schedules satisfying stronger fairness and efficiency properties, including envy-freeness up to any chore (EFX) together with maximality and EF1 together with Pareto optimality.

KEYWORDS

Fair Allocation; Chores; Scheduling; EF1; Interval Graphs

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1 INTRODUCTION

Fair allocation of indivisible resources has become a significant area of study within economics, operations research, and computer science [5, 6, 16]. The main objective is to distribute a set of discrete resources among agents with differing preferences such that the outcome satisfies rigorous guarantees of fairness and economic efficiency. This field has generated extensive theoretical and practical interest in recent years [1, 3, 8, 13].

Many real-world applications call for fair division of resources that are *undesirable*, also known as *chores* [12]. Common examples of such situations include the division of household tasks like cooking or cleaning [14], as well as the distribution of responsibilities for tackling global issues like climate change among countries [17].

The problem of fair division of indivisible chores involves a set of discrete resources for which agents have non-positive values. The goal is to assign each chore to exactly one agent such that the final allocation is fair. A well-studied notion of fairness is *envy-freeness* [10, 11] which requires that each agent weakly prefers its bundle over any other agent's. However, due to the discrete nature of the tasks, an envy-free allocation may not always exist. This has led to the study of approximations such as *envy-freeness up to one chore* (EF1) which bounds the pairwise envy by the removal of some chore in the envious agent's bundle [2, 7]. Unlike exact envy-freeness, an EF1 allocation of chores is guaranteed to exist even under general monotone valuations [4, 15].

A common assumption in the fair division literature is that any item can be feasibly assigned to any agent. This assumption may not hold in many settings of interest. For example, in course allocation, a student can only attend at most one course at any given time. Similarly, in assigning volunteers to conference sessions, temporal overlaps may need to be taken into account. In such settings, it is more natural to model *conflicts* among the items and allow only feasible (or non-conflicting) allocations.

We formalize the problem of fair and efficient scheduling of indivisible chores under conflict constraints. Each chore is associated with a start time and a finish time. Indivisibility dictates that a chore can be assigned to at most one agent. An agent can perform at most one chore at a time; furthermore, a chore once started must be performed until its completion. By modeling the chores as vertices of a graph and capturing temporal conflicts with edges, we obtain the problem of dividing the vertices of an *interval* graph among agents such that each agent gets an independent subset. Note that due to conflicts, it may not be possible to allocate all chores. Thus, we ask for schedules to be *maximal*, i.e., it should not be possible to assign any agent an unallocated chore without creating a conflict.

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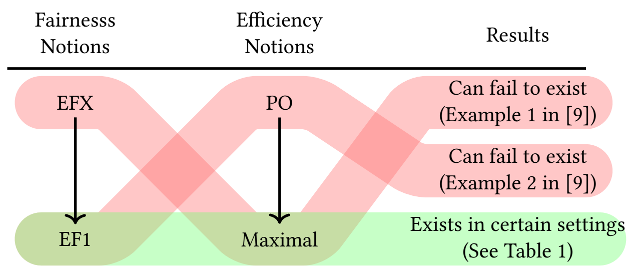


Figure 1: Summary of our results. The arrows denote logical implications between fairness and efficiency notions. The positive and negative results are shown in green and red, respectively.

2 OUR CONTRIBUTIONS

We formulate the problem of fair and efficient scheduling of indivisible chores under conflict constraints and make the following contributions:

Non-existence results. We show that the strongest approximation notion—envy-freeness up to *any* chore (EFX)—may not be compatible with maximality. By weakening the fairness requirement to envy-freeness up to one chore (EF1) but strengthening the efficiency requirement to Pareto optimality, we again obtain a non-existence result (see Figure 1).

Algorithms for two agents. We present a polynomial-time algorithm for finding an EF1 and maximal schedule for two agents under general *monotone* valuations and for *any* interval graph. Our analysis develops a novel notion of *adjacent* schedules and uses a *coloring* technique that may be of independent interest.

Algorithms for an arbitrary number of agents. We consider the case of an arbitrary number of agents. We show that under *restricted* valuations (specifically, identical dichotomous valuations), an EF1 and maximal schedule always exists for a path graph. Furthermore, for identical valuations (not necessarily dichotomous), we show that EF1 and maximality can be simultaneously achieved for a more general class of graphs, namely, any graph in which each connected component is of size at most n, where n is the number of agents.

The detailed statements of the results can be found in the full version of the paper [9].

3 AN ILLUSTRATIVE EXAMPLE

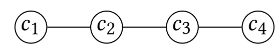
In this section, we will illustrate our framework by means of a simple example. The example will show that a schedule that satisfies *envy-freeness up to any chore* (EFX)[1] and maximality can fail to exist. Interestingly, this counterexample (as well as others in the full version of the paper [9]

Example 3.1 (EFX and maximal schedule may not exist). Consider an instance with two agents a_1 and a_2 and four chores c_1, c_2, c_3, and c_4 that are identically valued by the agents at -1, -1, -1, and -4, respectively. The conflict graph is as shown below:

[1]Under EFX, any pairwise envy can be eliminated by removing any chore from the envious agent's bundle.

No. of agents	Path graph	Interval graph
$n = 2$	✓ for monotone vals (Theorem 1 in [9])	✓ for monotone vals (Theorem 3 in [9])
arbitrary n	✓ for identical dichotomous vals (Theorem 4 in [9])	✓ for identical vals and bounded components (Theorem 5 in [9])

Table 1: Summary of our results for EF1 and maximality. In each cell, a ✓ denotes that an EF1 and maximal schedule always exists and is computable in polynomial time under the assumptions on the number of agents (rows) and the conflict graph (column).



Let **X** be the desired EFX and maximal schedule. Observe that due to maximality, the chore c_4 cannot remain unallocated under **X**. This is because if the neighboring chore c_3 is assigned to one of the agents, say a_1, then the chore c_4 must be assigned to the other agent a_2. Similarly, if c_3 is unassigned, then c_4 can be assigned to either of the agents without creating any conflict. Thus, we can assume, without loss of generality, that c_4 is assigned to agent a_1.

In order for the schedule **X** to satisfy EFX, agent a_1 cannot be assigned any other chore. Furthermore, feasibility dictates that the other agent a_2 can be given at most two of the three remaining chores c_1, c_2, and c_3. If agent a_2 gets exactly two chores, then it must be given c_1 and c_3; however, then c_2 must be assigned to agent a_1, violating EFX. On the other hand, if agent a_2 gets at most one chore, then once again by maximality of **X**, it will be required to get at least one chore out of c_1, c_2, and c_3, again violating EFX. Thus, an EFX and maximal schedule does not exist in the above instance. □

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